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Strategic Asset Allocation of Pension Funds; An Application of Markowitz Portfolio Theory

Research Report in Mathematics, Number 20, 2018

Obed Mokaya Menjeri

August 2018

Submitted to the School of Mathematics in partial fulfilment for a degree in Master of Science in Actuarial Science

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Master of Science Project

Submitted to the School of Mathematics in partial fulfilment for a degree in Master of Science in Actuarial Science

Prepared for The Director
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Monitored by School of Mathematics

Abstract

The management of pension funds is a very sensitive and important aspect that has a bearing on the quality of life given by retirees when they come to the end of their useful work life. Therefore their risk management is very crucial. The financial products n which the pension assets are invested in have different levels of risk that investment managers are loo of. Their aim is therefore to maximize the returns for their members while taking the minimum possible risk with their resources. Markowitz provided a solution to this problem through the mean-variance model, which has been critically analyzed and used to various investment portfolios. This study was therefore aimed at finding an optimal way of allocating the pension funds keeping in mind their risk characteristics. The results from the analysis done showed tremendous improvement in terms of efficiency in the allocation of the pension assets to various investment opportunities. The optimal restricted portfolio gave us a return of 9.47% while the unrestricted one gave us 13.45%. this came with a standard deviation of 10.45% and 13.74% respectively. Therefore, the investor can be able to invest 62% and 60% of the restricted and unrestricted portfolios respectively in the risky portfolio and 38% and 40% respectively in the risk free asset. From the data used for this study, this will give the best returns.

This thesis also explores the improvements that can be made to index funds by removing the link between pricing errors and portfolio weights and compared their performance with that of actively managed funds. Index funds today tend to overweight over valued companies, leading to serious performance lags especially during pricing bubbles. The Markowitz optimization formulation is used in combination with the fundamental metrics and the weightings from the solutions to the mean variance optimizations were used to calculate the expected returns. The Sharpe ratio for the S&P 500 was 0.07 and that of the Mean Variance Optimization was 0.57. The results clearly demonstrate that the MVO outperforms the S&P 500. By using fundamental metrics such as P/E ratio, sales, book value and dividends to evaluate the size of a company rather than the traditional market capitalization, significant improvements can be made to the value of a portfolio.

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Declaration and Approval

I the undersigned declare that this project report is my original work and to the best of my knowledge, it has not been submitted in support of an award of a degree in any other university or institution of learning.

Signature　　　　Date

Obed Mokaya Menjeri
Reg No. I56/5296/2017

In my capacity as a supervisor of the candidate, I certify that this report has my approval for submission.

Signature　　　　Date

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Dedication

To my loving parents David Menjeri and Susana Nyakoboke, my brother Seth Omwansa and and Sister Ruth Osebe for their unending and patient support. They have made me who I am today.

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Acknowledgments

I would like to thank the almighty God for sustenance and good health throughout the period of this study. I would also like to recognize the efforts of my university supervisor Prof. Richard Simwa who tirelessly guided me through the study and kept e on track for the entire period of the research. He always showed interest and dedication in my chosen area of study.

Obed Menjeri Mokaya

Nairobi, 2017.

1 Introduction

Strategies geared towards investment have over the years been used to come up with classes under which individuals and organizations can invest their monies so as to obtain optimal results. This is the case wit pension funds. These pension funds are left in the hands of investment managers who are believed to have the expertise to make investments on behalf of members of the individual schemes. This is in a bid to meet investment expectations of the members of these schemes. These investment managers are employed to minimize the risk of losing investments as done by the individual members depending on their risk appetites. These investors are full aware of the need to diversify in the choice of their investment decisions (Franzen, 2010). In order to be able to meet the future obligations of these members within the short term and the long term, the pension funds make their investments in capital markets. These pension funds are usually the backbone of long-term components of domestic capital (Economou & Manola 2013). These funds are initially invested in treasury bills and upon maturing they are invested in other areas where they realize good returns depending on the expected risks in those investment areas (Inderst & Stewart, 2014)

Its in this regard that there has been a significant growth of pension assets with respect to their investments. Pension funds do well in stock markets in terms of their performance and despite the differences in estimations, 20% of pension funds in developed countries are invested in hedge funds, and in some cases this goes to a high of 40%. Some pension funds are also invested in social projects geared towards improving social security like achieving better housing affordable energy. These projects increase returns financially and also improve social amenities for various communities.

Returns are usually maximized for given levels of risk and also risks minimized for given levels of return. One cannot achieve both at the same time. However, there is usually a challenge on the availability of information that can be used to assess the levels of return and risk. This is the challenge mostly in developing countries. The inadequacy of information with respect to these levels of risk and return make it very difficult to predict with precision the desired combinations of risk and return that can be used to maximize the investments of pension assets. Our main aim is to look for alternatives to prediction of these levels of risk ad return to be able to optimize the returns realized from investing pension assets in the various categories of investments in the capital markets.

1.1 Background of theStudy

Pension funds are primarily mandated to provide retirement benefits to members of the respective pension schemes so that these members can be able to meet their financial obligations especially in retirement or after their useful working life. A document called the trust deed and rules of the pension scheme is used to define the extent of risk or the appetite of risk a particular combination of a pension scheme can be able to take. Therefore the asset allocation as done by the pension fund manager has a bearing on the results achieved from the investment activities. Trustees of pension funds are the ones mandated with the function of carrying out investment decisions, a role they play through the fund managers. This is under the guidance of an investment policy statement, which is a document prepared by an actuary to guide investments of a particular pension scheme with respect to its risk characteristics. The risk characteristics are determined by the composition of the members in term of their ages and the money requirements of the scheme. This ensures good performance of the financial assets and availability of funds whenever they are needed by embers of a particular scheme.

The asset allocations as done by trustees in Kenya should conform and comply with the requirements as set out by the Retirement Benefits Authority of Kenya (RBA). These rules are used in governing investments and asset allocation decisions depending on the structure of the scheme undertaking the investments. The scheme structure can be a defined contribution one or a defined benefits scheme. There has been a growing need to provide good retirement packages to employees exiting employment hence the need for sound principles when it comes to asset allocation.

1.2 Statement of the problem

The financial performance of pension schemes in Kenya is very critical and hence the pension schemes should be able to meet their financial obligations to their members. Therefore of importance is how the pension assets are being managed in order to meet the financial obligations that they owe to their members. The asset classes that can be invested in include shares/equities, real estate/property market, fixed income deposits, money market, private investments and venture capital investments.

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Studies have been carried out locally on portfolio allocation but despite these studies on the performance of pension funds, very few have labored to explain in depth the direct effects of chocice of asset classes on the financial performance of these pension funds. This study therefore seeks to address this occasioned gap in research by addressing the research question; Can financial performance of pension funds be increased by optimal asset allocation?

1.3 General Objective

To investigate the Asset Allocation of pension funds using the Markowitz approach.

1.4 Specific objectives

1. To understand the mean-variance relationship and their implications in investment of a fund.
2. To understand the analysis and metrics used in assessing the financial performance of assets.
3. To establish portfolio management strategies to be employed by fund managers.
4. To assess asset allocation effects on financial performance of pension funds using the Markowitz approach.

1.5 Research question

Can an investor achieve a higher return on investment by choosing an optimal portfolio?

1.6 Justification

This study will help Board of Trustees of Pension schemes to know the extent to which regulations on various asset classes have an effect on the performance of their funds. The findings of this study are expected to contribute towards coming up with strategies that can guide investments of pension assets and advice the regulator on optimal limits of asset classes as guides for pension schemes investment categories.

The study is also aimed at assisting the trustees of pension schemes to know the asset classes that have the greatest influence pension schemes performance. The findings of this study will be helpful in increasing the existing pool of knowledge that can provide good ground to carry on more research in Kenya.

2 Literature Review

2.1 Introduction

Asset allocation strategies as developed by modern theories cannot be downplayed by investors seeking to increase their asset allocation strategies. The portfolio theory presents a unique sphere where investors needs and intuitions are met and sound decisions on optimal asset allocations are made. The investors are assumed to have made decision both short term and long term that can potentially improve their returns generally.

As at present, the modern portfolio theory has proposed useful solutions to the problem of maximizing returns in the optimal portfolio selection. Very important steps have been made by the Capital Asset Pricing Model (CAPM) derived by (Sharpe, 1964), the Arbitrage Pricing Theory of Ross (1976), the intertemporal CAPM by Merton (1973), Lintner (1965), Mossin (1966) and (Markowitz H., 1959).

Important research results have been achieved as regards the discussions in this project. The above authors have each developed various theories that support the idea of maximization of returns through the use of the modern portfolio theory. The foundations of this theory were established by Harry Markowitz in 1952 when he was writing his doctoral dissertation in the field of statistics. He emphasized the impact of portfolio diversification and the covariance relationships between securities that are within a portfolio. His findings that were titled *Portfolio Selection* were first published in the Journal of Finance. With time, they were expanded largely with the publication of his book Portfolio Selection: Efficient Diversification (Markowitz H.,1959). About 30 years later, Markowitz won a nobel pprice for his contributions on MPT in the field of Corporate Finance and Economics.

