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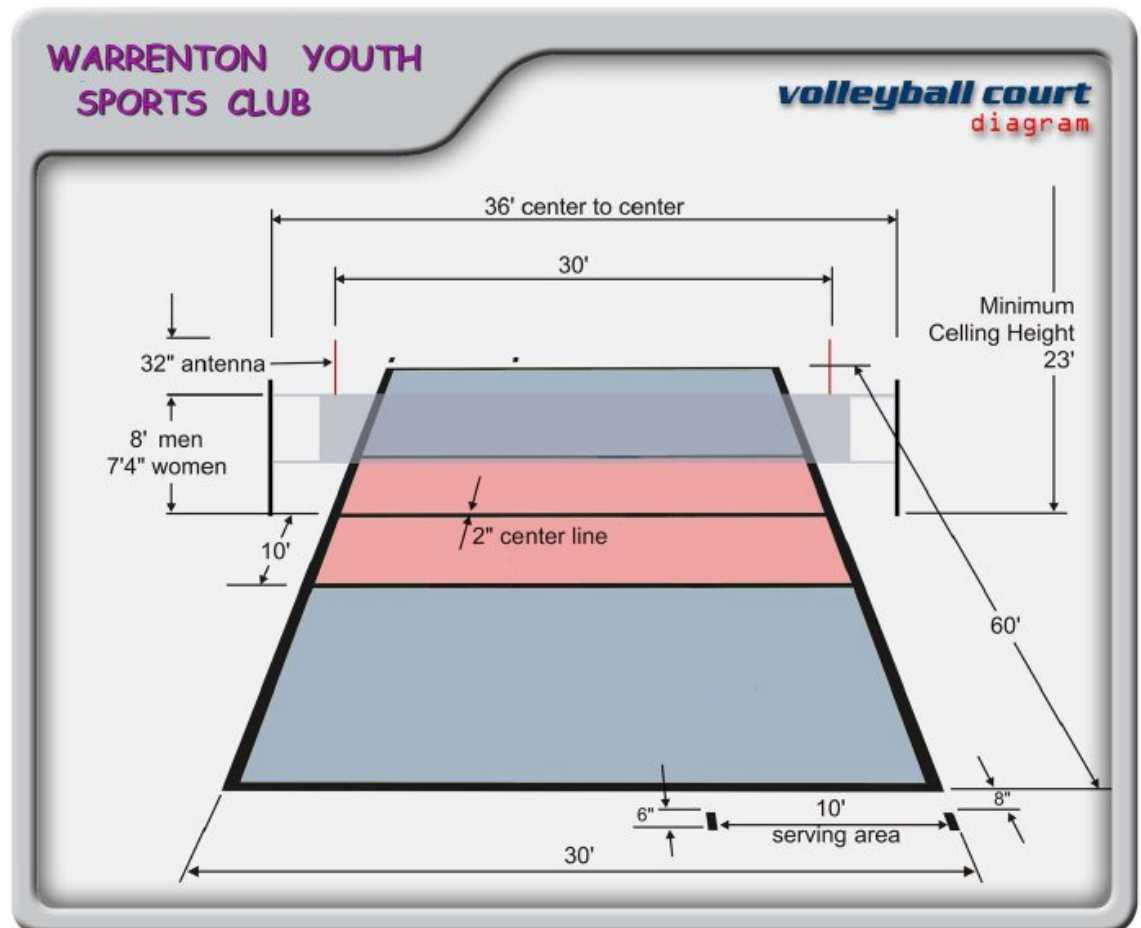
Master Project in Mathematics

MATHEMATICAL SWERVE ANALYSIS OF AN IN-FLIGHT VOLLEYBALL THROUGH THE AIR

Research Report in Mathematics, Number 14, 2017

MURIUKI K PATRICK
I56/82710/2015

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MURIUKI K PATRICK

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School of Mathematics
College of Biological and Physical sciences
Chiromo, off Riverside Drive
30197-00100 Nairobi, Kenya

Master of Science Project

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Monitored by: The Director
School of Mathematics
University of Nairobi

Supervised by: Prof.C.B. Singh
School of Mathematics

Abstract

In this project , we will study how a volleyball is swerved(laterally displaced from the main path) as it travels though the air as a result of induced side-spin. Our study will assume the ball is played in a controlled room to avoid the effect of wind to the swerving of the ball.

Declaration and Approval

I the undersigned declare that this project report is my original work and to the best of my knowledge, it has not been submitted in support of an award of a degree in any other university or institution of learning.

Signature

Date

MURIUKI K PATRICK

Reg No. I56/82710/2015

In my capacity as a supervisor of the candidate, I certify that this report has my approval for submission.

Signature

Date

Prof. C.B Singh
School of Mathematics,
University of Nairobi,
Box 30197, 00100 Nairobi, Kenya.
E-mail: singh@uonbi.ac.ke

Dedication

I dedicate this work to my parents Luka Kamuri and Lucy Kamuri together my brother Denis Karimi.

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Secondly I want to thank Prof. C.B Singh for his unending assistance in helping me to put my ideas together and guide me through this project.

I also want to thank my parents for their financial and moral support they have given me throughout my project work.

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MURIUKI K PATRICK

Nairobi, 2017.

1 Introduction

For those who study mathematics of sports, aerodynamic properties of various sport object is of interest. Some of these properties are roughness of balls' surfaces and introduction of dimples in golf balls. The research work on the physics of balls has been carried out over the decades and several wind-tunnel experiments and computer models have lead to even a better understanding of these study topics.

The phenomena revolving around volleyball in the air cannot be fully discussed without considering a non-dimensionless Reynolds number. So far there is no specific letter used universally to denote Reynolds number and hence in my case I am going to use Re to denote it. Reynolds number Re , is defined as

$$Re = \frac{vD}{\nu} \quad (1)$$

Where v is the air speed far from the ball in the balls rest frame (speed of the ball in the stationary air's frame). D represents the diameter of the ball, whereas ν denotes the Kinematic viscosity, defined as the ration of viscosity μ to the density of air ρ

Reynold numbers involved for a volleyball game is approximately 400,000.

It is interesting to note that as the Re increases through a critical value, air flow on the ball's boundary layer changes from laminar flow to turbulent one. This in turn drops the drag coefficient since the boundary layer separates further back on the ball.

Introduction of dimples in golf balls and Panel connections on base balls and volleyball induce turbulence at Reynolds numbers lower than that of a smooth ball.

