AESTETHICAL MATHEMATICS-PEDAGOGY: ANALYSIS OF THE CONSTRUCTIVIST PERSPECTIVE IN SECONDARY SCHOOLS IN TRANS NZOIA COUNTY - KENYA

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DECLARATION

This report is my original work and has never been presented for award in any other University.

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ABSTRACT

Lack of interest in mathematics could be as a result of the methodologies (pedagogies) used to deliver mathematical concepts. The methods used could be lacking aesthetic approaches which are those methodologies that are learner centered, learner involved or learner participatory. These are hands on or learning by doing pedagogies also known as constructivist-aesthetic pedagogical approaches. They focus on the interests of the student's ability and learning style, placing the teacher as a facilitator of learning. This study explored the benefits of these constructivist-aesthetic pedagogical approaches so as to understand their implications for mathematical pedagogy. The study sought for an understanding of aesthetic knowing in mathematics education as a way of changing learner attitude towards mathematics and consequently improving performance in mathematics examinations. The study's objectives were to: analyze the concept of "aesthetic mathematics pedagogy"; analyze aesthetic features of mathematics; and develop a model of aesthetic mathematics pedagogy. The study adopted constructivism theoretical framework which works on the premises that learners are not empty vessels and given the right environment can come up with their own ways of solving problems with teachers as facilitators. This study used the famous "Critical method". This is a method where learners do not accept concepts or theories the way they are presented without determining whether they are adequate, the only ones adequate or whether they are superior, inferior or equal to the solutions. This is the method used by the great philosopher, Socrates. The study found that if greater emphasis is placed on explicitly addressing the aesthetic dimension in the classroom practices by teachers to articulate and situate aesthetical pedagogy, then the teaching and learning of mathematics would be made pleasurable. It found that aesthetic features are very important in studying mathematics. It finally found that aesthetic mathematics models, in turn, produce new understandings about the original settings of interest and help students answer the questions that they have posed. The study recommends that a significant number of learning and teaching methods which work well in secondary education should be mapped against those desirable outcomes for which they are most suited, and those subjects for which they are best matched.

CHAPTER ONE

INTRODUCTION

1.0 Background to the study

Aesthetics is a branch of philosophy that deals with the nature of beauty, art, taste and with creation appreciation of beauty (National Council of Teachers of Mathematics NCTM, 2000). A study in beauty could be, for instance, the beauty of human creation, natural scene, art, nature or language among others (Dreyfus and Eisenberg, 1986). It is the study of the responses of the mind and emotions in relation to the sense of beauty. Aesthetics could also be looked at as a critical reflection on art, culture and nature with practical implications on art theory, literary theory, film theory and music theory. Monroe (1982) says of aesthetic experience as that stretch of time that a great part of a person's mental activity is united and made pleasurable by being tied to qualities of a sensuously presented or imaginatively intended object, that is, the image already formed in the mind on which their primary attention is concentrated. Art, literature and mathematics are some of the subjects that use aesthetics. In art, aesthetics is related with poetry, painting and music while in mathematics aesthetics is stressed on geometry which deals with shape, size, relative position of figures and properties of space, graphs which when well drawn carries a lot of information with them particularly in mathematics and are quite appealing and enhance the beauty of mathematics and patterns which stands for a particular organized way of doing things. Geometry, graphs and patterns easily capture human sense particularly that of sight; the sense of sight controls one's emotions hence patterns and graphs well-presented would be so magnetic to the eye thus granting mathematics extreme beauty. Unlike the way mathematics is taught at secondary school level where a text book is followed biblically without learners questioning hence denying them the aesthetic part of teaching.

Mathematics education as an educational discipline holds its own aesthetical values. The beauty of mathematics is found within mathematical patterns, set theory, logic, and the utilization of symbols, mathematical modeling, mathematical sequences and mathematical research that focuses on conceptualizing mathematics and a move towards abstraction (Monroe, 1982). In mathematics, aesthetical value is recognized in clever solution, geometric constructions, patterns or elegant tools, although these are subjective judgments which are content based. What could be given recognition in mathematics would be well presented lessons which include evaluations from different mathematical perspectives like mathematical content, pedagogical value and epistemological soundness (NCTM, 2000). However, Dreyfus and Eisenburg (1986) also insist that mathematics educators are not well equipped to properly bring out the issue of aesthetics with secondary mathematics whether it is in terms of students comparing their work, students seeking answers, teachers asking questions or evaluating student findings. In their findings, educators of mathematics think that students cannot solve problems on their own and therefore spoon feed them, a notion that erroneously gives students a perception that mathematics is boring and valueless thus leading to dislike towards the subject. It is common thought that mathematics cannot be only true but also beautiful, and some mathematicians have attached central importance to the aesthetical merit of their work (NCTM, 2000). Students could be made to understand that beautiful mathematics is meant to be approached with aesthetical pedagogies for them to have strong knowledge about mathematical concepts passed to them.

Carnine,Silbert and Kameenui (1997) observe that mathematics has the ability to confuse, frighten and frustrate learners of all ages. If a child has a negative experience in mathematics, that experience will affect their achievement and attitude towards mathematics in adulthood. The question is whether students' failure to learn mathematics could be ascribed to problems of curriculum, teaching or the student or combination of all (Carnine, Silbert and Kameenui, 1997). Carnine, Silbert and Kameenui (1997) argue that existing methods of teaching have not fulfilled the needs of vast majority of students; traditional methods have made students memorize information, conduct well organized experiments by educators which make them submissive and rule bound. In America, this pedagogy has shown to work well particularly for sharp students (NCTN, 1991). The British feel that the use of communicative strategies encourage pedagogic practices that are interactive in nature and are more likely to impact on learning outcomes and hence be effective. Constructivist learning theories argue that tasks and discourse models that explicitly incorporate students, former experiences can form powerful learning environments (NCTN, 1991)

The performance of mathematics in Kenya has persistently been wanting, an effect which could easily be associated with consistent use of pedagogies that do not attach beauty on themselves. The Kenya national examinations council cited motivation of mathematics teachers, their workload, attitude and poor approach towards delivery of mathematics concepts as being the major contributors in performance of mathematics at Kenya certificate of secondary examinations. This could have hinted that lack of aesthetic pedagogies could be one of the causes of poor performance in mathematics (KNEC Report, 2014). The table below shows the performance in mathematics at KCSE.

Year	Enrolment	Mean (%)
2009	333,516	12.25
2010	354,341	16.30
2011	410,586	19.75
2012	436,349	20.40
2013	446,696	18.50

Table 1: Showing year, enrolment, and mean percentage pass in mathematics at Kenya certificate of secondary education examination.

Source: KNEC Report, 2014

The mathematician, however, is not merely an ascetic, cold and austere. He or she is an expressive artist involved in the richly human struggle to create and discover. Constructivism aesthetic pedagogy is based on this statement. If mathematical concepts could be delivered using beautifully designed pedagogies, its delivery could be attractive to the learners which would lead to love for it. The mathematician experiences in this work the same pleasure as an artist; his pleasure is as great and of the same nature.

The most brilliant members of our species, the likes of Pythagoras, Andrew Wales, Isaac Newton, Gauss, Leonhard Euler and many others have exerted the noblest effort to give us this mathematics. They came up with their own beautiful pedagogies, to this, they embraced constructivism. Constructivism encourages creativity, own thinking and experimentation which is accompanied by success or failure that is gotten with pleasure or pain (Bruner, 1990). But once gotten, the discoverer will sit back and look at their work admirably because they got it by themselves. Anything discovered by one self is beautiful and one will have tacit knowledge on how they discovered it. This is what constructivism entails. The method would be fantastic in the discovery.

Attachment of beauty on pedagogy and content in mathematics is something that mathematicians of all ages will endeavor to do. In fact when Kenyan government in conjunction with Japanese government introduced a course to retrain secondary school mathematics and science teachers in the famous SMASSE (strengthening of mathematics and science in secondary education), it was thought that this would be the first line in treatment of hatred towards mathematics. They came up with the much praised learner centered, hands on constructivism pedagogy which embraces constructivist' philosophy in all its ways. This provides learners with experiences through learning.