2.2 The Modern Portfolio Theory

The Modern Portfolio Theory is an extension on the Portfolio Selection Theory as developed by Markowitz was first introduced in the year 1952. These theory received tremendous support and research by William Sharpe whose contributions were primarily on the financial asset price formation which was later introduced in the year 1964. This came to be known as the Capital Asset Pricing Theory (CAPM) (Veneeva, 2006). The modern portfolio theory is an investment framework used for selection and construction of investment portfolios which are based on maximizing the expected returns of the portfolio and subsequent minimizing of the risk involved in the investment. (Fabozzi, Gupta, & Markowitz, 2002).

Generally, the component of risk in the Modern Portfolio Theory can be measured using the various formulations mathematically and it can also be reduced through the concept of diversification which primarily is aimed at selecting the proper weighted collectionof the assets that have been invested to yield lower risk factors that could be achieved in investing in individual asset classes or singular asset mixes. The wisdom of the modern portfolio theory is such that it subscribes to the saying of never putting all the eggs in one basket (Fabozzi, Gupta, & Markowitz, 2002; McClure, 2010; Veneeya, 2006).

The concentration made on Markowitz's Portfolio Selecttion Theory are for puroses of this writing. Going forward, these contributions will be referred to as the collective modern pportfolio theory. They are in other spheres also known as the mean variance analysis. The term mean is used interchangeably with the term average or expected return. The variance here denotes the risk involved.

Markowitz showed that given certain cnditions, portfolio selection as done by a particular investor can be reduced by striking a balnce between two dimentions: (Royal Swedish Academy of Sciences, 1990)

1. The portfolio expected return and;
2. The variance/risk of a particular portfolio.

Because of the risk that can be reduced by diversification, the portfolio investment risk depends on both the variances of the individual asset returns and the covariances of the pairs of assets (McClure, 2010). This means that portfolio selection should be based on the average risk-reward characteristics and not simply by putting together portfolios with securities with individually attractive risk-rewward characteristics. The important modern portfolio terms are discussed in the pages that follow.

2.3 Discussions and Assumptions of the Modern Portfolio Theory

The modern portfolio theory has a number of assumptions with regards to markets behavior and investors decisions. The assumptions are both explicit and implicit and are follows; (Bofah, n; Wecker, d.; Markowitz, 1952):

- All investors are rational as they seek to maximize their returns while at the same time minimizing the risk
- Investors can only take higher amounts of risk if they are compensated by higher returns.
- The investors receive all relevant information in good time to make their investment decisions.
- The investors can lend and borrow an unlimited amount of capital at a risk free rate of interest.
- The efficiency of the markets is perfect
- There are no transaction/hidden costs
- Securities whose individual performance is independent of other portfolio investments can be selected.

These fundamental assumptions of the modern portfolio theory have however been challenged by a good number of scholars. Many its challenges are discussed in this study and opinions of various critics weighed

2.4 Portfolio risk and Return as presented by MPT

By definition, financial risk is any deviation from the expected historical returns during a given period of time (Bofah, n.d.; McClure, 2010). In his theory, Markowitz states that, "the essential aspect pertaining to the risk of an asset is not the risk of each asset in isolation, but the contribution of each asset to the risk of the aggregate portfolio". Therefore, in analysis of security, the following options can be taken;

1. Use of a stand alone basis that is usually considered in isolation
2. Use of a portfolio basis in which assets in consideration represent one of the group of many assets.

Therefore we can divide a security into two components when looking at it from the context of a portfolio. The component whose risk can be diversified and the one whose risk cannot be diversified. Th one with diversifiable risk is called unsystematic risk and the other one with undiversifiable risk is called systematic risk. This theory presented by Markowitz assumes that these risks are always common to all the portfolios.

2.5 Conclusion

The topic on Mean Portfolio Theory has been critically analyzed and the methodology used included an extensive literature review n the same and other related topics. The current and past reviews were included in this analysis and both present and past economic theorists' views were compared. Benniga was very useful in providing the data used for this literary works. His ideas and suggetsions especially on the application of Microsoft excel to numerous statistical computations of the mean portfolio theory are invaluable. They were modified, verified and tested against a number of previously proven mathematical models. He uses complicated mathematical models but regardless of this shortcomings, he has been able to synchronize well with the mean portfolio theory which now forms the backbone of the financial theory and practice used in the current world.

The mean portfolio theory inferences that the market is very difficult to beat given that all relevant information is known and available and that all those who succeed in doing so are those that can effectively diversify their investment portfolios and take higher risks in anticipation of higher returns. Its therefore important to note that this modern portfolio theory is just a tool for use in the financial markets in establishing the relationship between the risk involved in investments and the anticipated returns. Its application is so profound that it will be used both presently and in the future. It is unlikely that its importance will be faced off in the near future. The MPT has since led to the development of other theories in theoretical analysis of the field of portfolio theory. It is however subject to continuous criticizing and improvement.

3 Methodology

3.1 Introduction

The theoretical bit and the various applications of the Modern Portfolio Theory will be introduced here. The aspects of the various combinations of diversification will be discussed and the notion of not putting all eggs into one basket, which predates the economic theory will be explained and applied.

As we advance the application of Markowitz' portfolio theory, it will be shown that when you continuously add some assets into the investment portfolio of your choice, the ultimate total risk of that portfolio as determined by its variance or the standard deviation of that total return will reduce or decline steadily. Its important here to note that the expected return of this particular portfolio is a weighted average of the individual assets that form part of the portfolio. This mans that if investors were to invest in portfolios as opposed to individual assets, they could significantly lower the total risk of their investments and this will not have a bearing on the return expected.

3.2 The Markowitz Portfolio-Selection Model

Harry Markowitz in the year 1952 through to 1959 developed a portfolio selection model that later developed and earned the name modern portfolio theory as currently known. This model's primary focus was on the returns that were being generated by the various investment opportunities that investors were interested in. The practice was majorly identifying the securities that gave the best returns with the minimum risk and then come up with a portfolio from these assets. However, the modern portfolio theory retained the emphasis on return, but laid some equal level importance on the risk factor. Markowitz was therefore the pioneer who showed that the variance of a portfolio can be reduced through diversification. He suggested that investors can focus their portfolio selection on the basis of their expectation in terms of the overall risk and reward characteristics as opposed to putting together portfolios that have individual securities with good risk and return characteristics.

When no amount of diversification can reduce the risk associated with a portfolio for a given level of reward expected we have the Markowitz portfolio model. It is characterized by the following:

1. Estimation of distribution of returns over a given time frame by investors

2. Maximization of utility by investors through a single period utility function
3. Possible variation in terms of the values of reward expected by the individual investors
4. The only important thing to the investors is the mean and variance of their portfolios over a given period of time
5. The first two moments of the pprobability distribution of the return's varance and expected value are used to measure and determine the expected return and associated risk.
6. While reward is desired, risk is avoided at all cost.
7. There is no friction in the financial markets

3.3 Index Funds

3.3.1 Introduction

Every year, millions of dollars are invested in index funds by institutions and individuals looking for a safe, reliable and simple investment strategy. Index funds are an investment vehicle designed to track the movement of the stock market by holding a representative basket of equities. People oflen choose index funds over actively managing a portfolio because they require less work and are considered secure. Even so, a large proportion of investors do not truly understand how an index is constructed and the inherent weaknesses of a market cap weighted portfolio. They know even less about the potential alternatives available to them and the impact that this could have on their investments Investors tend to have too much faith in the free market pricing mechanism; they never stop to contemplate alternative tactics that can account for fluctuations and instability in stock prices. Once you start examining some of the fundamental tenets that aKect the construction of indices, it becomes very clear that current index funds suKer from performance limitations.

3.3.2 Traditional Investment Strategies

Equity investing is the practice of buying, selling and holding publicly traded stocks in order to generate a profit. Investors generally have quite a few options when it comes to selecting an investment strategy. One of the first decisions they have to make is whether to manage their funds on their own or to entrust the work to professionals. Afler that has been resolved, they must then select between two basic approaches, active or passive management.

On the other side of the spectrum, adherents of passive management are convinced that stock price is a reasonable estimate of a company's value and so the best way to invest is through an aggregate index that tracks the market because there is no way to beat it.

3.3.3 Active Management

Practitioners of active management believe that they are able to outperform the market by distinguishing between undervalued and overvalued stocks. Their main strategy is to conduct detailed analyses to discover which companies will have higher growth than expected to invest in and short sell companies that they project to lose money. Active management involves constantly updating a portfolio by buying, selling and/or holding the stocks of companies. These additions and deletions can be made as many as multiple times in one day for day traders and less for those interested in more stable, long term gains.