Now considering the trajectory of any ball in the air we will note that the paths followed by a spinning ball and another one by a non-spinning one are different. There are three ways in which a ball can be made to spin; topspin, down-spin and side-spin. When a player imparts a spin on the ball it tends to arc more than it would if it were not rotating. This can clearly be seen in a video where Roberto Carlos' free kick curved more than expected earning him goal against France in 1997.

[<https://www.youtube.com/watch?v=2zcXJBINuTU>].

This phenomenon has motivated me to further investigate the effect of inducing spin on a volleyball. I will also discuss the forces associated with the spinning volley ball.

Its also important to note that the forces associated with the spinning ball are parametrized by the Reynolds number and by the dimensionless spin Parameter, S_p . S_p is defined as the ratio of the tangential and center-of-mass speeds of the rotating ball at the equator with respect to the air.

If we consider a ball of radius r , angular speed ω , and center-of-mass speed v , S_p is given by

$$S_p = \frac{r\omega}{v}$$

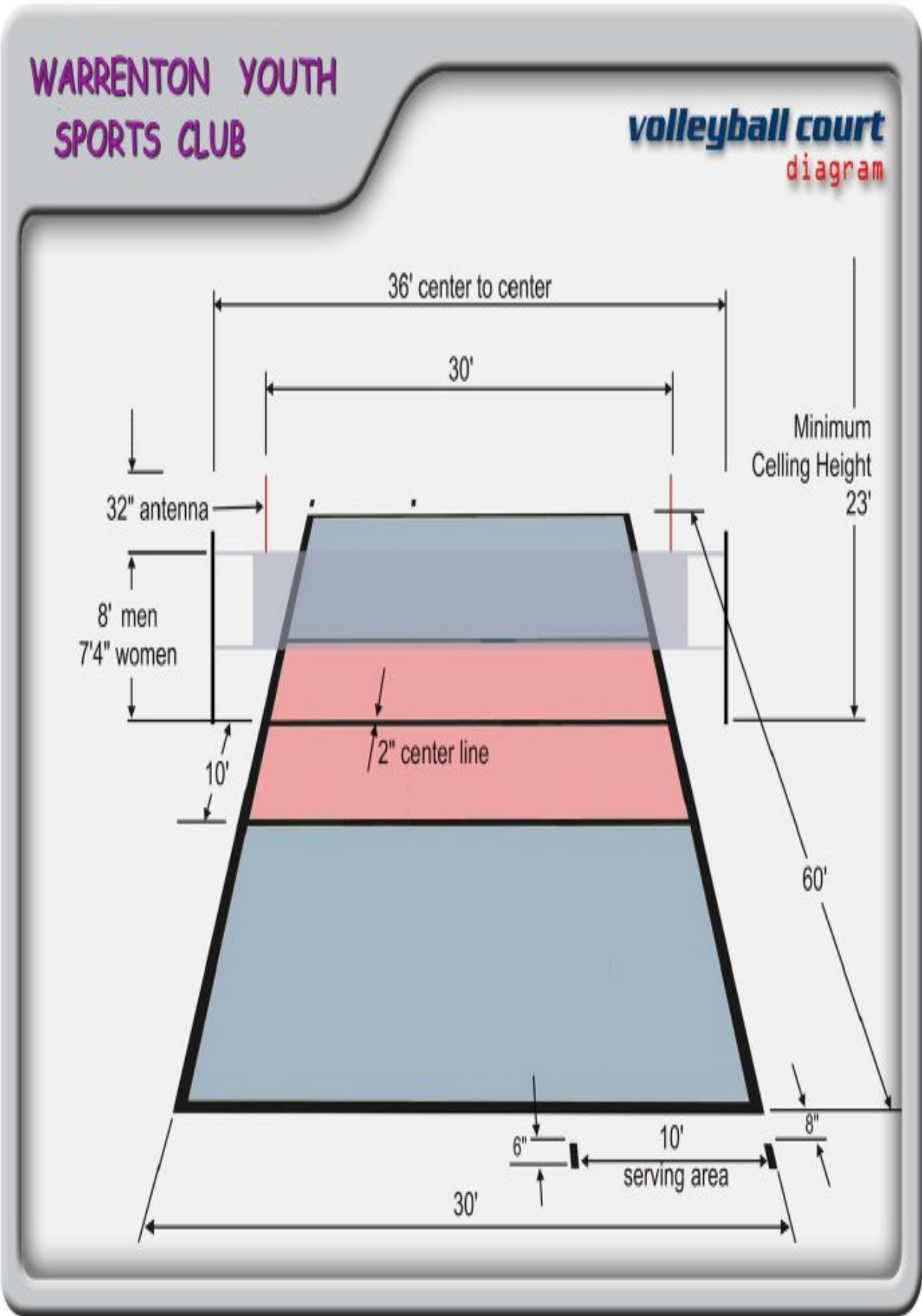


Figure 1. A standard volleyball pitch

2 FORCES INVOLVED ON A BALL IN FLIGHT

In order to quantify the trajectory of the volleyball, we will derive the equation of motion from the first principle namely Newton 2nd law of motion. And to fully understand these forces, consider the following figure

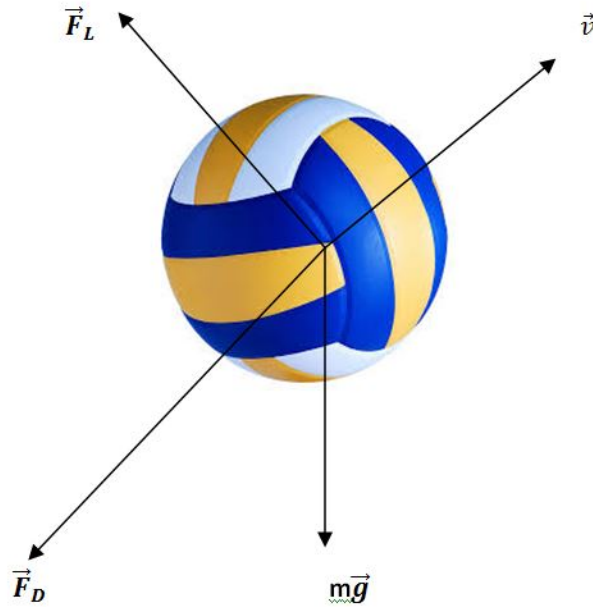


Figure 2. $m\vec{g}$ directed downwards is the ball's weight, \vec{F}_D directed opposite to the motion of the ball is the drag force, \vec{F}_L acting perpendicular to the direction of motion and in the plane formed by the velocity and the ball's weight is the lift force. Finally is the sideways force (not shown) directed into the page.