Constructivism is a philosophical view point about the nature of knowledge, specifically representing an epistemological stance. It is a theory that focuses on how humans make meaning in relation to the interaction between their experiences and their ideas. It bases its argument on human development that is influenced by other humans. The concept of constructivism has roots in classical antiquity going back to Socrates dialogues with his followers in which he asked directed questions that led his students to realize for themselves the weakness in their thinking. The Socratic dialogue is still an important tool in the way constructivist educators assess their students learning and plan new learning experiences.

The major theme in the constructivism theoretical framework is that learning is an active learning process in which learners construct new ideas or concept based upon their current and past knowledge. Constructivism encourages students to uncover concepts and discover principles themselves. The instructor and the students engage in an active dialogue (e.g. Socratic learning). The task of the instructor is to translate information to be learned to be formal and appropriate to the learners' current state of understanding. A common practice in curriculum design therefore is its spiral manner. In this way, the students continually build upon what they have orally learned (Bruner, 1996).

1.2 Statement of the problem

Mathematics as a subject has had its own share of challenges, one of them being lack of aesthetic pedagogies that could lead to less interest towards it and hence dismal performance in the subject. The methods used could be lacking constructivism aesthetical approaches and as such render the subject dull. In Kenya certificate of secondary education (KCSE), the mean score is hardly 20% and this could possibly be due to inadequate student centered methodologies used in the teaching of mathematics whereby teachers take learners through traditional pedagogies that embrace procedural textbook processes that do not employ methodologies that are learner centered, hands on, learning by doing or learner participatory otherwise known as constructivist-aesthetic pedagogies.

Educators of mathematics have assumed that learners know nothing and therefore have tended to give them procedures to follow without giving them a chance to think on their own and come up with solutions. This has made the teaching of mathematics lack aesthetic approaches. This study looks at this problem with the intention of providing alternative pedagogies.

1.3 The purpose

The study looks into the beauty of mathematics and the approach that could be used to deliver the same with a view to analyze the effect of aesthetic pedagogy in the learning of mathematics and recommend teaching methods that could make the subject interesting if used.

1.4 Objectives of the study

Objectives

i. analyze the concept of "aesthetical mathematics pedagogy";

ii. analyze aesthetic features of mathematics, and

iii. develop a model of aesthetical mathematics pedagogy.

1.5 Research questions

i. What ideas explicate the concept of aesthetical mathematics pedagogy?

ii. What are the aesthetic features of mathematics?

iii. What is involved the development of a model of aesthetical mathematics pedagogy?

1.6 Significance of the study

The findings of the study could be used to encourage the policy makers come up with alternative training methodologies that would empower educators to approach the subject aesthetically and constructively to allow learners to joyfully and creatively solve mathematical problems.

1.7 Theoretical framework

The study uses the theoretical framework of constructivism. Constructivism is a philosophical view point about the nature of knowledge, specifically representing an epistemological stance that learning is an active constructive process. It is a theory that focuses on how humans make meaning in relation to the interaction between their experience and their ideas. It bases its argument on human development that is influenced by other humans. The concept of constructivism has roots in classical antiquity going back to Socrates dialogues with his followers in which he asked directed questions that led his students to realize for themselves the weakness in their thinking. The Socratic dialogue is still an important tool in the way constructivist educators assess their students learning and plan new learning experiences.

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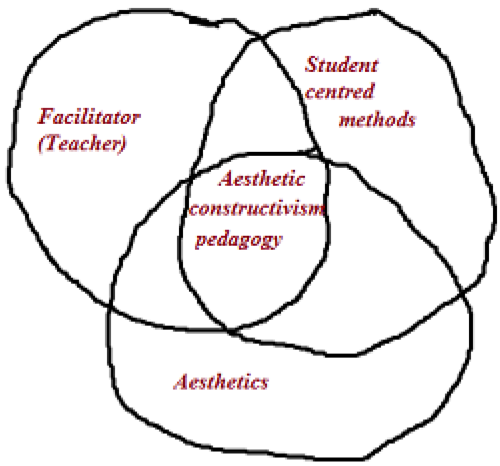
1.8 Conceptual framework

Conceptual framework also known as idea context is the system of concepts, assumptions, expectations, beliefs and theories that supports and informs ones research. It is a visual or written product that explains either graphically or in narrative form the main things to be studied. It explains the key factors, concepts or variables and the presumed relationship among them. It links the actual ideas and the beliefs (Robson, 2011). The conceptual framework under consideration works on the premises that learners are not empty slates to be pumped with knowledge but rather be guided to construe their knowledge. The belief that students do not know mathematics and therefore be taught using traditional methods of rote learning which might not be aesthetical are rife among secondary school teachers of mathematics,(Robson,2011)



Educators of mathematics on the other hand could be there to facilitate, guide and direct the learners to discover for themselves mathematical concepts. Teachers would be there to provide conducive learning environment as illustrated in the diagram above.

1.9 Limitation of the study

Some of the limitations to the study include syllabus coverage, Most a time, secondary school administrations and managements find out how much work is covered and not how it is covered. In a normal learning situation therefore, educators would be more interested in syllabus coverage and not the aesthetical pedagogy or beauty of the subject. This type of teaching method has the potential for students to draw unclear or untrue conclusions if the facilitator is not available or willing to give directions and feedback. It will work well with facilitators who are committed. The method can be problematic for certain students with disabilities who are included in the classroom settings.

1.9.1 Delimitations

The study looks into philosophical aspects of the general theory and practice of mathematics pedagogies in secondary schools in Kenya. Though mathematics has universal rules, the study looks at the Kenyan case because each nation has its own unique way of making sure that concepts passed to learners are well presented and given a special treatment in context for learners to capture them. Generally the study looks at how aesthetics could enhance the teaching and learning of mathematics in secondary schools in Kenya.

1.9.2 Methodology

This study used the "Critical method". Critical thinking is an intellectually disciplined process of actively and skillfully conceptualizing, applying, analyzing, synthesizing and interpreting ideas, concepts or theories generated by observation, experience, reflection, reasoning or communication as a guide to belief and action. It is a form of cooperative argumentative dialogue between individuals based on asking and answering questions to stimulate critical thinking and to draw out ideas and underlying presumptions. It involves discussion in which the difference of a point of view is questioned. One participant may lead the other to contradict themselves in some way, thus weakening the defenders point (Monroe, 1982). It involves thinking about ones thinking in a manner designed to organize and clarify, raise the efficiency of, and recognize errors and bases in one's own thinking. Critical thinking is not hard thinking but an inward directed thinking with the intent of maximizing the rationality of the thinker (Bruner, 1996).

Critical method is significant in the learning process particularly that of internalization. Internalization involves the construction of basic ideas, principles and theories inherent in content. It is important in the process of application whereby those ideas, principles and theories are implemented effectively as they become relevant in the learners lives thus embracing constructivism theoretical framework. Good teachers cultivate intellectually engaged thinking at every stage of learning (Walshaw, 2008). This process of intellectual thinking is at the heart of constructivism theory. The teacher questions student often in a Socratic manner hence fostering critical thinking that incalculates reflectiveness in students. Constructivists value critical thinking method because academically it enables one to analyze, evaluate, explain and restructure their thinking thereby decreasing the risk of adopting, acting on or thinking with a false.

1.9.3 Organization of the study

Chapter one is a research process shown by looking at the background to the study, the statement of the problem, purpose of the study, research objectives and research questions, limitations, theoretical framework, conceptual framework and methodology of the of the study. In chapter two, an elaborate analysis of aesthetical mathematics pedagogy is considered and in chapter three, the concept of aesthetic features of mathematics together with their characteristics make the main body. Chapter four dwells on developing a model of mathematical pedagogy while chapter five gives a summary and conclusion of the research. Have conclusion and indicate what the next chapter will deal with.

Conclusion

Chapter one is a research process shown by looking at the background to the study,

the statement of the problem, purpose of the study, research objectives and research questions, limitations, theoretical framework, conceptual framework and methodology of the of the study. In chapter two, an elaborate analysis of aesthetical mathematics pedagogy is considered.