3.3.4 Passive Management

Managing a portfolio passively implies that the investor has little if anything to do. Changes to the portfolio should be infrequent and predictable. Index funds are the investment vehicle of choice for those who innately believe in the pricing mechanism of the free market. The assumption here is that pprice is related to the fair value. With this belief, it becomes impossible to buy low and sell high, since the true worth of a company is exactly its market price. It wouldn't mean anything to select undervalued stocks anymore because by definition, stocks are always perfectly valued.

Stock market indices attempt to mirror a given market by holding a representative collection of assets. The traditional method in constructing indices simply takes the largest companies by market capitalization in any market. Each asset's weight in the fund is calculated based on its capitalization (Ferri).

Individual investors can purchase an index fund through Exchange Traded Funds (ETFs). An ETF is similar to a publicly traded stock in that shares can be bought, sold, traded and held on the market (Index ETFS – Know Your Funds). ETFs allow investors to own a collective basket of assets without having to individually purchase each item.

3.3.5 Advantages of Index Funds

There are many advantages for investing in index funds over an actively managed fund, including ease of use and low costs. Picking an index fund is very straightforward. Since they are all based on the same market indices, there is very little difference between a well managed and a poorly managed index. Index fund managers do not need any special skills or abilities to predict the future performance of their assets. Turnover is low because the policies and guidelines are constant, which means that assets rarely get added or removed. This results in significantly lower costs involved in operating and managing an index fund.

3.3.6 Motivation

Because of the direct relationship between price and the allocation of funds in an index, any pricing errors affect portfolio performance. For this exact reason cap-weighted index funds tend to over weigh overvalued companies and under weigh undervalued companies. If these pricing errors are significant enough, the index will no longer be an accurate measure of the market. By constructing a portfolio in such a way, you must have a lot of faith in the pricing mechanism of the free market. If the market is wrong about the future value of a company and mistakenly overprices its shares, this will create a bias in the index and inevitably when price self corrects, the value of the entire portfolio will drop.

4 Data Analysis and Results

4.1 Calculation of the efficient frontier

A number of software packages can be used to come up with the efficient frontier. Here we will use the common tool of Microsoft excel.

4.2 Microsoft excel

Microsoft excel is one of the programs we can use for data analysis in terms of generating the efficient frontier but is limited when it comes to the number of assets it can be able to handle. The solution here is therefore to work with a simple tool used for portfolio optimization which can be used to illustrate vividly how the calculations can be done in a simpler way as opposed to the sophisticated programs. It can be confidently said that doing the computations of portfolio optimization in excel is quite easy.

For instance, if a pension fund manager forms a stock index combination of six stocks. This stock therefore consists of six stock indices for various countries say United States, UK, Switzerland, Hongkong, Korea and Singapore. Their indices are S&P500, FTSE100, Swiss Market Index – SMI, Hang Seng Index – HIS, Korea Composite Stock Price Index – KOSPI, and Straits Times Index – STI.

Monthly prices from January 1990 to December 2006 from all the six stock indices was used in the data analysis. From this data we can use excel to determine the optimal allocations for these assets/indices.

From our discussions above we can now employ the Markowitz portfolio selection theory and see how the optimization problem can be solved. We can therefore divide it into three parts; one is the determination of the efficient frontier; two we come up with an optimal portfolio given the risky assets and a capital allocation line for a particular investor. Here we determine the point of tangency between the capital allocation line and the efficient fronntiier. Finally, we use what we come up with in two above to allocate the funds between the risk free asset and the risky portfolio. To do this we use the optimal complete portfolio

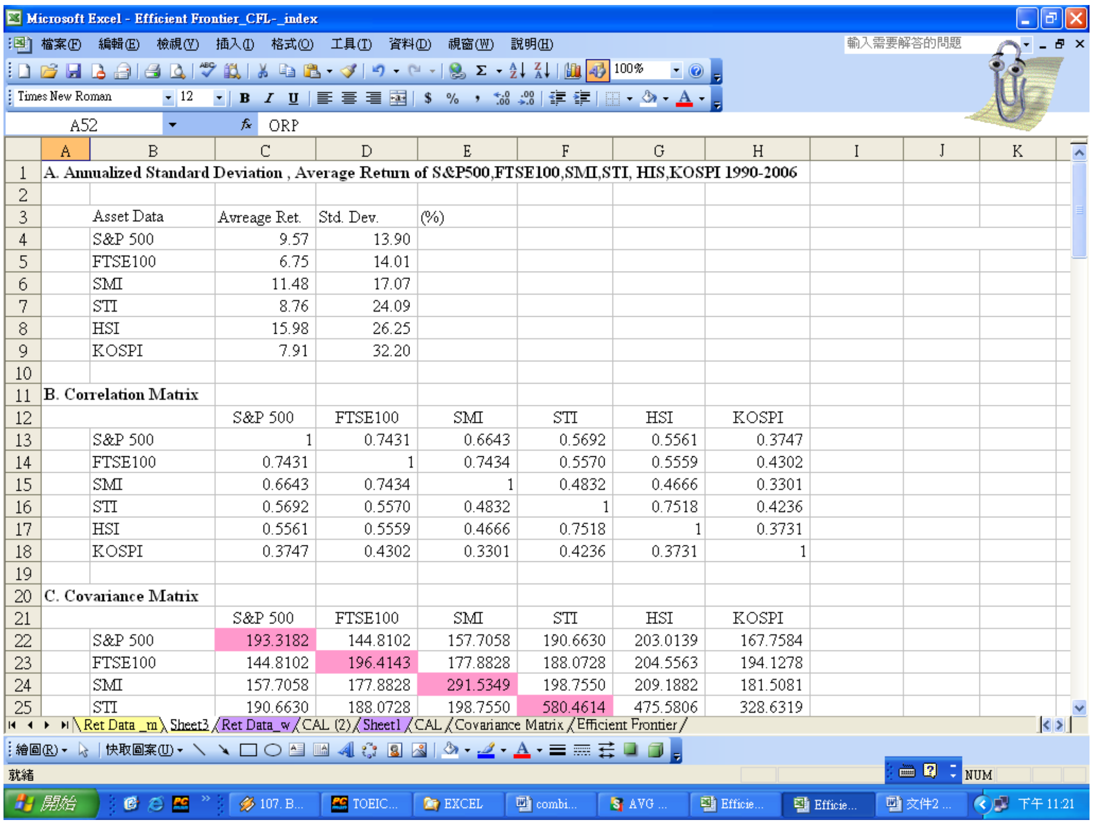
4.2.1 Determination of the Efficient Frontier

The first thing we do here is to determine the return expected from the investments, iits standard deviation and the covariance. From our excel formulae we can easily determine the expected return and the standard devation by applying the STDEV and Average under the functions tab of excel. We apply this to the historical data presented as percentages for the years of interest.

The tables below, table A and Table B are representing the average returns as determined from the historical data, their respective standard deviations and their correlations in terms of the rates of return gotten from the stock indices. Table A is put into the excel spreadsheet and then we obtain Table B which is a representation of the relationship $Cov(r_i, r_j) = \rho_{ij}\sigma_i\sigma_j$.

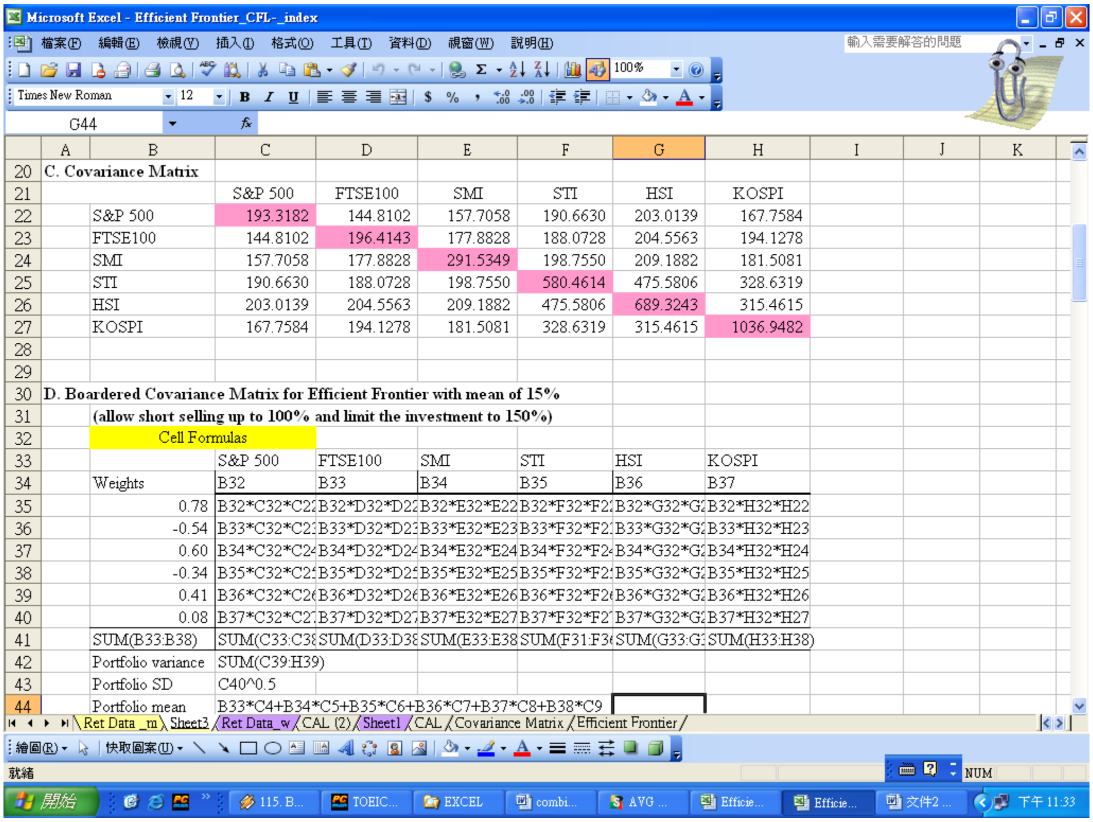
The tables below are on the performance of the stock indices. Their explanations are provided after the last table.