Here we will assume the ball's trajectory is close to the earth's surface so that the gravitational force on the ball $m\vec{g}$ is constant. the mass of the ball is $m = 0.268\text{kg}$

Forces acting on the volleyball of mass m , and with acceleration \vec{a} is given by

$$F = m\vec{a} = m\frac{dv}{dt} \quad (2)$$

These forces are: earth's gravitational force $m\vec{g}$, drag force F_{drag} and magnus force F_{magnus} .

Substituting these forces into 2nd Newton law of motion we get;

$$F = m\vec{g} + F_{\text{drag}} + F_{\text{magnus}} \quad (3)$$

The magnus effect results from the asymmetric flow of air around a spinning ball. This effect displaces the ball in the direction perpendicular to the spin axis. This is to mean that, if the spin axis is horizontal the magnus effect provides back lift and if the spin is vertical, the magnus effect causes the ball's trajectory to bend sideways.

We may resolve this force into two components namely the lift and lateral motion.

If the motion of the ball is to the direction defined by the unit vector \hat{v} then we can define another vector \hat{l} perpendicular \hat{v} and in the same plane as \hat{v} . Let \hat{g} be the acceleration due to gravity and z be a positive component of \hat{l} .

We can then define a right-handed system through the 3rd vector $\hat{l} \times \hat{v}$ as shown in figure (3)

Now after splitting the magnus force into lift and spin forces our equation looks like this

$$F = m\vec{g} + F_{\text{drag}} + F_{\text{lift}} + F_{\text{spin}} \quad (4)$$

For convenience we are going to abbreviate

$$F_{drag} \quad \text{as} \quad F_D$$

$$F_{lift} \quad \text{as} \quad F_L$$

and

$$F_{spin} \quad \text{as} \quad F_S$$

where the drag force, F_d is directed opposite to the vector \hat{v} direction, F_L is the lift force and is directed in the same direction as \hat{l} whereas the lateral force F_s has a sense similar to that of the vector $\hat{l} \times \hat{v}$.

2.1 THE DRAG FORCE

This force is directed opposite to the motion of the ball or simply opposite to the velocity \vec{v} and maybe written as

$$\vec{F}_d = -\frac{1}{2}\rho Av^2 C_D \hat{v} \quad (5)$$

where $\rho = 1.23\text{kg}/\text{m}^3$ is the density of air.

$A = 0.36\text{m}^2$ if the cross-sectional area of the volleyball.

$v = |\vec{v}|$ is the ball's speed.

C_D denotes dimensionless drag coefficient and

$$\hat{v} = \frac{\vec{v}}{v}$$

2.1.1 THE LIFT FORCE

The lift force which points perpendicular to the drag force and remains in the plane formed by \hat{v} and the ball's weight is given by

$$\vec{F}_L = -\frac{1}{2}\rho Av^2 C_L \hat{l} \quad (6)$$

where C_L is the dimensionless lift coefficient and \hat{l} is a unit vector normal to \hat{v} and lies on the plane formed by \hat{v} and the balls weight.

2.1.2 SIDEWAYS FORCE

This force is given by

$$\vec{F}_S = \frac{1}{2} \rho A v^2 C_S (\hat{l} \times \hat{v}) \quad (7)$$

where C_S is the dimensionless sideways coefficient.

It is also important to note that Lift force F_L and Spin force F_S occur as a result of the same phenomena and hence, finding lift coefficient C_L as a function of Reynolds number and Spin parameter means also knowing spin coefficient C_S as a function of Reynolds number and spin parameter.

For pure topspin or back-spin, spin coefficient,
 $C_S = 0$

For pure side-spin, lift coefficient $C_L = 0$

It is also important to note that we can have a situation where the ball is made to move in the air without any kind of rotation. This phenomenon is known as "knuckle-ball" effect and when we have such,

$$C_L = C_S = 0$$

We will also ignore the force exerted by the air particles on our ball because its contribution to the air force from buoyancy is small.

2.2 ASSUMPTIONS

- We assume the ball to be primarily played in the direction of the y -axis. Due to this reason, the velocity in the y -direction must be relatively higher than the velocities in the x - and z - directions. Thus the ball travels furthest in the y -direction.
- We also assume that the force applied on the ball is such that it causes a side-spin meaning that the the motion of the ball is dominated by sideways swerve.
- Serving the ball is done in a way such that the vertical velocity will be greater than zero. i.e $w \gg 0$. This means the ball will acquire some vertical length before it start falling to the ground.

3 DRAG COEFFICIENT OF A VOLLEYBALL

In our case we will take drag coefficient as a constant. However Beatrice Hahn and David Mc Culloch[1999], two students at the University of Michigan did a wind-tunnel experimental study to determine the drag on non-spinning volleyball as a project in Aeronautical engineering under the direction of Dr. Don Geister[2001]. Their results were excellent set of data that were included along with other sport balls in a paper by Dr. Rabi Mehta of NASA Ames and Dr. Jani Pallis of Cislinar. Their result is shown below.

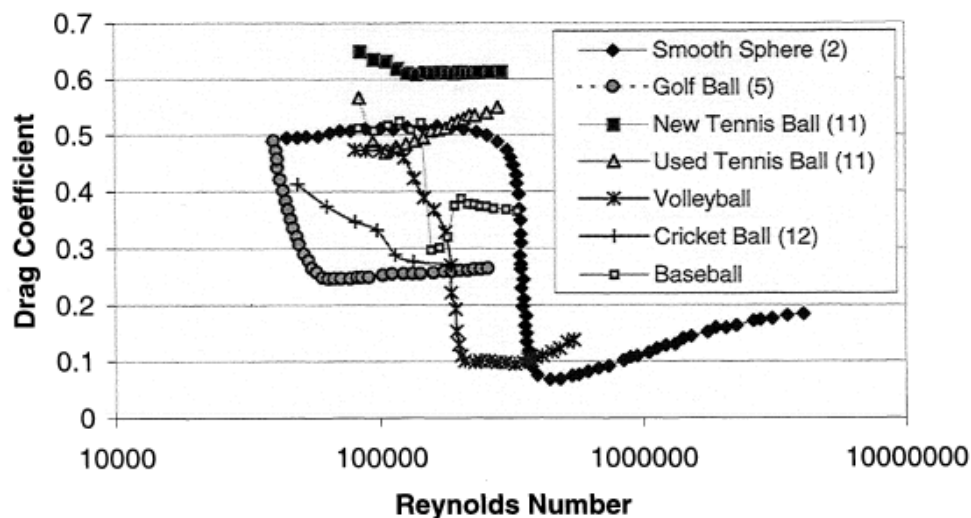


Figure 3. Drag coefficients versus Reynolds number for different sport balls. Volleyball data courtesy of Don Geister [2001], Aerospace Department, University of Michigan.