CHAPTER TWO

AESTHETIC MATHEMATICS PEDAGOGY

2.0 Introduction

This chapter discusses the origin and meaning of the term pedagogy as used in philosophy. It discusses pedagogical models which include productive pedagogy, middle school inquiry based model and multiliteracies model. The chapter further discusses the mathematical pedagogy, aesthetical mathematics pedagogy, significance of aesthetical mathematics pedagogy, learning arrangement and the challenges that hinder effective aesthetical mathematics pedagogy.

2.1 Pedagogy

As noted by Corbett and Norwich (1999), the word pedagogy comes from a Greek word paidagogia in which paidos means child and gogio means lead. The word pedagogy thus literally means “to lead a child”. According to Thomas Coram research unit of education, University of London, pedagogy is a wholistic personal approach to work with children and young people across services. Although pedagogy is sometimes seen as a confusing concept, it is essentially a combination of knowledge and skills required for effective teaching (Wallach, 2015). Pedagogy is described as either the theory or practice of teaching that makes a difference in the intellectual and social development of students (Lovat, 2013). Wallach and Even (2015) define it as ‘any conscious activity by one person designed to enhance the learning of another’. Akinpelu (2013) has his own preferred definition which suggests that pedagogy is the act of teaching together with its attendant discourse. It is what one needs to know, and the skills one needs to command in order to make and justify the many different kinds of decisions of which teaching is constituted.

However, the study of pedagogy is one of confusion, ambiguity and change as no one pedagogy has been examined and established for universal use. Every culture and generation has had its own style of teaching (Fletcher, 2012). The failure to examine pedagogy limits the potential for effecting change through education since to advance teacher reform, it is essential to develop codified representations of the practical pedagogical wisdom of able teachers.

As such, effective pedagogical practice promotes the wellbeing of students, teachers and the school community. It improves students and teachers confidence and contributes to their sense of purpose for being at school. It builds community confidence in the quality of learning and teaching in the school (Claxton, 2015). A pedagogy that embraces constructivism is deemed the best during the teaching or learning as students are not empty vessels to be filled with expert knowledge. They ought to construct their own understanding through considered learning experiences.

2.2 Pedagogical models

The term pedagogical model is often used in the context of e-learning and indeed of course more broadly for learning and teaching. Models such as Kolb's learning cycle which works on the principle that learning is the process whereby knowledge is created through the transformation of experience, that is new concepts are provided by new experiences, Salmon's e-moderating framework whose essential role is promoting human interaction and communication through the modeling, conveying and building of knowledge and skills by using mediation of on line environments designed for interaction and collaboration and Lauriallard's conversational framework which insists that complex learning involves iterative dialogue between teacher and student which reveals the participants conceptions and the variations between them are much quoted (Pimm, 2006). These are often used as an analytic lens to frame a research study or as a support for guiding educational innovation. There already exists a range of established models and theories relating to teachers' professional and pedagogical knowledge and skills. While Shulman (2010) suggests different types of pedagogical knowledge, he does not reflect in detail on the interrelationship between them or influences that may affect teachers' pedagogy (Bhatia, 2012). This section covers models such as Productive Pedagogies, Primary and Middle School Inquiry Based Model and Multiliteracies Model.

2.2.1 Productive pedagogy

Productive Pedagogy is a theoretical framework that teachers can use to reflect critically upon their current classroom practice; that is, a vehicle to use as a professional ‘vocabulary’ around which to have conversations about teaching practice with colleagues and to focus on individual student needs. There are four groups of productive pedagogy models under dimensions of classroom practice which are potentially necessary conditions for improved and more equitable student outcomes. These dimensions include high degree of intellectual quality which involves high order thinking and critical analysis, high levels of demonstrable relevance or connectedness whose main aim is knowledge integration, highly supportive classroom environments where students have control and say in the pace, direction or outcome of the lesson and strong recognition of difference which involves cultural knowledge (Illich, 1994).

2.2.2 Inquiry based model

Inquiry based model is inquiry based, which reflects the belief that “active involvement on the part of students in constructing their knowledge is essential to effective teaching and learning” (Murdoch, 2011). Inquiry is a framework for developing understandings about the world and has become a powerful tool in the contemporary classroom. Outcomes based curriculum documents continue to advocate the process of inquiry as a vehicle for achieving effective learning in areas such as science, health and social and environmental education. As a means of meeting the productive pedagogies dimensions of classroom practice, this model would appear to be an effective planning framework to cater for students’ different learning styles and for engaging them in cognitively challenging and relevant curriculum.

Within this framework, units of work are integrated across curricula and based around topics of relevance and interest to students. Skills, values and understanding are taught and assessed within meaningful, ‘connected’ contexts (Murdoch, 2011). The essence to this approach is the relationship between those learning areas concerned with ‘the world around us’ such as science, technology, health and environmental and social education and those areas through which we explore and come to understand that world which include language, mathematics, art, drama, dance, music and aspects of technology. Furthermore, the inquiry approach reflects the belief that active involvement on the part of students in constructing their knowledge is essential to effective teaching and learning.

2.2.3 Multiliteracies model

Multiliteracies refer to use of language. It refers to the variability of meaning making in different cultural, social and domain specific contexts which are becoming more important to the communication environment. Multiliteracies provide a framework for re-thinking curriculum in all learning areas and mainly focus on how literacy has been redefined by social, technological and economic change (Anstey, 2002). Multiliteracies also refer to different modes of meaning to address some of the major aspects of change in our contemporary communication environment. These days, exchanges of meaning are rarely just linguistic but multimodal which include visual, audio, gestural tactile, and spatial patterns of meaning. The Multiliteracies framework supports teachers across all learning areas to develop curriculum which ensures sound pedagogy with in-built quality assurance and which responds to the diversity of students in their classes.

2.3 Mathematical pedagogy

Mathematics pedagogy is the practice of learning what mathematics is and the methodology of passing it onto the learners (Bhatia, 2012). It involves the method and practice of learning. Good mathematics pedagogy brings about competence in the subject. Mathematical competence is a fundamental skill for personal fulfillment, active citizenship, social inclusion and employability in the modern world (Feynman et al., 1985). It is further argued that in mathematical pedagogy, the goals are different from those of the "ordinary" math class or of a more conceptual approach to mathematics teaching. In this ideal, learning what mathematics is and how one engages in it are goals coequal and interconnected with acquiring the "stuff" such as concepts and procedures of mathematics. Therefore, mathematical pedagogy is not only concerned with the computational knowledge of the subject but is also concerned with the selection of the mathematical content and communication leading to its understanding and application. The nature and quality of instructional material, the presentation of content, the pedagogic skills of the teacher, the learning environment, the motivation of the students are all important and must be kept in view in an effort to ensure quality in teaching-learning of mathematics (Bhatia, 2012). Eurydice (2011) further argues that moving away from the traditional teacher-dominated way of learning, active learning approaches encourage pupils to participate in their own learning through discussions, project work, practical exercises and other ways to help them reflect upon and explain their mathematics learning.

Mathematical pedagogy assumes that students must be actively involved in constructing their own understandings, in discovering and inventing mathematics (Checkland, 1999). The basis for this emerges directly from a largely constructivist knowledge of the discipline. Mathematical pedagogy also takes a group orientation to classroom learning. The model is that of a teacher facilitating the learning of students. This approach uses the classroom as a mathematical community; learning involves collaboration among individuals.

Globally, teachers are the major players in guiding students' mathematical development by engaging them in problems, facilitating the sharing of their solutions, observing and listening carefully to their ideas and explanations, and discerning and making explicit the mathematical ideas presented in the solutions (NCTM, 2010). Claxton (2015) points out that when teachers attend to their students' mathematical thinking there are many benefits which include higher levels of conceptual understanding by students and more positive attitudes held by both teachers and students towards mathematics. In particular, the encouragement of students' methods of solution requires that the teacher develops a listening orientation. Such an orientation promotes a learning environment conducive to and respectful of students' own sense making and intellectual autonomy (Davis, 2010). Listening to students' mathematical thinking is one of the central tasks of mathematics teaching. However, listening to students' thinking is hard work, especially when students' ideas sound and look different from standard mathematics like adding zero to zero to get two zeros or adding one and one (1+1 =4) to get four (Wallach & Even, 2015).