Figure 4.6. Annual Standard deviation and the average of the return and correlation matrix



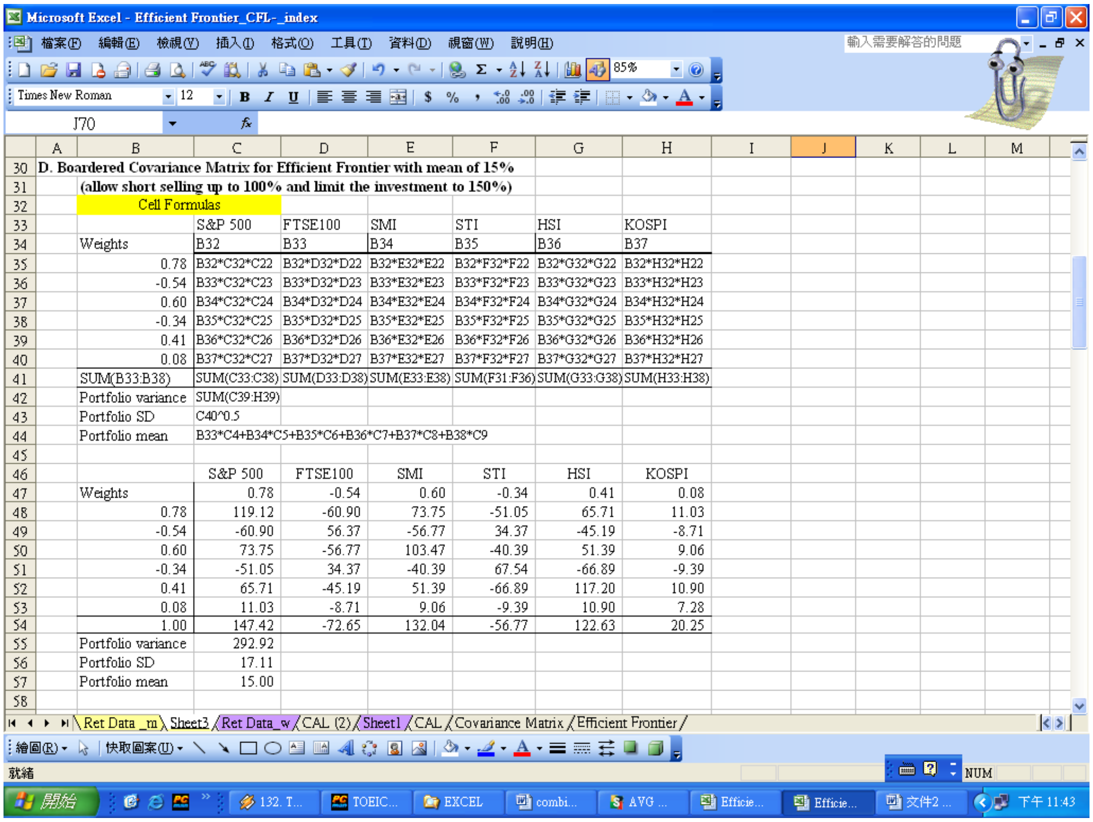
	A	B	C	D	E	F	G	H
1	A. Annualized Standard Deviation , Average Return of S&P500,FTSE100,SMI,STI, HIS,KOSPI 1990-2006							
2								
3		Asset Data	Avreage Ret.	Std. Dev.	(%)			
4		S&P 500	9.57	13.90				
5		FTSE100	6.75	14.01				
6		SMI	11.48	17.07				
7		STI	8.76	24.09				
8		HSI	15.98	26.25				
9		KOSPI	7.91	32.20				
10								
11	B. Correlation Matrix							
12			S&P 500	FTSE100	SMI	STI	HSI	KOSPI
13		S&P 500	1	0.7431	0.6643	0.5692	0.5561	0.3747
14		FTSE100	0.7431	1	0.7434	0.5570	0.5559	0.4302
15		SMI	0.6643	0.7434	1	0.4832	0.4666	0.3301
16		STI	0.5692	0.5570	0.4832	1	0.7518	0.4236
17		HSI	0.5561	0.5559	0.4666	0.7518	1	0.3731
18		KOSPI	0.3747	0.4302	0.3301	0.4236	0.3731	1
19								
20	C. Covariance Matrix							
21			S&P 500	FTSE100	SMI	STI	HSI	KOSPI
22		S&P 500	193.3182	144.8102	157.7058	190.6630	203.0139	167.7584
23		FTSE100	144.8102	196.4143	177.8828	188.0728	204.5563	194.1278
24		SMI	157.7058	177.8828	291.5349	198.7550	209.1882	181.5081
25		STI	190.6630	188.0728	198.7550	580.4614	475.5806	328.6319

Figure 4.7. The Covariance Matrix



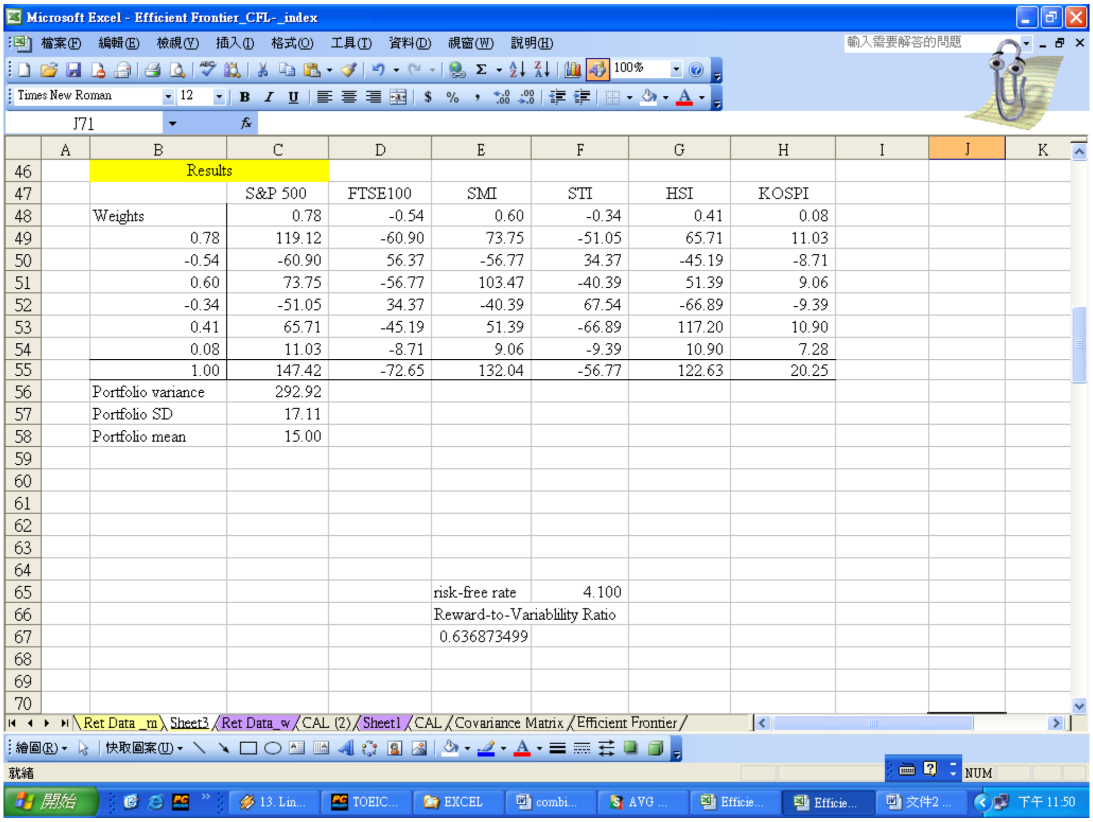
	A	B	C	D	E	F	G	H
20	C. Covariance Matrix							
21			S&P 500	FTSE100	SMI	STI	HSI	KOSPI
22		S&P 500	193.3182	144.8102	157.7058	190.6630	203.0139	167.7584
23		FTSE100	144.8102	196.4143	177.8828	188.0728	204.5563	194.1278
24		SMI	157.7058	177.8828	291.5349	198.7550	209.1882	181.5081
25		STI	190.6630	188.0728	198.7550	580.4614	475.5806	328.6319
26		HSI	203.0139	204.5563	209.1882	475.5806	689.3243	315.4615
27		KOSPI	167.7584	194.1278	181.5081	328.6319	315.4615	1036.9482
28								
29								
30	D. Boardered Covariance Matrix for Efficient Frontier with mean of 15%							
31		(allow short selling up to 100% and limit the investment to 150%)						
32		Cell Formulas						
33			S&P 500	FTSE100	SMI	STI	HSI	KOSPI
34		Weights	B32	B33	B34	B35	B36	B37
35		0.78	B32*C32*C2	B32*D32*D2	B32*E32*E22	B32*F32*F2	B32*G32*G	B32*H32*H22
36		-0.54	B33*C32*C2	B33*D32*D2	B33*E32*E23	B33*F32*F2	B33*G32*G	B33*H32*H23
37		0.60	B34*C32*C2	B34*D32*D2	B34*E32*E24	B34*F32*F2	B34*G32*G	B34*H32*H24
38		-0.34	B35*C32*C2	B35*D32*D2	B35*E32*E25	B35*F32*F2	B35*G32*G	B35*H32*H25
39		0.41	B36*C32*C2	B36*D32*D2	B36*E32*E26	B36*F32*F2	B36*G32*G	B36*H32*H26
40		0.08	B37*C32*C2	B37*D32*D2	B37*E32*E27	B37*F32*F2	B37*G32*G	B37*H32*H27
41		SUM(B33:B38)	SUM(C33:C3	SUM(D33:D3	SUM(E33:E38	SUM(F31:F3	SUM(G33:G	SUM(H33:H38)
42		Portfolio variance	SUM(C39:H39)					
43		Portfolio SD	C40^0.5					
44		Portfolio mean	B33*C4+B34*C5+B35*C6+B36*C7+B37*C8+B38*C9					