It is also important to note that in the course of the ball's flight, the angular velocity keeps on vanishing at slow rate so that we can comfortably consider it to be a constant.

However, we realize that $||v||$, decreases with time due to the force of drag so that the spin rate

$$S_p = \frac{R\omega}{|v|} \text{ increases.}$$

Both Re and C_D are dimensionless quantities and Re is very important in fluid dynamics in that it shows us the relationship between physical situations that appear to be different.

The drag coefficient versus Reynold number for volleyball is displayed below as Thomas W. Cairns [2004], a Ph.D student at the University of Tulsa obtained it

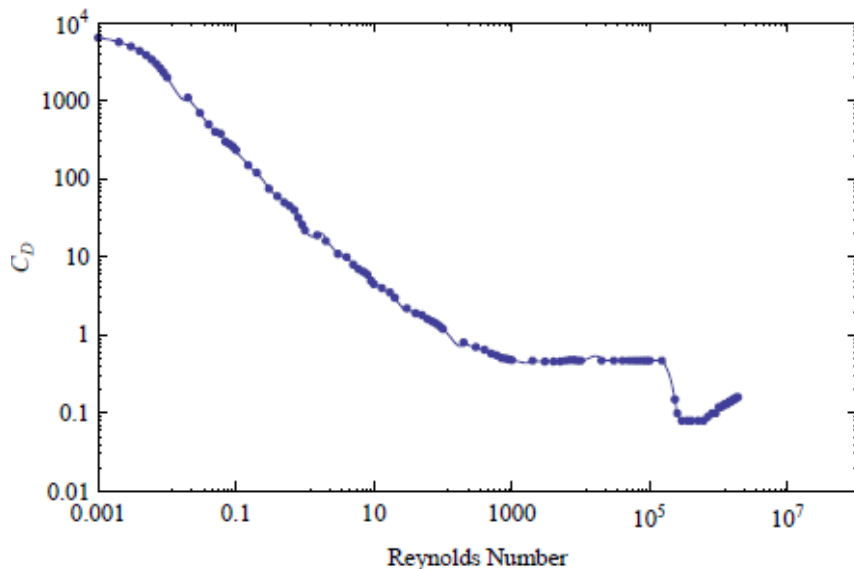


Figure 4. Drag coefficients of volleyballs

Here we will take Reynolds number to be the scalar multiple of the ball's speed given by

$$Re = \frac{\rho L}{\mu} v$$

where

ρ – mass density of air = $1.23\text{kg}/\text{m}^3$

L – diameter = 0.214m

μ dynamic viscosity of air = $1.79 \times 10^{-5}\text{Ns}/\text{m}^2$

v ball's speed in m/s

Thus we will have $Re = 1470v$

The phenomenon at $Re = 100,000$ where C_D drops an order of magnitude is called **Drag crisis**. This occurs when a thin boundary layer of air and next to the ball switches from laminar to turbulent. Different balls experience drag crisis at different Reynolds numbers because of the roughness of their surfaces.

The region of rapidly falling C_D is called the **Critical region**. The significance of the C_D values for a volleyball is that much of the game is played in the critical region. A spike at 30m/s ($Re = 400,000$) is a very hard hit while serves at 25m/s are hard and at 20m/s are common. From Hahn Mc Culloch [1999] data, we can say that there is no volleyball ever hit faster than 34m/s which is $Re = 500,000$. Below is a table showing four data points in the volleyball critical regions. v denotes velocity in m/s.

v	C_D
10.2	0.47
15.7	0.15
17.0	0.10
19.7	0.08

4 DERIVATION OF THE GOVERNING EQUATION

From our previous computations of the forces affecting the ball, we were able to calculate drag force (5), lift force (6) and sideways force (7) and now we will put them in Newton's second law equation to obtain the total force acting on the ball.

$$ma = -\frac{1}{2}\rho Av^2 C_D \hat{v} - \frac{1}{2}\rho Av^2 C_L \hat{l} + \frac{1}{2}\rho Av^2 C_S (\hat{l} \times \hat{v})$$

From here we will divide all through by m and then find accelerations in x , y and z directions.

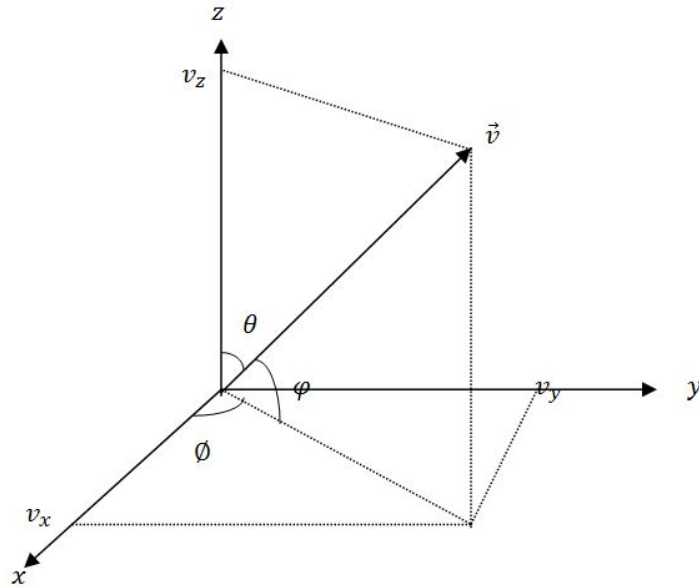


Figure 5. The polar angle θ and the azimuthal angle is ϕ of the velocity vector. The angle measured from the horizontal is $\phi = \frac{\pi}{2-\theta}$.