Implementing this vision of mathematics classes where students' autonomous sense making and problem solving are facilitated challenges previous held notions of what it means to teach mathematics (Silver, 2015). The notion of teaching as telling such as speaking, explaining, demonstrating rather than listening, hearing, seeing and interpreting still pervades most mathematics classrooms. Despite the many benefits seen by listening to students' mathematical thinking, focusing on students' thinking is challenging (Wallach & Even, 2015). This could be due to students presenting a variety of ways of thinking about a mathematical problem and teachers worrying whether they will recognize mathematical understanding in all of the representations presented. Although a student may not appear to a teacher to understand a concept, there may actually be sense in their thinking and explanation. When teachers do not attend to student thinking they tend to dismiss what students bring to the mathematical community and instead impose traditional formalized procedures on students (Silver et al., 2013).

There are several approaches to professional development that have been viewed as positively supporting teachers in shifting practice. In most cases, mathematical instruction often is approached in terms of starting and exemplifying rules. The "tell, show and do" model, based on the under assumptions that information can be presented by telling and that understanding will result from being told. Such an approach does not work because it frequently overlooks two crucial developmental components, the process of assimilation and the issue of readiness. Essentially in this approach, students are ready intellectually when the teacher is ready for them to receive the information. Learning through such an approach often fails to promote a lack of transfer of mathematical information to new situations. Teachers need to employ alternative forms of instruction that permit students to build their repertoire of mathematical problems and concept. Promising models for such instruction are highly interactive. In such models teachers acting as facilitators, both model and elicit mathematical discourse by asking questions, following lead, and conjecturing rather than presenting faultless products which is in line with constructivism (Noddings, 1990).

Educators need to focus on creating learning environments that encourage students' questions and deliberations, environments in which the students and teacher are engaged with one another's thinking and function as members of a mathematical community. In such a community, the teacher student interaction provides teachers with opportunities for diagnosis and guidance and for modeling mathematical thinking while at the same time it provides students with opportunities to challenge and defend their constructions. Educators of mathematics need to employ strategies that will help them develop the participation essential to engage students in mathematics. Increasing the amount of time students spend working together supporting the development of discourse and community. Working in groups, learners gradually internalize the discourse that occurs, challenging themselves by asking for reasons and in general, accounting for their own mental work. Another practice that supports learners' participation involves shifting responsibility from educators to learners to make commitments to their answers. Further, learners' reflective processes could be developed by focusing their efforts on interpreting problems, describing strategies for solutions and justifying and defending the results.

Mathematical pedagogy cannot be complete without an elaborate assessment program. Tests given out must measure what is of value, not just what is easy to test. If learners have to investigate, explore and discover, then the assessment should not measure just mimicry mathematics (Noddings, 1990). By confusing means and ends, by making testing more important than learning, present practice holds today's students hostage to yesterday's mistakes (National Research Council, 1989). Through assessment, educators of mathematics learn how the recipients think about mathematics and what ways to use in order to assist them. Moreover, learners obtain feedback in order to make adjustments and deepen their understanding of mathematics (Sternmark, 2009).

Every educator gets a wealth of information during the process of assessment. Many act on this information but few document it. It is through the documented assessment that one can communicate most clearly to learners which behaviors and learning outcomes are valued (Clarke, 2003). Assessment plays a key role in effective teaching of mathematics. Too often educators' experiences with methods of assessment are limited to the more traditional "testing and measurement". Strategies provided through a previous course. Give the growing awareness and efforts for change; there is a strong need to integrate the understanding and use of alternative methods of assessment as an ongoing topic throughout the teacher's educational life.

For effective learning of mathematics, the aspects stated above as "mathematic pedagogy" are quite integral. Educators' knowledge and their ability to use and evaluate these components develop overtime. Decisions over instructional materials are intimately associated with decisions about ways to represent mathematics concepts and procedures. The discourse of the classroom and need for ongoing assessment also are part of process of dynamic interaction that results in knowing mathematical pedagogy. Therefore, whatever pedagogical strategies teachers believe, their role is always a determinant factor of effective classroom session. Teacher quality is the single greatest factor in explaining student achievement, more important than classroom related issues such as resources, curriculum guidelines and assessment practices, or the broader school environment such as school culture and organization.

A review of literature reveals that a lot of research on analysis of pedagogical perspective has been undertaken in developed countries context and their applicability in the developing countries such as Kenya is yet to be explored. Developing countries in Asian continent have carried some studies on pedagogical perspective while in Kenya the studies have focused on reasons for pedagogical perspective failures rather than what determines effective mathematical pedagogy according to the Ministry of Education and Technology Republic of Kenya (MoEST) (2014) National Educational

Sector Plan Volume One. Ashley et al. (2007) did a study on the analysis of modeling units of study from a pedagogical perspective of the pedagogical meta-model behind the Educational Modeling Language EML. Their study did not look at the determinants of effective mathematical pedagogy. Torp et al (2004) also carried a study on pedagogical models and online pedagogy in Norway. The objective was to ensure provision of learning environments that are based on constructivist insights. The study still did not identify the determinants of effective pedagogical models. Gharashe (2009) looked at content based teacher centered model in Kenya and concluded in his study on analysis of factors affecting pedagogical models in Kenya that the quality of inadequate teacher preparation, gender and fear as factors affecting pedagogical models. These studies have focused on the reasons for failure. None has attempted to analyze the implementation of pedagogical models in Kenya. In order to fill this gap, the study investigates the aesthetical mathematics pedagogy in Kenya.

2.4. Learning arrangement

It is the role of the teacher to provide students with working arrangements that are responsive to their needs. All students need some time to think and work quietly by themselves, away from the varied and sometimes conflicting perspectives of other students (Ashley, 2001). At other times, partners or peers in groups can provide the context for sharing ideas and for learning with and from others. Group or partner arrangements are useful not only for enhancing engagement but also for exchanging and testing ideas and generating a higher level of thinking. In supportive, small-group environments, students learn to make conjectures and learn how to engage in mathematical argumentation and validation (Ashley, 2001).

Whole class discussion can also provide a forum for broader interpretations and an opportunity for students to clarify their understanding. It can also assist students in solving challenging problems when a solution is not initially available. Teachers have an important role to play in the discussion. Focusing attention on efficient ways of recording, they invite students to listen to and respect one another‘s solutions and evaluate different viewpoints. In all forms of classroom organization, it is the teacher's task to listen, to monitor how often students contribute, and to keep the discussion focused. When class discussion is an integral part of an overall strategy for teaching and learning, students provide their teachers with information about what they know and what they need to learn (Ding, Li, Piccolo and Kulm, 2007).

With the emphasis on building on students' existing proficiencies, rather than remediating weaknesses and filling gaps in students' knowledge, effective teachers are able to be both responsive to their students and to the discipline (Carpenter, Fennema, &Franke, 2012). They understand that learners make mistakes for many reasons. Some mistakes happen because students have not taken sufficient time or care; others are the result of consistent, alternative interpretations of mathematical ideas that arise from learners' attempts to create meaning. To help students to learn from their errors, teachers organize discussions with peers or the whole class that focus students' attention on the known difficulties. Asking students to share a variety of interpretations or solution strategies enables learners to compare and reevaluate their ideas.

2.5 Barriers to effective aesthetic mathematics pedagogy

Teaching is a lively process in which a person shares information and ideas to make behavioral changes (Banks, 2000). Learning is the process of assimilating information with a resultant change in behavior (NCTM, 1989). Teaching-learning process is a planned interaction that promotes behavioral change that is not a result of maturation or coincidence. However, mathematics has become a nightmare for most of the teachers and students. According to Snoeyink and Ertmer (2001), teachers face these challenges due to the barriers that exist as either external or internal barriers.