Figure 4.8. The Covariance Matrix for the efficient Frontier at a mean of 15%



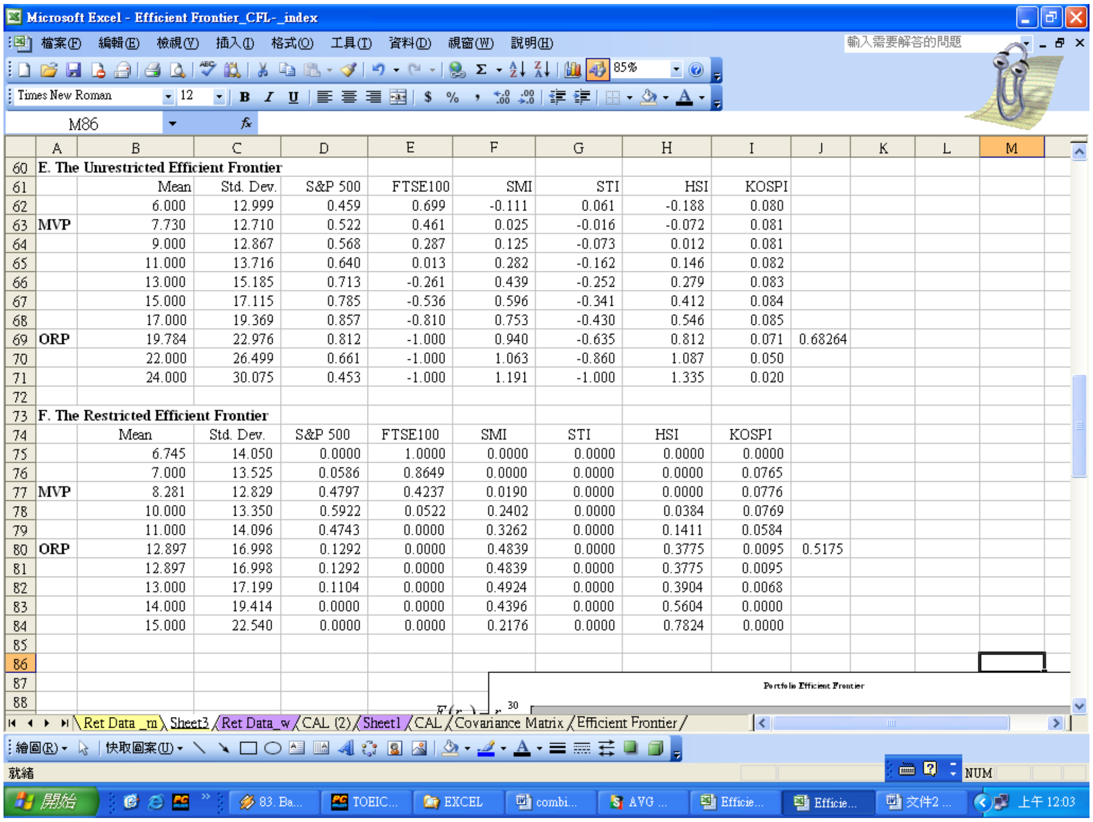
	A	B	C	D	E	F	G	H
30	D. Boardered Covariance Matrix for Efficient Frontier with mean of 15%							
31		(allow short selling up to 100% and limit the investment to 150%)						
32		Cell Formulas						
33			S&P 500	FTSE100	SMI	STI	HSI	KOSPI
34		Weights	B32	B33	B34	B35	B36	B37
35		0.78	B32*C32*C22	B32*D32*D22	B32*E32*E22	B32*F32*F22	B32*G32*G22	B32*H32*H22
36		-0.54	B33*C32*C23	B33*D32*D23	B33*E32*E23	B33*F32*F23	B33*G32*G23	B33*H32*H23
37		0.60	B34*C32*C24	B34*D32*D24	B34*E32*E24	B34*F32*F24	B34*G32*G24	B34*H32*H24
38		-0.34	B35*C32*C25	B35*D32*D25	B35*E32*E25	B35*F32*F25	B35*G32*G25	B35*H32*H25
39		0.41	B36*C32*C26	B36*D32*D26	B36*E32*E26	B36*F32*F26	B36*G32*G26	B36*H32*H26
40		0.08	B37*C32*C27	B37*D32*D27	B37*E32*E27	B37*F32*F27	B37*G32*G27	B37*H32*H27
41		SUM(B33:B38)	SUM(C33:C38)	SUM(D33:D38)	SUM(E33:E38)	SUM(F31:F36)	SUM(G33:G38)	SUM(H33:H38)
42		Portfolio variance	SUM(C39:H39)					
43		Portfolio SD	C40^0.5					
44		Portfolio mean	B33*C4+B34*C5+B35*C6+B36*C7+B37*C8+B38*C9					
45								
46			S&P 500	FTSE100	SMI	STI	HSI	KOSPI
47		Weights	0.78	-0.54	0.60	-0.34	0.41	0.08
48		0.78	119.12	-60.90	73.75	-51.05	65.71	11.03
49		-0.54	-60.90	56.37	-56.77	34.37	-45.19	-8.71
50		0.60	73.75	-56.77	103.47	-40.39	51.39	9.06
51		-0.34	-51.05	34.37	-40.39	67.54	-66.89	-9.39
52		0.41	65.71	-45.19	51.39	-66.89	117.20	10.90
53		0.08	11.03	-8.71	9.06	-9.39	10.90	7.28
54		1.00	147.42	-72.65	132.04	-56.77	122.63	20.25
55		Portfolio variance	292.92					
56		Portfolio SD	17.11					
57		Portfolio mean	15.00					
58								

Figure 4.9. Analysis on Weights



	B	C	D	E	F	G	H
46	Results						
47		S&P 500	FTSE100	SMI	STI	HSI	KOSPI
48	Weights	0.78	-0.54	0.60	-0.34	0.41	0.08
49	0.78	119.12	-60.90	73.75	-51.05	65.71	11.03
50	-0.54	-60.90	56.37	-56.77	34.37	-45.19	-8.71
51	0.60	73.75	-56.77	103.47	-40.39	51.39	9.06
52	-0.34	-51.05	34.37	-40.39	67.54	-66.89	-9.39
53	0.41	65.71	-45.19	51.39	-66.89	117.20	10.90
54	0.08	11.03	-8.71	9.06	-9.39	10.90	7.28
55	1.00	147.42	-72.65	132.04	-56.77	122.63	20.25
56	Portfolio variance	292.92					
57	Portfolio SD	17.11					
58	Portfolio mean	15.00					
65				risk-free rate	4.100		
66				Reward-to-Variablility Ratio			
67				0.636873499			