With the aid of fig 5, we will easily calculate unit vectors as follows. The unit vector along \hat{v} may be written as

$$\hat{v} = \sin \theta \cos \phi \hat{i} + \sin \theta \sin \phi \hat{j} + \cos \theta \hat{k} \quad (8)$$

The unit vector \hat{l} is found by taking \hat{v} and rotating the angle θ back by $\frac{\pi}{2}$ and keeping ϕ the same.

$$\text{Since } \sin(\theta - \frac{\pi}{2}) = -\cos \phi$$

and

$$\cos(\theta - \frac{\pi}{2}) = \sin \phi$$

then the unit vector \hat{l} is given as

$$\hat{l} = -\cos \theta \cos \phi i - \cos \theta \sin \phi j + \sin \theta k \quad (9)$$

The third vector we need is $\hat{l} \times \hat{v}$ from (8) and (9) we obtain

$$\hat{l} \times \hat{v} = -\sin \phi i + \cos \phi j \quad (10)$$

These angles can be expressed in terms of Cartesian velocity components as

$$\sin \theta = \frac{|v_p|}{|v|}, \quad \cos \theta = \frac{\dot{z}}{|v|}, \quad \cos \phi = \frac{\dot{x}}{|v_p|}, \quad \sin \phi = \frac{\dot{y}}{|v_p|} \quad (11)$$

where $v_p = (\dot{x}, \dot{y})$ denotes projection of v on the $x - y$ plane. (see fig 5)

We then write the motion equation in form of components as:

$$\ddot{x} = -k_d |v| \dot{x} + k_l \frac{|v|}{|v_p|} \dot{x} \dot{z} + k_s \frac{|v|^2}{|v_p|} \dot{y} \quad (12)$$

$$\ddot{y} = -k_d |v| \dot{y} + k_l \frac{|v|}{|v_p|} \dot{y} \dot{z} - k_s \frac{|v|^2}{|v_p|} \dot{x} \quad (13)$$

$$\ddot{z} = -g - k_d |v| \dot{z} - k_l |v| |v_p| \quad (14)$$

The scaled drag coefficients are

$$k_d = \frac{\rho A C_D}{2m}$$

$$k_l = \frac{\rho A C_L}{2m}$$

$$k_s = \frac{\rho A C_S}{2m}$$

5 NON-DIMENSIONALIZING THE GOVERNING EQUATIONS

We realize there are many variables in our equations and since we want to make analytic progress in our computations we would need to work with a consistent set of units. For example time can be measured in seconds(sec), distance in meters(m), and mass in (kg). We notice that these quantities time,length and mass are dimensions . For our equations to make sense , we need to measure all dimensions with consistent units. Also we need to realize that the dimension of a variable is an inherent property of the variable, but the units are something we can choose. For example x , y and z are lengths , but we may choose it in terms of furlongs ,miles,meters and centimeters or even kilometers. The idea is to measure our variables in "units" that are intrinsic to our problems.

The following procedure for non-dimensionalizing a differential equation will be followed.

- We will list all the variables and parameters along with their dimensions.
- For each variable, say x , form a product (or quotient) L of parameter that has the same dimensions as x and define a new variable $X = \frac{x}{L}$, where X is the "dimensionless" variable. We notice that its numerical value is the same no matter what system of units is used.
- Rewrite the differential equation in terms of the new variables.
- Since our aim is also to reduce the number of parameters, we will group them into non-dimensional combinations, and define a new set of non-dimension parameters expressed as the non-dimensional combinations of the original parameters.
- Also we will make as many non-dimensional constants equal to one as possible

So let

$$X = \frac{x}{L_1}, \quad Y = \frac{y}{L_2}, \quad Z = \frac{z}{L_3}, \quad T = \frac{t}{\tau}. \quad (15)$$

where L_2 is the distance moved by the ball in a direction parallel to the y -axis, $\tau = \frac{L_2}{v}$ is the symbol used to represent the chosen time-scale in the course of motion of the

ball until it reaches the target. $L_3 = g\tau^2$ represents the length scale vertically and X, Y, Z, T are non-dimensional length variables

Due to assumptions made the length-scale L_1 is to be determined but $L_1 \ll L_2$.

Writing velocity vectors in non-dimensional form

$$|v| = \sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2} \quad (16)$$

Using (15) equation (16) simplifies to

$$|v| = \frac{L_2 \dot{Y}}{\tau} \sqrt{\frac{L_1^2 \dot{X}^2}{L_2^2 \dot{Y}^2} + 1 + \frac{L_3^2 \dot{Z}^2}{L_2^2 \dot{Y}^2}}$$

$$|v| = \frac{L_2 \dot{Y} r_1}{\tau} \quad (17)$$

Where r_1 is the value under the square root

$$|v_p| = \sqrt{\dot{x}^2 + \dot{y}^2}$$

$$|v_p| = \frac{L_2 \dot{Y}}{\tau} \sqrt{1 + \frac{L_1^2 \dot{X}^2}{L_2^2 \dot{Y}^2}}$$

$$|v_p| = \frac{L_2 \dot{Y}}{\tau} r_2 \quad (18)$$

Where $r_2 = \sqrt{1 + \frac{L_1^2 \dot{X}^2}{L_2^2 \dot{Y}^2}}$

We have isolated $\frac{L_2 \dot{Y}}{\tau}$ since it is the highest magnitude in the expression for velocity

and therefore r_1, r_2 have the order of unity i.e.

$$r_1, r_2 = O(1).$$

$k_d = 0.013$	$\rho = 1.23 \text{kgm}^{-3}$
$k_s = 0.0108$	$A = 0.036 \text{m}^2$
$K_l = 0.004$	$D = 0.214 \text{m}$
$C_D = 0.3$	$v = 20, 25, 30 \text{m/s}$
$C_L = 0.1$	$m = 0.268 \text{kg}$
$C_S = 0.25$	

We can now substitute our scaled factors into the governing equations (12)(13)(14) to get

$$\ddot{X} = -k_d L_2 \dot{X} \dot{Y} r_1 + k_l L_3 \frac{r_1}{r_2} \dot{X} \dot{Z} + k_s \frac{L_2^2 r_1^2}{L_1 r_2} \dot{Y}^2 \quad (19)$$

$$\ddot{Y} = -k_d L_2 \dot{Y}^2 r_1 + k_l L_3 \frac{r_1}{r_2} \dot{Y} \dot{Z} - k_s \frac{L_1 r_1^2}{r_2} \dot{X} \dot{Y} \quad (20)$$

$$\ddot{Z} = -1 - k_d L_2 r_1 \dot{Y} \dot{Z} - k_l \frac{L_2^2}{L_3} r_1 r_2 \dot{Y}^2 \quad (21)$$

Since our study is focusing on the arching of the ball in the x - direction, it is important to note that this arching is determined by spin component in the y - direction and initial velocity in the same direction.