2.6 Inadequate teacher preparation

Mathematics education relies very heavily on the preparation that the teacher has, in their own understanding of mathematics, of the nature of mathematics, and in her bag of pedagogic techniques. Textbook-centered pedagogy dulls the teacher's own mathematics activity. At two ends of the spectrum, mathematics teaching poses special problems. At the primary level, most teachers assume that they know all the mathematics needed, and in the absence of any specific pedagogic training, simply try and uncritically reproduce the techniques they experienced in their school days. Often this ends up perpetuating problems across time and space. At the secondary level, some teachers face a different situation as compared to primary school. The syllabi have considerably changed since their school days and in the absence of systematic and continuing education programmes for teachers, their fundamentals in many concept areas are not strong. This encourages reliance on 'notes' available in the market, offering little breadth or depth for the students.

2.7 Curricular acceleration

The rate at which mathematics curriculum changes is high, with higher level mathematics being brought to lower levels**.** A generation ago, calculus was first encountered by a student in college. Another generation earlier, analytical geometry was considered college mathematics. But these are all part of secondary school curriculum now. Such acceleration has naturally meant pruning of some topics: there is far less solid geometry or spherical geometry now. One reason for the narrowing is that calculus and differential equations are critically important in undergraduate sciences, technology and engineering, and hence it is felt that early introduction of these topics helps students proceeding further on these lines. Whatever the logic, the shape of mathematics education has become taller and more spindly, rather than broad and rounded.

2.8 Gender

Mathematics tends to be regarded as a 'masculine domain'. This perception is aided by the complete lack of references in textbooks to women mathematicians, the absence of social concerns in the designing of curricula which would enable students questioning perceived gender ideologies and the absence of reference to women's lives in problems. A study of mathematics textbooks found that in the problem sums, not a single reference was made to women's clothing, although several problems referred to the buying of cloth (Ahmedabad Women's Action Group, 2010).

According to Feynman et al. (1985), there is a fairly systematic devaluation of girls as incapable of 'mastering' mathematics, even when they perform reasonably well at verbal as well as cognitive tasks in mathematics. It has been seen that teachers tend to address boys more than girls, which feeds into the construction of the normative mathematics learner as male. Also, when instructional decisions are in teachers' hands, their gendered constructions cover the mathematical learning strategies of girls and boys, with the latter using more invented strategies for problem-solving, which reflects greater conceptual understanding (Feynman et al., 1985).

Studies have shown that teachers tend to attribute boys' mathematical 'success' more to ability, and girls' success more to effort. Classroom discourses also give some indication of how the 'masculinising' of mathematics occurs, and the profound influence of gender ideologies in patterning notions of academic competence in school. With performance in mathematics signifying school 'success', girls are clearly at the losing end (Weisbeck, 2012).

2.9 Fear and failure

Negative attitude of mathematics means having an aversion towards learning mathematics and using it in their daily life and discouraging students from choosing mathematics as their major subjects. Hostile feelings and negative attitudes toward Mathematics and science, therefore, have a great influence on general behavior and values. These feelings and attitude that sustain a dislike of Mathematics or hamper any interest in mathematics and are great barriers to the development of Mathematical literacy than any lack of particular concepts, skills, or thinking abilities' (Smart and Rahman, 2012).

Many students develop fear towards Mathematics due to their misunderstanding, non-understanding and failure during previous lessons. Bergeson (2010) stated that mathematical anxiety is developed as a result of having a poor image of mathematics due to general lack of comfort in t [illegible] might experience mathematical when required to perform. Children with negative attitudes towards Mathematics have performance problems because they develop anxiety. Such fear is closely linked to a sense of failure. By Class III or IV, many children start seeing themselves as unable to cope with the demands made by mathematics. In high school, among children who fail only in one or two subjects in year-end examinations and hence are detained, the maximum numbers fail in mathematics.

Failure in mathematics could be read through social indicators as well. Structural problems in education, reflecting structures of social discrimination, by way of class, caste and gender, contribute further to failure in mathematics education as well. Prevalent social attitudes which see girls as incapable of mathematics, or which, for centuries, have associated formal computational abilities with the upper cadres deepen such failure by way of creating self-fulfilling expectations (Bergeson, 2010).

Conclusion

This chapter covers the relationship between understandings about pedagogy and views about learning and the purpose of education. It discusses pedagogical models as used in mathematics teaching. Current theorizing has radically altered the way the teacher–student relationship is perceived and gives status to personal experiences as a source of knowledge.

The chapter also discusses literature review on mathematical pedagogy and effective mathematical pedagogy measures. Furthermore, the chapter covers aesthetical mathematical pedagogy. From the a fore going discussion, it follows that if greater emphasis on explicitly addressing the aesthetic dimension in the classroom practices by teachers to articulate and situate aesthetical pedagogy, then the teaching and learning of mathematics would be made pleasurable. It finally explains in details the significance of aesthetic mathematics pedagogy and barriers in effective aesthetic mathematics pedagogy. The next chapter which is chapter three will cover the aesthetic features of mathematics. The next chapter divulges into characteristics of aesthetic mathematics.

CHAPTER THREE

AESTHETIC FEATURES OF MATHEMATICS

3.0 Introduction

This chapter elaborates in details the aesthetical features of mathematics. The features covered include elegance*, exemplification, beauty and wonder, symmetry and philosophy of proof. Finally the chapter covers the conclusion for the study.*

3.1 Elegance

In mathematics, elegance refers to simplicity and consistence (Dijkstra, 2004). Simplicity is a desired future of elegant explanations. It uses arguments which are simple ease to understand but effective and constructive. This elegance is seen in mathematics and mathematical thoughts which are obviously directed towards beauty as one profound characteristic. As stated by Dreyfus and Eisenberg (1986), elegance plays the most central role in the process of mathematical thinking. The appreciation of mathematical elegance by students should thus be an integral component of mathematical education (Dreyfus and Eisenberg, 1986). Enzensberger and Csiszar (2011) points out that the mathematics has certain archaic elegant features that have changed very little in the past two thousand years, some that speak more to the Baroque or the Classical than to any more contemporary or post-modern sensibilities. An elegant strategy has the property that it is recognized as a “very good” method by other problem solvers once they become aware of it (Dreyfus and Eisenberg, 1986). Elegance implies not only a deeper than usual awareness of the structure of the problem, but also a creative ability to apply a procedure not suggested by the structure. While the construction of an “elegant” solution is a personal achievement, it is something which is readily recognized as “worthy” by others in a position to appreciate it (Dreyfus and Eisenberg, 1986). Del Campo and Clements (1990) offered the following “elegant” response to the task of writing numbers 1 to 999999:

"Instead of writing 1 to 999999, I'll write 0 to 999999 as follows: 000000, 000001, 000002 ... and so on, to ... 999998, 999999. Now, altogether there are a million 6-digit numbers here, so there are 6 million digits in total (1000000 6). Each digit 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 appears exactly the same number of times. Therefore: the number of "1s" is 6000000 ÷ 10 = 600000" (Dreyfus and Eisenberg, 1990).

According to Del Campo and Clements (1990), this was the best possible way of writing these numbers that has a genius evident. Dijkstra (2004) also adds that mathematics teachers should not aim to produce learners who will merely strive to reproduce teacher methods and 'basic facts 'but those who can give elegant explanations as well. He further claims that school students who have learned to think of mathematics as "formulae and rules" rarely generate elegant solutions to mathematics problems or elegant mathematical proofs. Furthermore, mathematicians and mathematics educators have often referred to the concept of "elegance" in mathematics and have urged teachers to encourage their students to strive for elegance in mathematical problem solving and in mathematical proof (Dijkstra, 2004). Elegance may well be the driving subconscious force behind mathematical creativity.