Figure 4.10. The Efficiient Frontier: Restricted and Unrestriicted



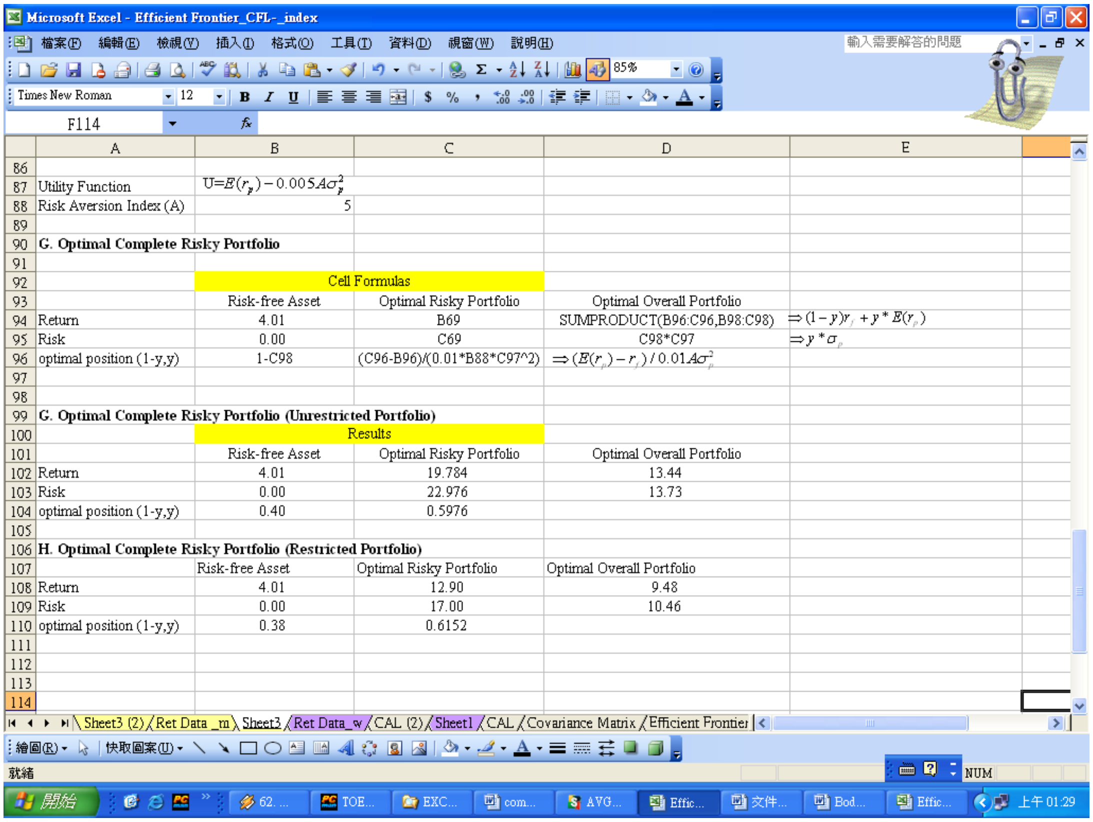
E. The Unrestricted Efficient Frontier

	Mean	Std. Dev.	S&P 500	FTSE100	SMI	STI	HSI	KOSPI	
	6.000	12.999	0.459	0.699	-0.111	0.061	-0.188	0.080	
MVP	7.730	12.710	0.522	0.461	0.025	-0.016	-0.072	0.081	
	9.000	12.867	0.568	0.287	0.125	-0.073	0.012	0.081	
	11.000	13.716	0.640	0.013	0.282	-0.162	0.146	0.082	
	13.000	15.185	0.713	-0.261	0.439	-0.252	0.279	0.083	
	15.000	17.115	0.785	-0.536	0.596	-0.341	0.412	0.084	
	17.000	19.369	0.857	-0.810	0.753	-0.430	0.546	0.085	
ORP	19.784	22.976	0.812	-1.000	0.940	-0.635	0.812	0.071	0.68264
	22.000	26.499	0.661	-1.000	1.063	-0.860	1.087	0.050	
	24.000	30.075	0.453	-1.000	1.191	-1.000	1.335	0.020	

F. The Restricted Efficient Frontier

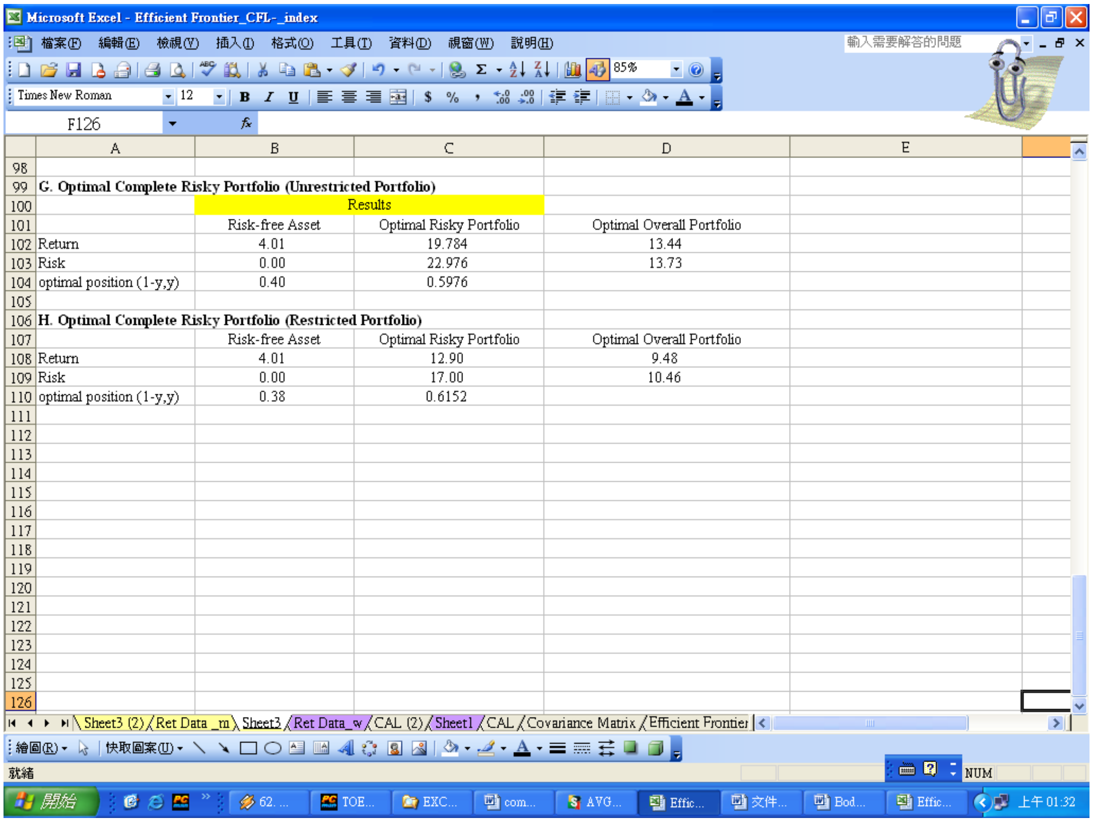
	Mean	Std. Dev.	S&P 500	FTSE100	SMI	STI	HSI	KOSPI	
	6.745	14.050	0.0000	1.0000	0.0000	0.0000	0.0000	0.0000	
	7.000	13.525	0.0586	0.8649	0.0000	0.0000	0.0000	0.0765	
MVP	8.281	12.829	0.4797	0.4237	0.0190	0.0000	0.0000	0.0776	
	10.000	13.350	0.5922	0.0522	0.2402	0.0000	0.0384	0.0769	
	11.000	14.096	0.4743	0.0000	0.3262	0.0000	0.1411	0.0584	
ORP	12.897	16.998	0.1292	0.0000	0.4839	0.0000	0.3775	0.0095	0.5175
	12.897	16.998	0.1292	0.0000	0.4839	0.0000	0.3775	0.0095	
	13.000	17.199	0.1104	0.0000	0.4924	0.0000	0.3904	0.0068	
	14.000	19.414	0.0000	0.0000	0.4396	0.0000	0.5604	0.0000	
	15.000	22.540	0.0000	0.0000	0.2176	0.0000	0.7824	0.0000	

Figure 4.11. The Optimal Complete Risky Portfolio



	A	B	C	D	E
86					
87	Utility Function	$U = E(r_p) - 0.005A\sigma_p^2$			
88	Risk Aversion Index (A)	5			
89					
90	**G. Optimal Complete Risky Portfolio**				
91					
92		Cell Formulas			
93		Risk-free Asset	Optimal Risky Portfolio	Optimal Overall Portfolio	
94	Return	4.01	B69	SUMPRODUCT(B96:C96,B98:C98)	$\Rightarrow (1-y)r_f + y * E(r_p)$
95	Risk	0.00	C69	C98*C97	$\Rightarrow y * \sigma_p$
96	optimal position (1-y,y)	1-C98	(C96-B96)/(0.01*B88*C97^2)	$\Rightarrow (E(r_p) - r_f)/0.01A\sigma_p^2$	
97					
98					
99	**G. Optimal Complete Risky Portfolio (Unrestricted Portfolio)**				
100		Results			
101		Risk-free Asset	Optimal Risky Portfolio	Optimal Overall Portfolio	
102	Return	4.01	19.784	13.44	
103	Risk	0.00	22.976	13.73	
104	optimal position (1-y,y)	0.40	0.5976		
105					
106	**H. Optimal Complete Risky Portfolio (Restricted Portfolio)**				
107		Risk-free Asset	Optimal Risky Portfolio	Optimal Overall Portfolio	
108	Return	4.01	12.90	9.48	
109	Risk	0.00	17.00	10.46	
110	optimal position (1-y,y)	0.38	0.6152		
111					
112					
113					
114					