We thus take the initial velocity in the direction of the variable x (swerving direction) to be zero, i.e. $\dot{x}(0) = \dot{X}(0) = 0$

We scale the velocity vectors so that $r_1 r_2 = O(1)$ (order unity)

It is clear the dominant term in (19) is the one involving \dot{Y}^2 .

Thus $k_s \frac{L_2^2}{L_1} = 1$.

It implies that

$$L_1 = k_s L_2^2$$

Our volleyball pitch is 18m long and since we want the ball to travel long enough past the net, we will chose L_2 to be between 9.1 m -18 m and for convenience we will work with 18 m. From the table above $L_1 \approx 2m$ i.e. we expect the ball to have a swerve of the order 2m laterally in the course of flight.

The dominant term in (20) involve \dot{Y}^2

$$k_d L_2 = 0.2 .$$

Lets denote this by $\varepsilon = k_d L_2$ which is the small parameter and

$$(\varepsilon^2 \approx 0.04)$$

Our vector components contains ratios $\frac{L_1^2}{L_2^2} \approx 0.02$ and

$$\frac{L_3^2}{L_2^2} \approx 0.1$$

We notice that in the z -direction, L_3 , the ball can be analyzed from any height that is above 2.43m for men and 2.24m for women (wysc.org/page.asp?n=32591).

Lets denote

$$\frac{L_1^2}{L_2^2} \text{ as } c_1 \varepsilon^2 \quad \text{and} \quad \frac{L_3^2}{L_2^2} \text{ as } c_1 \varepsilon^2$$

Notice also

$$k_s L_1 = \frac{L_1^2}{L_2^2} \approx c_1 \varepsilon^2$$

$$k_l L_3 \approx 0.02 \quad \text{denoted as } c_3 \varepsilon^2$$

$$\frac{k_l L_2^2}{L_3} = c_4 \varepsilon \approx 0.18$$

Substituting these in the governing equations we end up getting

$$\ddot{X} = -\varepsilon \dot{X} \dot{Y} r_1 + c_3 \varepsilon^2 \frac{r_1}{r_2} \dot{X} \dot{Z} + \frac{r_1}{r_2} \dot{Y}^2 \quad (22)$$

$$\ddot{Y} = -\varepsilon \dot{Y}^2 r_1 + c_3 \varepsilon^2 \frac{r_1}{r_2} \dot{Y} \dot{Z} - c_1 \varepsilon^2 \frac{r_1^2}{r_2} \dot{X} \dot{Y} \quad (23)$$

$$\ddot{Z} = -1 - \varepsilon r_1 \dot{Y} \dot{Z} - c_4 \varepsilon r_1 r_2 \dot{Y}^2 \quad (24)$$

Factoring $\dot{Y} r_1$ in equation (22),(23) and (24) we obtain

$$\ddot{X} = -\dot{Y} r_1 \left[\varepsilon \dot{X} - c_3 \varepsilon^2 \frac{\dot{X} \dot{Z}}{r_2 \dot{Y}} - \frac{1}{r_2} \dot{Y} \right] \quad (25)$$

$$\ddot{Y} = -\dot{Y} r_1 \left[\varepsilon \dot{Y} - c_3 \varepsilon^2 \frac{\dot{Z}}{r_2} + c_1 \varepsilon^2 \frac{r_1}{r_2} \dot{X} \right] \quad (26)$$

$$\ddot{Z} = -1 - \dot{Y} r_1 \left[\varepsilon \dot{Z} + c_4 \varepsilon r_2 \dot{Y} \right] \quad (27)$$

Scaled velocity component is given by

$$r_1 = \sqrt{1 + c_1 \varepsilon^2 \frac{\dot{X}^2}{\dot{Y}^2} + c_2 \varepsilon \frac{\dot{Z}^2}{\dot{Y}^2}}$$

$$r_2 = \sqrt{1 + c_1 \varepsilon^2 \frac{\dot{X}^2}{\dot{Y}^2}}$$

At this point we can be able to observe that all terms in equation (26) involve ε which is a parameter of small magnitude, this indicates that the motion is dominant in the Y direction and is we describe this situation by $\ddot{Y} = 0$ and the drag represented by ε has a negligible effect.

Now we have the initial conditions such that the ball is kicked with dimensional velocity $(0, v, \omega)$. We have chosen the velocity in x - direction (our arching direction) to be zero since we assumed our ball is dominantly moving in the y direction. If we had chosen x - direction as our dominant path, then we could have chosen velocity in y - direction to be zero. In non-dimensional form , these conditions become

$$X(0) = Y(0) = Z(0) = 0,$$

$$\dot{X}(0) = 0,$$

$$\dot{Y}(0) = 1,$$

$$\dot{Z}(0) = W$$

where

$$W = \frac{\omega\tau}{L_3}$$

6 METHODOLOGY

PERTURBATION METHOD

Now we will employ perturbation method based on the small parameter ε to obtain our solution and we will solve up to second order for convenience.

Regular Perturbation expansion: Our familiarity with Taylor expansion principle for an analytic function $f(x)$ tells us that we can expand close to a point $x = a$ as

$$f(a + \varepsilon) = f(a) + \varepsilon f'(a) + \frac{1}{2}\varepsilon^2 f'' + \dots$$

But we realize that for general functions $f(x)$, the expansion can fail if it fails to converge or simply if the series is unable to capture the behavior of the function. The paradigm of the expansion in which a small change to x makes a small change to $f(x)$ is a powerful tool and it forms the backbone of regular perturbation expansions.

Below are the steps to be followed in our work in order to obtain a perturbation solution;

- Set $\varepsilon = 0$ and solve the resulting system.
- Perturb the system by allowing ε to be non-zero (but small in some sense).
- Formulate the solution to the new perturbed system as a series

$$f_0 + \varepsilon f_1 + \varepsilon^2 f_2 + \dots$$

- Expand the governing equations as a series in ε , collecting terms with equal powers of ε .
- Solve the equation in turn and as far as the solution will be required. In our case we will expand up to $O(\varepsilon^2)$.