3.2 *Exemplification*

There is evidence from earliest historical records that examples play a central role in both the development of mathematics as a discipline and in the teaching of mathematics. It is not surprising therefore those examples have found a place in many theories of learning mathematics. Nelson (2011) maintains that exemplification is a relation by which a sample refers to what it samples as a symptom of the aesthetic. *Examples in the form of worked solutions to problems are key features in virtually any instructional explanation (Peled and Zaslavky, 1997) and examples of all kinds are some of the principle devices used to illustrate and communicate concepts between teachers and learners (Peled & Zaslavsky, 1997).*

A study by Piaget (1970) on genetic epistemology assumes that individuals actively try to make sense of their world of experience, supported by social groupings (Confrey, 1991) in which they find themselves. It underpins many current theories of mathematics learning, by assuming the impact of new examples on existing mental schema through assimilation and accommodation. Piaget's notion of reflective abstraction (Dubinsky, 1991) implies experiences and actions performed by the learner through which abstraction is possible.

Building on Piaget's notion of schema, Skemp (1969) wrote about the learning of mathematical concepts through abstraction from examples, which meant that the teachers' choice of which examples to present to pupils was crucial. His advice on this topic includes consideration of noise, that is the conspicuous attributes of the example which are not essential to the concept, and of non-examples, which might be used to draw attention to the distinction between essential and non-essential attributes of the concept and hence to refine its boundaries. This is totally in line with constructivism where examples are encouraged as a way of eliciting enthusiasm in learners.

3.3 Beauty and wonder

Mathematics essentially comprises an abundance of ideas. Number, triangle and limit are just some examples of the myriad ideas in mathematics. Mathematical ideas like number can only be really 'seen' with the 'eyes of the mind' because that is how one 'sees' ideas. One can appreciate music without reading the sheet of music. Similarly, mathematical notation and symbols on a blackboard are just like the sheet of music; they are important and useful but they are nowhere near as interesting, beautiful or powerful as the actual mathematical ideas they represent *(Dubinsky, 1991).*

Additionally, modern technology would be impossible without mathematics. There is probably not a single technical process which can be carried through without more or less complicated calculations; and mathematics plays a very important role in the development of new branches of technology *(Dubinsky, 1991).* It is true that every science, to a greater or lesser degree, makes essential use of mathematics. The "exact sciences," mechanics, astronomy, physics, and to a great extent chemistry, express their laws by means of formulas and make extensive use of mathematical apparatus in developing their theories. The progress of these sciences would have been completely impossible without mathematics. For this reason the requirements of mechanics, astronomy, and physics have always exercised a direct and decisive influence on the development of mathematics (Cairbre, Watson and McKeon, 2006).

The quest for beauty in mathematics is what has motivated many of the great mathematicians and yet their mathematics has turned out to be incredibly powerful in science and many other areas. Very often this search for beauty in mathematics has led to new ideas and discoveries of new theories that have fundamentally changed the understanding of the physical world and are now indispensable in the physical world. It's clear from the history of mathematics that the practical power of mathematics is often an offspring of the quest for beauty in mathematics. For example, a study by Copernicus in the sixteenth century, the Polish mathematician led to the discovery of universe as a systematic harmonious structure framed on the basis of mathematical principles designed by God (Cairbre, Watson and McKeon, 2006). This pursuit for an aesthetic harmonious mathematical structure led Copernicus to his famous Heliocentric Theory which stated that the earth and the planets revolved around the sun as opposed to the earlier belief that the earth was the centre of the universe with the sun revolving around the earth. Copernicus had no experimental evidence for his theory. The motivation for his theory was purely aesthetic because the mathematics describing the sun centered universe was more aesthetically pleasing than the mathematics describing the earth centered universe. Galileo and Kepler would later pursue Copernicus' ideas and provide experimental evidence that the earth revolved around the sun. This shocked the world and revolutionized science and society (Cairbre, 2006).

A study by Hamilton in mathematics also turned out to be incredibly powerful when applied to science and many other areas. Hamilton's Fundamental Theory of Dynamics was indispensable for the creation of Quantum Mechanics which is how we now understand the physical world at the microscopic level. According to Hamilton (1805-1865) in regard to Fundamental Theory of Dynamics, “the difficulty is therefore at least transferred from the integration of many equations of one class to the integration of two of another; and even if it should be thought that no practical facility is gained, yet an intellectual pleasure may result from the reduction of the most complex and, probably, of all researches respecting the forces and motions of body, to the study of one characteristic function, the unfolding of one central relation.” Therefore, the application of the theory had 'intellectual pleasure' which is aesthetic pleasure (Cairbre, 2006).

3.4 Symmetry

Symmetry plays an important role in some areas of mathematics and has traditionally been regarded as a factor to visual beauty. Symmetry is a fundamental part of geometry, nature, and shapes. It creates patterns that help organize our world conceptually. People use concepts of symmetry, including translations, rotations, reflections, and tessellations as part of their careers. Examples of careers that incorporate these ideas are artists, craftspeople, musicians, choreographers, and not to mention, mathematicians (Montessori, 2002).

It is important for students to grasp the concepts of geometry and symmetry while at the elementary level as a means of exposing them to things they see every day that aren’t obviously related to mathematics but have a strong foundation in it (Ma and Kishor, 2015). According to the National Council of Teachers of Mathematics learners should be able to apply transformations and use symmetry to analyze mathematical situations. This includes predicting and describing the results of sliding, flipping, and turning two-dimensional shapes. They should also be able to describe a motion or a series of motions that will show that two shapes are congruent, and identify and describe line and rotational symmetry in 2 and 3-dimensional shapes and designs. The Montana State Standards for Mathematics are in line with NCTM’s standards indicating that students should be able to identify lines of symmetry, congruent and similar shapes, and positional relationships (Murdoch, 2011).

Montessori (2002) did a study and found that many of the students were concerned with filling in the gaps of their patterns. She concluded that while she was not attempting to teach or emphasize any particular area of mathematics, “ideas regarding size, symmetry, tessellation and representation of 3-D objects were arising spontaneously and, given more time, could have been further developed” (Montessori, 2002). Montessori (2002) provides an easy way of introducing the topic of symmetry by taking what students create on their own with no instruction and showing them what they have created and how they did it in mathematical terms which are in line with constructivism.

Murdoch (2011) also likes the idea of using dynamic geometry software as a visual learning tool in the mathematics classroom. Her first objective was to “first and foremost to enable the students to build on their previous knowledge of rotation and extend their skills in the topic with confidence and enjoyment”. She wanted students to be able to do the activity by hand using the computer as an active helper. Her entire article focuses on two lessons in rotations, the second one building on what students learned in the first lesson. The activities presented could easily be expanded on and give rise to other symmetrical concepts such as translations and reflections (Murdoch, 2011).

In their study, Ma and Kishor (2015) focus a lesson on transformations using the book A Cloak for the Dreamer by Aileen Friedman. It contains links to tessellation, tiling, and symmetry and incorporates the importance of predicting, guessing, and thinking of all possible solutions for a problem, ultimately finding the best answer. Using literature as a teaching tool captures the students’ attention and engages them in the learning opportunity at hand. It also gives kids a chance to build on their previous knowledge and apply what they know to learn more, coinciding with Montessori’s (2002) idea that these concepts will come out of little instruction and a lot of exploration.

Ma and Kishor (2015) also want students to be able to create their own designs and then verbalize what they did using mathematical terms giving other students a chance to hear and see the concepts over and over again.

3.5 Philosophy of proof

One of the basic starting points of the characteristics of mathematics seems to be that mathematics is unique among intellectual disciplines because of the definitive nature of its results. Mathematics makes a steady advance, while philosophy continues to flounder in unending bafflement at the problems it confronted at the outset (Dummett, 1998). According to the dialogical perspective, the defining criteria for what counts as a mathematical proof can be explained in terms of the ultimate function of a mathematical proof, namely that of convincing an interlocutor that the conclusion of the proof is true by showing why that is the case. Thus, a proof seeks not only to force the interlocutor to grant the conclusion if she has granted the premises; it seeks also to reveal something about the mathematical concepts involved to the interlocutor so that she also apprehends what makes the conclusion true its causes, as it were. On this conception of proof, aesthetic considerations may well play an important role, but they will be subsumed to the ideal of explanatory persuasion (Dummett, 1998).