Figure 4.12. Optimal Complete Risky Portfolio, Restricted and Unrestricted



	A	B	C	D
99	G. Optimal Complete Risky Portfolio (Unrestricted Portfolio)			
100		Results		
101		Risk-free Asset	Optimal Risky Portfolio	Optimal Overall Portfolio
102	Return	4.01	19.784	13.44
103	Risk	0.00	22.976	13.73
104	optimal position (1-y,y)	0.40	0.5976	
105				
106	H. Optimal Complete Risky Portfolio (Restricted Portfolio)			
107		Risk-free Asset	Optimal Risky Portfolio	Optimal Overall Portfolio
108	Return	4.01	12.90	9.48
109	Risk	0.00	17.00	10.46
110	optimal position (1-y,y)	0.38	0.6152	

We now determine the efficient frontier but before we compute this we have to go back to our data and come up with a benchmark against which we can establish and evaluate our portfolios in terms of their efficiency. To do this we come up with a border multiplied covariance matrix. We then choose a target mean, for instance 15%. From here we look for a way to compute the target mean and variance of the portfolio. We do this by using excel as shown in the screen shot below. The cells are all numbered and calculations are shown for determining the variance and the mean. Also included are two columns for calculating the expected return and the standard deviation.

In doing the above computations we use the solver function in excel as shown in Table D above. You first need to open the solver function in excel. You will then be prompted to enter the cell containing the objective function. This in our case is the cell containing the variance of our portfolio. The aim is for solver to minimize this target. We now come to the decision variables where we will input the cell range. These are normally the portfolio weights. These portfolio weights are contained in cell B49 through to cell B54. Finally, we collect together all necessary constraints and enter them into solver. When there is an unrestricted efficient frontier that gives room for short selling, we take two constraints. First we have the sum of the weights and second, we enter the constraint that the overall portfolio expected return is equal to the target return of 15 %. Once these two constraints have been entered, we then prompt solver to give us the optimal portfolio weights.

Solver normally makes some sounds whenever it has reached a solution.it then automatically changes the weights contained in the cells covered by row48 and column C. This shows what the efficient portfolio is composed of. Solver also adjusts the contents contained in the border multiplied covariance matrix to show how these new eights are multiplied and also shows the means and the variances represented by the optimal portfolio, which is the 15% minimum variance portfolio. Table D clearly shows these results.

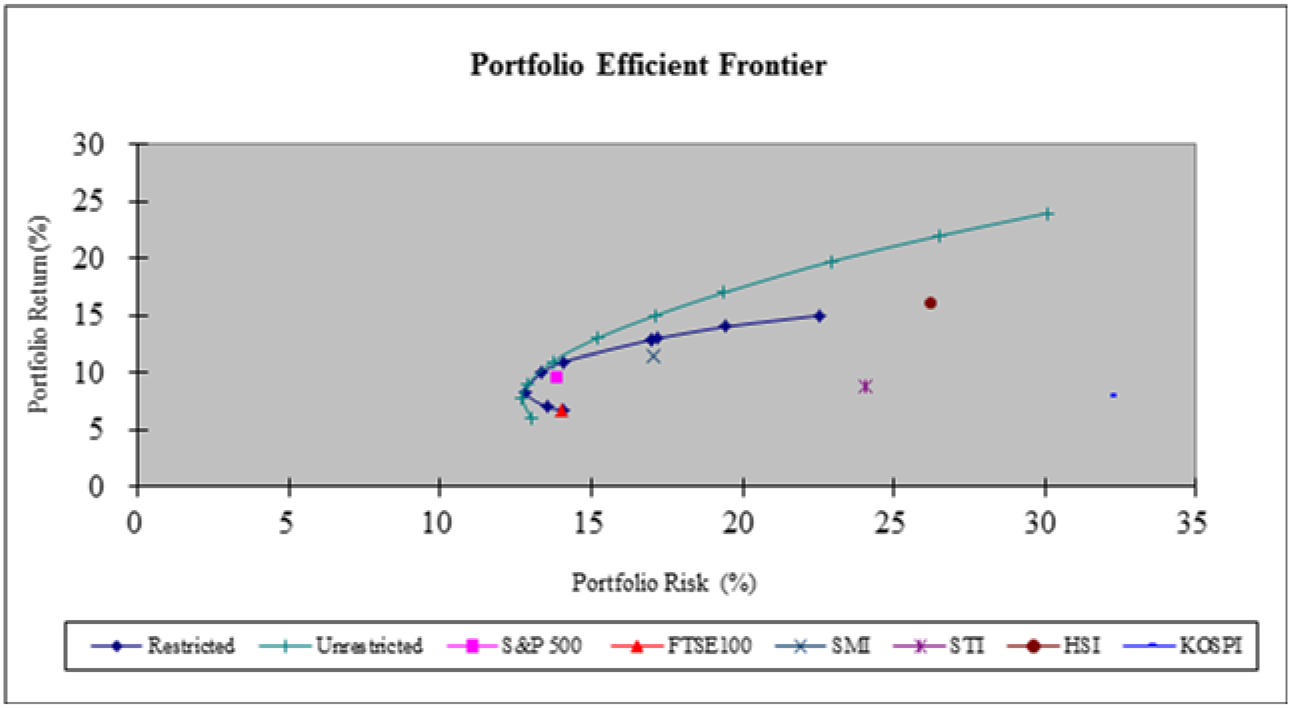
Solver also has an option for no short sales whenever short selling is not allowed. This is accompanied with other constraints. We impose an extra constraint detailing that we must have nonnegative numbers for all the elements in column B and row 49. These are the weights. These once entered, we repeat the task on variance minimization until we come up with a restricted frontier. Assuming that the investor or a group of investors might wish to maintain weights in his portfolio that are negative, the inner frontier will be such that it will allow short sales. For the two frontiers, tables E and F give us a number of points on the frontiers. Clearly, it can be seen that in the restricted portfolio, weights are always positive and that for the two frontiers, the minimum variance portfolios are not at all similar.

The Efficient Frontier

Restricted and unrestricted Portfolio

Efficient frontier of unrestricted and restricted portfolio

Figure 4.13. Efficient Frontier



4.2.2 The Optimal Risky Portfolio

Under the optimal risky portfolio, we go a notch higher after deriving the efficient frontier. Here, we find the portfolio at the tangency point of the efficient frontier and the capital allocation line. Here again we use solver to determine this. We first use the target function, which in this case is the maximum reward to variability ratio $\left(\frac{E(r_p)-r_f}{}\right)$, and then we make an inference or assumption on the risk free rate at 4.01% as the slope of the capital allocation line. We then specify he cell range which are the portfolio weights we input in cells B49 to B54. We will also include other necessary and important constraints. For instance, the weights must all add up to one. From here, solver then calculates the optimal weights of the portfolio.

4.2.3 Capital allocation

Risk is an important aspect in capital allocation. The degree of risk aversion will influence the decision to be made by an investor on his allocation of investment in the various asset classes available to him. We now use the idea behind the allocation of funds under the complete portfolio between the riskless and the risky assets. Referring to equation 11 above we use it as the utility function and set our level of risk aversion to 5 and our risk free rate to 4.1%. to do this we first come p with a complete portfolio with a risk free asset and an optimal risky portfolio. Drawing from equation 14 above, we determine from the risky portfolio the optimal weight as

$\frac{E(rp)-r_f}{0.01A\sigma_p^2}$ and from here what gives us the optimal position when it comes to the risky asset is $1 - \frac{E(r_p)-r_f}{0.01A\sigma_p^2}$.

It follows from equation 5 and 6 that we can calculate what we expect as our return and risk measure i.e. standard deviation of generally the whole/overall optimal portfolio. We can then relate to the information provided in tables G and H as our results. Our optimal restricted/unrestricted portfolio gives us 9.48%/13.44% as our results with an expected return of 10.46/13.73% standard deviation. From here the investor can comfortably invest 62%/60% of his portfolio value in the risky assets and the remaining 38%/40% in the riskless assets combination. We can refer to the table below for a summary of the above results.

4.3 Index Funds Analysis

4.3.1 Potential Factors

There are many metrics available that can be used to measure a company's value. The following section explains some potential factors and the impact of using each.