Having said that, we will let

$$X = X_0 + \varepsilon(X_1) + \varepsilon^2(X_2) + \dots$$

$$Y = Y_0 + \varepsilon(Y_1) + \varepsilon^2(Y_2) + \dots$$

$$Z = X_0 + \varepsilon(Z_1) + \varepsilon^2(Z_2) + ..$$

After perturbing to $O(\varepsilon^2)$ the solution becomes

$$X = \frac{T^2}{2} + \varepsilon \left[\frac{c_2 W^2 T^2}{2} - (3 + 2c_2 W) \frac{T^3}{6} + \frac{c_2 T^4}{12} \right]$$

$$\begin{aligned}
& + \varepsilon^2 \left[\left(-\frac{9}{2} c_2 W^2 + 3c_3 W - 2c_2 c_4 W \right) \frac{T^3}{6} \right. \\
& \left. + \left(6c_2 W + \frac{11}{2} - 2c_3 - \frac{c_1}{2} + 2c_2 c_4 \right) \frac{T^4}{12} - \frac{13c_2 T^5}{120} \right] \quad (28)
\end{aligned}$$

$$Y = T - \varepsilon \frac{T^2}{2} + \varepsilon^2 \left[- (c_2 W^2 - 2c_3 W) \frac{T^2}{4} + (2 - c_1 + c_2 W - c_3) \frac{T^3}{6} - \frac{c_2 T^4}{24} \right] \quad (29)$$

$$\begin{aligned}
Z &= WT - \frac{T^2}{2} + \varepsilon \left[- (W + c_4) \frac{T^2}{2} + \frac{T^3}{6} \right] \\
& \varepsilon^2 \left[-c_2 W^2 (W + C_4) \frac{T^2}{4} + (4W + 6c_4 + 3c_2 W^2 + 2c_2 c_4 W) \frac{T^3}{12} \right] \\
& - \left(3 + 3c_2 W + c_2 c_4 \right) \frac{T^4}{24} + \frac{c_2 T^5}{40} \quad (30)
\end{aligned}$$

Bray and Kerwin[2003] presented a 2-D system which was more simplified. This was achieved equating the acceleration to the largest term on the RHS of the equation and from equation (25, 26) we will have $\ddot{X} = \dot{Y}^2$ and $\ddot{Y} = -\varepsilon\dot{Y}^2$

with $r_1 \approx r_2 \approx 1$

Using the initial conditions we obtain

$$Y = \frac{1}{\varepsilon} \ln |1 + \varepsilon T|$$

$$X = -\frac{1}{\varepsilon}(T - T)$$

If we set $W = 0$, $c_i = 0$ to remove z -dependence in order to appear two dimensional, The Tylor series expansion for $\varepsilon \ll 1$ is applied to obtain that

$$X = \frac{T^2}{2} - \varepsilon \frac{T^3}{3} + \varepsilon^2 \frac{T^4}{4}$$

$$Y = T - \varepsilon \frac{T^2}{3} + \varepsilon^2 \frac{T^3}{3}$$

We realize that x solution differs at $o(\varepsilon)$ while the y solution differ at $o(\varepsilon^2)$

Now expressing our equations in dimension form as we had done earlier

$$\begin{aligned} X &= \frac{x}{L_1} , & Y &= \frac{y}{L_2} \\ Z &= \frac{z}{L_3} , & T &= \frac{t}{\tau} \end{aligned}$$

We also need to express our constants in terms of dimensional parameters

$$\varepsilon = k_d L_2 \quad , \quad c_1 = \frac{k_s^2}{k_d^2}$$

$$c_2 = \frac{g^2 L_2}{k_d v^4} \quad , \quad c_3 = \frac{k_l g}{v k_d^2 L_2}$$

$$c_4 = \frac{k_l v^2}{g k_d L_2} \quad , \quad W = \frac{\omega v}{g L_2}$$

Where we note that

$$L_1 = k_s L_2^2 \quad , \quad L_3 = g\tau^2 \quad , \quad \tau = \frac{L_2}{v}$$

L_2 is the initial distance and v the initial velocity in the y - direction thus the dimensional solution is thus given by

$$x = \frac{k_s(vt)^2}{2} \left[1 - \left\{ k_d vt - \frac{g^2 t^2 - 4gwt + 6w^2}{6v^2} \right\} \right] \quad (31)$$

$$y = vt \left[1 - \left\{ \frac{k_d vt}{2} \right\} \right] \quad (32)$$

$$z = wt - \frac{gt^2}{2} + \left\{ \frac{k_d g vt^3}{6} - (k_d w + k_l v) \frac{vt^2}{2} \right\} \quad (33)$$

7 RESULTS, DISCUSSIONS AND CONCLUSION

7.1 RESULTS AND DISCUSSIONS

In our study of an in-flight volleyball, we have perturbed our governing equations to order $o(\epsilon^2)$. From equation (31) it can be seen that the deflection of the ball in the x direction is dominantly influenced by the spin coefficient k_s and (vt) , where (vt) denotes the initial approximation of the distance traveled in the direction of y .

Ball's design and atmospheric conditions are the major contributors to the spin coefficient k_s . This has been greatly discussed by Rabindra D.Mehta and Jani Macari Pallis[2001] on their work on sport balls aerodynamics: Effect of Velocity, Spin and Surface roughness.

It is also important to note that the quadratic $g^2t^2 - 4gwt + 6w^2$ is positive provided $w \gg 0$ and that's why we assumed the ball in our case will not be kicked into the ground but rather will be served in such a way that the velocity in the z direction will not be equal to zero. A 2-D analysis will be free of this effect.

All of our equations (31 ,32 & 33) are dependent on time and we realize that as time increases they will become less accurate. Hörzer [2010] in his experiment showed that drag coefficient shows a weak dependence on spin parameter and to prove his point we have used our data to justify our next step of holding drag coefficient a constant as we vary the spin coefficient.