Mathematical fact has an elevated status over other kinds of fact. It’s revered as a very certain kind of truth in a way that makes me feel uneasy, and sometimes even fraudulent (Corfield, 2003). Mathematical truth is revered because of proof. Due to the notion of “proof”, there exist an utterly rigorous way of knowing what is and isn’t true in mathematics. The wonderful thing about formal mathematical proof is that it eliminates the use of intuition in an argument. And the trouble with formal mathematical proof is that it eliminates the use of intuition in an argument. That is, formal mathematical proofs may be wonderfully watertight, but they are impossible to understand (Gowers, 2000).

Mathematicians make constant use, to assist them in the discovery of their theorems and methods, of models and physical analogues, and they have recourse to various completely concrete examples. These examples serve as the actual source of the theory and as a means of discovering its theorems, but no theorem definitely belongs to mathematics until it has been rigorously proved by a logical argument (Cairbre, Watson and McKeon, 2006). If a geometer, reporting a newly discovered theorem, were to demonstrate it by means of models and to confine himself to such a demonstration, no mathematician would admit that the theorem had been proved. The demand for a proof of a theorem is well known in high school geometry, but it pervades the whole of mathematics. Angles could be measured at the base of a thousand isosceles triangles with extreme accuracy, but such a procedure would never provide a mathematical proof of the theorem that the angles at the base of an isosceles triangle are equal *(Dubinsky, 1991).* Mathematics demands that this result be deduced from the fundamental concepts of geometry, which at the present time, in view of the fact that geometry is nowadays developed on a rigorous basis, are precisely formulated in the axioms. To prove a theorem means for the mathematician to deduce it by a logical argument from the fundamental properties of the concepts occurring in that theorem. In this way, not only the concepts but also the methods of mathematics are abstract and theoretical *(Dubinsky, 1991).*

Conclusion

This chapter covers information on aesthetical features of mathematics which include elegance, *exemplification, symmetry, beauty and wonder, aesthetic value of reasoning and philosophy of proof. These aesthetic features are very important in studying mathematics. However, critics of current schooling practices point to the fragmentation of learning in general and of mathematics specifically. The fragmentation of mathematics has divorced the subject from reality and from inquiry. The aesthetic aspect of mathematical inquiry has been stripped from educational practice in favor of sequences of procedural learning outcomes. The aesthetic nature of the activities of a mathematician may provide coherence to mathematical knowledge, and thus, increase learners' appreciation and understanding of mathematics.* The next chapter which is chapter four will cover information on a model of aesthetical mathematics pedagogy that will be developed by the researcher.

CHAPTER FOUR

MODEL OF AESTHETICAL MATHEMATICS PEDAGOGY

4.0 Introduction

This chapter develops model of aesthetical mathematics pedagogy. It defines mathematical modeling. It further discusses pedagogy of modeling which include assessment, resource information, affect and persistence, understanding and supporting group efforts and constructivism.

4.1 Mathematical modeling

Mathematical modeling is the process of using mathematics to study a question from outside the field of mathematics (Blais, 1988). Doty (1995) defines mathematical model as a representation of a particular phenomenon using structures such as graphs, equations, or algorithms. This course gives students practice in formulating interesting questions from fields such as science, entertainment, politics, or design. It teaches the specific skills used in creating and interpreting mathematical models. These models, in turn, produce new understandings about the original settings of interest and help students answer the questions that they have posed (Hugh, 1989).
Modelers seek to gain understanding, predict outcomes, make decisions, and develop designs. Furthermore, Doty (1995) posit that specific reference to traditional mathematical content areas is omitted not because new mathematics topics will not be addressed, but because they are the means and not the ends of the course. These ends could be met through the study of a range of mathematics subjects. Topics are chosen in order to deepen students' understandings of central mathematical ideas and to broaden their views of what mathematics is and what mathematicians do.

4.2 Assessment

Assessment is the process of gathering and discussing information from multiple and diverse sources in order to develop a deep understanding of what students know, understand and can do with their knowledge as a result of their educational experiences. The process culminates when assessment results are used to improve subsequent learning (Abbot and Ryan, 2011). The goals of assessment are for the student and the teacher to be informed about the degree to which the student is meeting the objectives of the course, the ways in which the student accomplishes their work and thinks about the material, and the skills and understandings that need further practice, refinement, or reformulation (Hugh, 1989).

Informal assessment occurs in several ways. Homework is, in general, not collected. Students are instructed to check the validity of their efforts using methods suitable to the work (Doty, 1995). According to Doty (1995), for assignments that involve questions in the text, they are to check all answers that appear in the back of the book and rework incorrect exercises. There are many students who require continual monitoring to get them to take advantage of these answers. For skills studied in the text, students are required to come to class with specific questions, knowing what difficulties they have or have not encountered (Doty, 1995).

There is considerable variety in the design of class activities and major assignments. Class time can be devoted to discussion, computer labs, investigations of a question, time for groups to work together on a project, or peer evaluations. Homework is a mixture of one-day readings or text exercises with long-term problem sets, research papers, group projects, and experiments. The variety and length of assignments can be stressful in their newness, but also contribute to helping the students accept the goals of the course (Doty, 1995). Timed-tests are by their nature stressful as is the search for some particular right answer or approach to a problem that one may think should be thought of differently. In contrast, open-ended problems may permit multiple means of solution, multiple interpretations and answers, or even multiple new questions. It is this latter habit of creatively modifying and extending problems so that a single problem becomes an area of investigation that is cited as a pinnacle of achievement for the students (Doty, 1995).

4.3 Resources: Information and research

Textbooks frequently determine the content, sequence, and methods of the course with which they are associated (Hugh, 1989). The fatal flaw for most texts is that, in their attempt to be both curriculum and teacher, they necessarily have to explain everything to try to avoid confusion. This incessant explaining preempts conjecturing and discourages the asking of interesting questions. Even for inquiry-based texts, the physical presence of a bound sequence of investigations sends a message that student ideas need to follow a particular path. It can still be helpful to have a traditional text handy as a reference for students (Hugh, 1989).

Brown (1992) states that almost every assignment the students submit throughout the year is completed “open-book.” Thus, they need to learn how to find information in their text, in the library, and on-line as appropriate. This use of resources moves their focus from memorization to the identification of what they do not know. Then they must figure out what parts of what they need to know might be available in the literature and how to obtain that information. Researching needed information includes a range of tasks that require persistence and creativity (Brown, 1992).

Mathematical modeling is inherently interdisciplinary. Modeling endeavors require the modeler to combine their mathematical expertise with knowledge from one or more other fields. Frequently students have to become "instant experts" in their area of interest in order to craft an effective mathematical representation (Logan, 1993). Thus, as they are developing as mathematicians, they are also learning how to be independent learners in all fields. The motivation to do this work arises from the fact that they are answering questions that they themselves have posed. The diversity of interdisciplinary connections made is inspiring. Student problems are drawn from the realms of physics, political science, sports, education, public policy, economics, religion, biology, architecture, and even gastronomy (Gatto, 1992).

The most important resource that the students have is each other. They stimulate each other’s interest in the discipline and in the challenge of problem solving. They are encouraged to work on homework and many of the major assignments in groups and class activities are usually cooperative in design. Students report a considerable amount of spontaneous peer tutoring during study times and by so doing, they will be embracing constructivism (Gatto, 1992).

4.4 Affect and persistence

Brown (1992) argue that the habits of problem posing, creating representations, explaining connections, and testing and checking are central to the development of interesting new mathematics and applications. Students need to see these habits as worthwhile activities. Real world applications often involve many variables, incomplete information, and multiple methods of solution and answers that vary according to the assumptions and simplifications made and approach taken (Brown, 1992). Gatto (1992) indicates further that encounters with such settings dispel students' notion that the trademark of mathematics is the exactness and uniqueness of results. Rather, recognition of underlying structure and abstraction become dominant features of the discipline. Students must accept that creativity and clear communication are part of active learning and discovery. Successful students in traditional math courses are rewarded for speed and technical accuracy. A different type of confidence is required when they begin posing problems with no immediately clear method of solution and no guarantee that a solution can be found (Gatto, 1992).