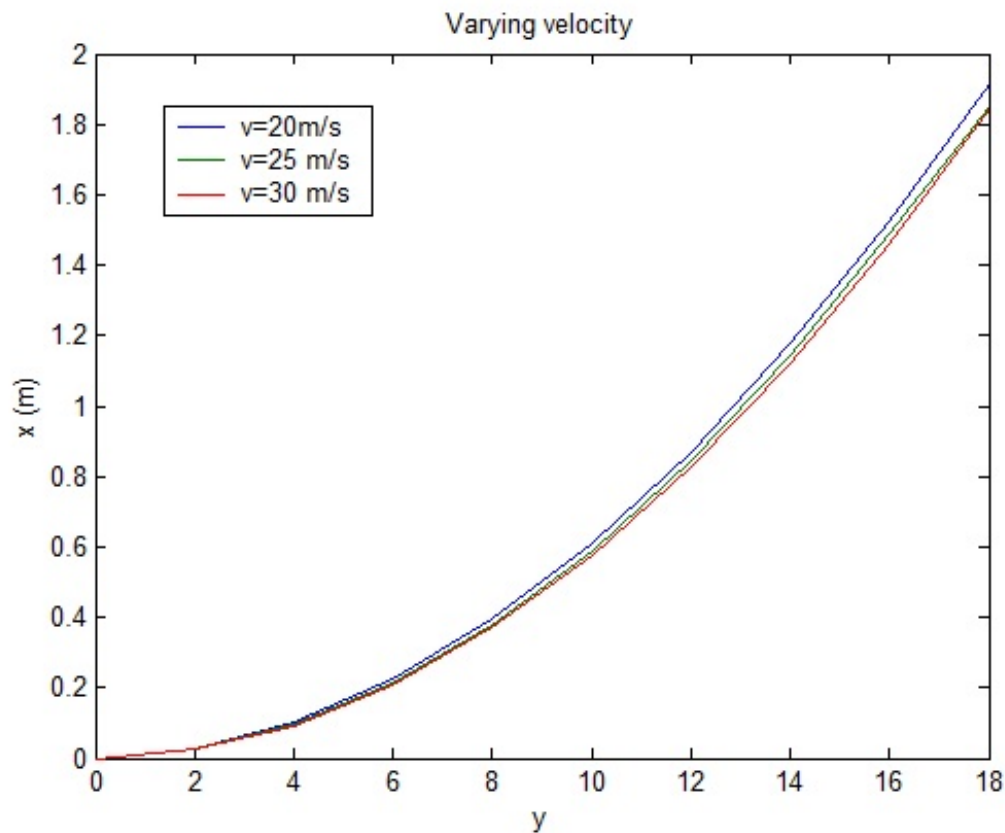


Figure 6. Graph showing the effect of varying initial velocity to the swerving of the volleyball.

From fig 6 We can assert Hörzer's [2010] claims that varying velocity which in turn vary the drag coefficient has a minimal effect on the lateral displacement of the ball and we can hold drag coefficient as a constant without having affected the final results greatly.

In volleyball, serves range from 20 m/s to 30 m/s and we have used this range to draw the above graph. Again we note that if we can decide to vary both drag and spin coefficients at the same time, our problem will become very complex to solve.

Spin coefficient increase with increase in the rate of spinning of the ball and reduces as the ball spins slowly. That is to mean that, the higher the number of revolutions made per second the greater the angular velocity and the larger the spin coefficient. Numerous experimental studies have shown that spin coefficients of balls ranges from ranges $0 < C_s < 0.35$ and in **scaled** form $0.01 < k_s < 0.035$ and we have the following graph;

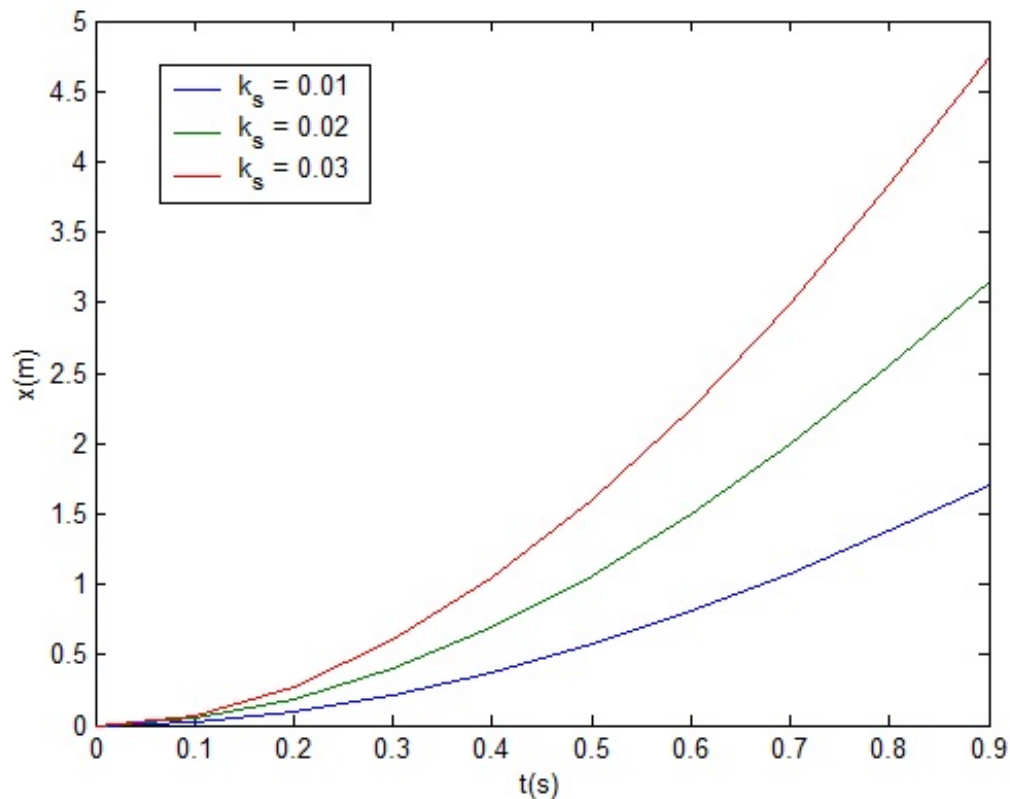


Figure 7. Graph showing the effect of varying spin coefficient to the arching of the volleyball.

From our graph it is clear that the larger the spin coefficient the greater the swerve and the smaller the magnitude of the spin the lesser the swerve. Thus from our research study we can conclude that volleyball players who want the ball to swerve more they should induce faster spin and if they are aiming for less swerve, then they need induce slower spin. If they don't want the ball to swerve at all they should not induce any spin.(This condition is known as knuckle-ball effect).

7.1.1 CONCLUSION

We have been investigating how inducing a side-spin on volleyball affect the movement of the ball. We have have seen that increasing spin rate of the ball increases the spin coefficient and this cause the ball to deflect more from the main path a phenomenon we have termed as swerving. Thus we can conclude that swerving rate increases as spin coefficient increase. This means that swerving increase as the ball spins faster.

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