Realizing goals for students to engage in lengthy, complex projects, to take risks and grapple with deep ideas requires persistence and a tolerance for frustration. Students (and adults) can become paralyzed, depressed, or antagonistic when faced with open-ended, incompletely defined tasks (Gatto, 1992). An example is the quote below;

> *It may be that when we no longer know what to do, we have come to our real work, and that when we no longer know which way to go, we have begun our real journey. The mind that is not baffled is not employed. The impeded stream is the one that sings. - Wendell Berry*

Many students interpret the above quote as a message that they are being left to their own devices. Students are much more open to meeting unsettling expectations if they, in turn, have some control over how the course is structured (Logan, 1993). Students are understandably fearful of failure. Students can also receive an extension when they have worked hard on a problem and want to continue their investigation provided the extension is requested in advance. They are allowed to redo efforts that were not successful to see if they can perfect a skill or discover a solution to an unsolved effort (Healy, 1993).

Healy (1993) also argues that the development of persistence in both individuals and a class requires careful planning. It is important that students have some early successes, but these need not always be immediate or complete. Challenges of greater length and difficulty should be introduced gradually and students need to have most of the skills and understandings required to solve a problem. Students are not only frustrated by the difficulty of the work (Doty, 1995). Modeling introduces some students for the first time to the subjective nature of mathematics used in context. The possibility of multiple interpretations in a field heralded for its precision and lack of bias can be unsettling. Work on projects follows rhythms that are not present in shorter exercises and to which it takes time to adapt. Both teacher and student need to be patient with these changes (Blais, 1988).

4.5 Understanding and supporting group efforts

Collaboration, when effective, is capable of producing outcomes that are more interesting, creative, and thorough checking each other's assumptions and computations than any of the group's members could produce on their own. Group work requires the development of listening and communication skills that have importance outside of formally structured group endeavors. It facilitates the sharing of ideas and the gaining of different perspectives. It brings together the different backgrounds, and therefore different strengths, of the group members leading to a greater appreciation for each other's talents. It is more fun than working individually.

Group work exposes students intimately to the ideas and approaches of others. In small group settings, students are more likely to be really listening and less likely to be distracted than during whole group discussions. Quieter students who hesitate to participate in class become more involved and everyone is more willing to experiment, take risks, make mistakes, and analyze their reasoning.

There is no one best way to create groups. They can be formed based on which research question each student wants to explore. Students can be allowed input into the formation of groups. Groups can be formed randomly. Avoid attempts at tracking students into groups that are all supposedly of the same or different ability. There are too many important skills problem posing, representation, writing, organization that proof that contribute to research for there to be one tidy ranking within a class.

Research efforts are worthwhile, in part; because they make apparent to the students that mathematics is not primarily a test of computational accuracy and that they each have a range of skills that they can bring to meaningful mathematics explorations. A well-balanced group will give each student a chance to be the expert at some task. A group that uniformly lacks a skill such as using technology will be forced to work, as a group, to gain some mastery in that area.

4.6 Constructivism

Constructivism holds that learning is essentially active. Constructivists argue that by definition, a person who is truly passive is incapable of learning. In constructivist learning, each individual structures his or her own knowledge of the world into a unique pattern, connecting each new fact, experience or understanding in a subjective way that binds the individual into rational and meaningful relationships to the wider world (Wilson and Daviss, 1994).

As scientists study learning, they are realizing that a constructivist model reflects their best understanding of the brain's natural way of making sense of the world (Logan, 1993). This is in total contrast to the behaviorist model that dominated learning theory in the late 19th and early 20th centuries – that is, "people expected rewards to do tasks, their brains were blank sheets awaiting instruction and intelligence was innate and largely inherited" (Abbot and Ryan, 2011). "Constructivism is not only an open-ended form of learning; it is essentially about reality, connectivity and the search for purpose" (Abbot and Ryan, 2011). Growing evidence suggests that a constructivist form of learning aligns with brain based learning. Brain-based learning stresses the importance of patterning, that is, the fact that the brain does not easily learn things that are not logical or have no meaning. Because our natural tendency is to integrate information, we resist learning isolated bits of information. Because the specifics of instruction are always tied to larger understandings and purposes, teachers and students should use stories and complex themes and metaphors to link information and understanding – and ICT should be integrated into all these types of work (Abbot and Ryan, 2011).

Brain-based learning also stresses the principle that the brain is a parallel processor – it performs many functions simultaneously. Therefore, all meaningful learning is complex and non-linear. This means that teachers must use all available resources - including community resources to orchestrate dynamic learning environments (Abbot and Ryan, 2011). Teachers ought to overcome the natural preference for conveying information tied to clear directions and opportunities for students to "do it right" rather than to explore and experiment.

Conclusion

This chapter covers information on model of aesthetical mathematics pedagogy. It defines mathematical modeling and discusses pedagogy of modeling which include assessment, resource information, affect and persistence, understanding and supporting group efforts and constructivism. The next chapter which is chapter five covers summary, key findings and recommendations.

CHAPTER FIVE

SUMMARY, CONCLUSION AND RECOMMENDATION

5.0 SUMMARY CONCLUSION

Chapter One defines aesthetic mathematics and the attendant key topics. It gives a background of the aesthetic mathematics thus placing it in context and providing the statement of the problem we hope to dissipate. It then gives a purpose and significance, looks at the aesthetic mathematics in light of a conceptual framework, provides a literature review, limitations of the study, scope, methodology and organization of the study. Chapter Two provides a concept of aesthetical mathematics pedagogy. It also looks at issues of aesthetical mathematics pedagogy and the barriers to effective aesthetical mathematics pedagogy. It closes with the conclusion from an academic point. Chapter Three identifies aesthetic features of mathematics. It looks at features which include elegance, exemplification, beauty and wonder, symmetry and philosophy of proof. The chapter covers the conclusion for the study. Chapter Four covers development of a model of aesthetical mathematics pedagogy. Chapter five provides a summary, key finding from the study, recommendations and provides scholarly suggestions on areas for further study.

5.1 Key findings

The analysis teases out what it can mean for a teacher to be compelled by and passionate about the subject and students engaging with the subject, to have a coherent and unified sense of what the subject is about and how to bring it to life for students, and to be transformed by what they know and believe in a way that aligns them to personally and professionally identify with the subject. The teachers' construction of the subject, their students and teaching is not simply cognitive but has an aesthetic dimension. An implication of this is that teachers who teach outside of their subject area may be lacking an appreciative aesthetic understanding. Their aesthetic response to the content matter and how to teach it may be unlike that of someone who has an appreciative aesthetic understanding of the subject. The findings of this study is that such teachers may: attempt to bring in a style appropriate for a subject that has a different set of demands; have a limited set of experiences with relevant phenomena, processes, ways of thinking and attitudes that can feed into their teaching; and fail to exhibit a passion for the subject and what the subject can do for their students. Consequently, the study reveals that any efforts to improve mathematics and science education should be aware that allowing teachers to experience the subject in a way that results in aesthetic appreciation for the beauty and elegance of mathematics and science is just as valuable as them developing conceptual and pedagogical knowledge associated with the subject. A teacher may then experience content in ways that allow them to more clearly see themselves in relation to subject matter ideas.

The study further reveals that the teaching attributes outlined above are what keep teachers grounded in their day-today dealings with students. However, good teachers also appreciate the value and power of research by colleagues at all levels in the educational field to broaden perspectives and enhance teaching practice. They can exchange ideas and knowledge about teaching and learning to the benefit of their students. In so doing, they become confident users of shared language and understandings associated with all aspects of pedagogy. Despite what is seen by some as educational jargon, many teachers enjoy talking the ‘teacher talk’ or a ‘professional parlance’ about what they do. A discussion about what is educationally appropriate for their students and their learning isn’t ‘dumped down’. There is common ground when speaking to colleagues at all educational levels, whether from colleges and universities or pre-schools and middle schools. It should be as much a code for professional acceptance and credibility as it is for other professional colleagues in law, medicine and other tertiary fields of endeavor.

5.2 Recommendation

There is a significant number of learning and teaching methods which work well in vocational education. These should be mapped against those desirable outcomes for which they are most suited, and those vocational subjects for which they are best matched.

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