1. Capitalization on the market.

Investment Portfolio	Unrestricted			Restricted		
	Portfolio minimum Varance	Optimal Risky Portfolio	Overall Optmal Portfolio	Portfolio minimum Varance	Optimal Risky Portfolio	Overall Optmal Portfolio
Return of portfolio	7.74%	19.79%	13.34%	8.27%	12.88%	9.84%
Risk of the Portfolio	12.72%	22.97%	13.74%	12.84%	16.98%	10.74%

Table 4.2. Optimization problem results

When you multiply the number of shares of a company and its share price you obtain the total market capitalization of that particular company. This is actually the worth of a company when looking at its market valuation. It shows how much a buyer is willing to give in consideration for buying a particular company in which he is interested. To measure how big a company is, most of the indices used in our markets use the market capitalization as the single most important measure of determining company size. This tends to overweigh those companies that for some reason are overvalued and on the other hand underrepresent those companies that are undervalued.

2. Company Sales

When looking at the total amount of goods sold by a company, this represents the total sales. If we were to use company sales as an indicator of the true value of a company this will be very unfair and would bring as this will bring a bias whenever we are dealing with those companies with very small margins. There are those companies that place an emphasis on volume of sales as a way to go when looking at profitability. Such companies could include automobile companies, trading companies and airlines.

3. Company Dividends

When shareholders are paid from proceeds from their companies, these are called dividends. Company dividends can be considered as a way of showing the profitability of a company. However, there are those of the opinion that these are purely optional and in most cases, organizations tend to keep some profits as retained earnings. These profits are reinvested in the company and are not redistributed to the shareholders. Here we cannot use a dividend weighted index because quite a good number of profitable companies which mostly do not issue dividends will not be given any consideration under this arrangement.

4. Company Shares Buyback

When we talk about a buyback in shares/stock we refer to a the process in which a company purchases its own shares to make the remaining shares more valuable. If a company makes a profit, they can either issue dividends to stockholders or use the money to reinvest if there is an opportunity to do so. If not, a company may choose to repurchase shares which can then be retired or held for re-issuance later on (A Breakdown of Stock Buybacks). This helps increase the value of the remaining publicly held shares and also helps to drive up the earnings per share ratio. Using stock buyback as a measure of company value may create a tilt towards companies who are mature and are no longer growing.

5. Book Value

This is the allocated worth of a company in terms of its balance sheet. We use the book value as a way of measuring the size of a company and avoid an unfair bias that may lean towards those companies that have assets that are old and do not have an actual cash value were they to be sold at market value. It also favors companies that rely on capital assets over companies that rely on human or intellectual capital . In addition, depending on the accounting practices across companies, reported book value may or may not be an accurate measure of size.

6. The Price Earnings Ratio

To determine the price to earnings ratio we take the price per share and divide it by the earnings per share, taken as an annual figure/vaue. We translate this value we derive here as the proportion that a shareholder has paid for a single dollar that the company has reported as its earning. This is the price earning ratio and those companies whose future growth is seen to be exponential are said to be having a higher price to earning ratio than those companies that have achieved a certain level of maturity and stability. Along the same line of thinking, a company with a low P/E may be undervalued. Using high P/E as a metric of company value has the same potential setbacks as using the market capitalization weighting. Overvalued companies will be overrepresented, while undervalued companies will be underrepresented. Another option here is to look at those companies that have experienced a smaller profit to earnings ratio by putting a more significant emphasis/importance in the current prevailing conditions than focusing primarily on growth. In this thesis, four fundamental factors were chosen for analysis along with market capitalization based on availability of data.

4.3.2 Simplifying Assumptions

Certain assumptions had to be made in order to make the analysis possible with the data that was available. They include:

1. Time period of historical data

9 years of historical data was used in the analysis, ranging from 1995 to 2003. This time period because it encapsulates the technology bubble from 1997 to 2001. The pricing bubble is especially interesting because the performance ineKiciencies of a cap weighted portfolio are especially evident. Several years before and afler the bubble are included to show that a fundamental index is equally valid in tracking the market even when prices are good estimates of value.

2. Data refresh rate

In evaluating the role and eKect of each fundamental factor in the construction of the portfolio, the criteria for inclusion in the index were assessed for all companies every 6 months. In the S&P 500, there is no regularly scheduled reconstitution. However, to replicate this practice with the fundamental index in this thesis would require data reported on a daily basis. This was not feasible for the purposes of this thesis because only freely available data sources were used. In addition, certain fundamental factors being measured (sales, book value, dividends) are only released every quarter or even less frequently.

3. Standard and Poor 500: Composition

The components of this ndex usually keep changing constantly as new companies are added and others are removed. To get this list of changes for the 1995-2003 time periods would require purchasing the data from a vendor. Instead, the composition of the S&P was assumed to be constant. An arbitrary date within the window of interest was chosen and the constituent list from that point in time was used as the S&P 500. In this manner, the composition of the S&P 500 was taken from December 31, 2000 going both backwards and forwards in time. This set of companies is only used from which to choose the subset of equities to form the fundamental index. Therefore, we will measure the returns of this index and they will be soley based on the actual price one unit of the index for the required periods.

4. Exclusion of companies which have been de-listed

The data source for the fundamental factors does not provide any information on companies that have been de-listed. For this reason and others such as ticker symbol changes, and company mergers and acquisitions, data could only be found for 373 out of 500 companies. This limitation means that there is a built in bias that favours the fundamentally weighted indices over the S&P, since the laNer incorporates companies that may have failed.

5. Selection of assets to be included in multifactor portfolio Depending on the desired portfolio size, companies were ranked every six months or every year based on fundamental metrics. If the desired portfolio size was 100 for a book value weighted index, the top 100 companies by book value in the S&P 500 were selected for inclusion. The ranking system in Excel allows for two companies to tie, so occasionally there may not be exactly 100 companies in the portfolio.

Tools and Data Sources Used

The list of constituents for the S&P 500 was downloaded from Standard and Poor's website.

1. Market Value
2. Price/Earnings Ratio
3. Price/Book Value
4. Dividends per share
5. Total Annual Sales

Excel was selected as the primary tool in the analysis stage. The decision was made because of Excel's flexibility and friendly user interface. Also, the Solver Add-in found in Excel was used in the mean variance optimization stage.

4.3.3 Results

Construction of the fundamental index

The fundamental index is constructed in a similar fashion to a traditional cap weighted market index. Companies are valued based on the fundamental factor in question, whether it is P/E ratio, book value, dividends or sales. The largest companies, as measured by the fundamental factors, are then selected for inclusion in the index. The weight of the investment in each company is also derived by its fundamental size. The following table illustrates a simple case where three companies make up a sales weighted portfolio.

Company	Sales	Final Weight
Company A	250MM	250/500 = 50%
Compant B	100MM	20%
Company C	150MM	30%
Total	500MM	100%

Table 4.3. Sales Weighted Portfolio

This process is repeated with updated data every 6 months at which point companies are added and removed in the index.

The two factor scenario is slightly more complicated, where the final weight is calculated as an average and renormalized so that the portfolio sums to 100%. Table below shows how a two factor (sales and dividend) portfolio is constructed.

Calculation of Portfolio Return and Variance

Company	Sales	Sales Weight	Dividend	Dividend Weight	Unnormalized Weight	Normalized Weight
Company A	250MM	50%	$1.00	33%	(50 + 33)/2 = 41.5%	41.5/115 = 36%
Company B	100MM	20%	$2.00	67%	43.5%	38%
Company C	150MM	30%	730%	26%		
Total	500MM	100%	$3.00	100%	115%	100%

Table 4.4. Construction of a two factor Portfolio

Once it is decided which companies to include and their weights in the index are established, the return on investment is computed for the entire portfolio.

The formula for return for any stock in a given time period is as follows:

$$Return = \frac{(p2 - p1}{p1}$$

where $P1$ is the share price in the first timer period and $P2$ is the price for the second time period.

We now determine the return of the whole portfolio which is the sum of the weighted returns of its constituents. Table below demonstrates how the total portfolio return is calculated for a simple three company scenario over two consecutive time periods.

Company	Weight	2000 Share Price	2001 Share Price	Return	Weighted Return
Company A	20%	$15.00	$17.50	(17.5 – 15)/15 = 17%	−17 * 20 = 3.3%
Company B	35%	$10.00	$9.00	-10%	-3.5%
Company C	45%	$20.00	$21.00	5%	2.25%
Total	100%				2.08%

Table 4.5. Total Portfolio Return

The variance of the returns for the entire portfolio over 1995-2003 is then:

$$Variance\ \frac{N\sum_x 2 - (\sum_x 2)}{N(N-1)}$$

From the above equation n is taken to be the number of data points and that x is taken to be the total portfolio return for one time period. Variance is a measure of volatility. A low variance implies less deviation. This is deviation from the expected vale.

The Sharpe Ratio

It is important to understand the importance of the sharpe ratio. As depicted herein, the sharpe ratio will tell us whether or not a portfolio's return ca be soley attributed to the optimization of the investor in his choice of assets or whether it's as a result of the investor taking up more than normal risk. The sharpe ratio can be determined as shown by the formula below:

The Sharpe ratio tells us "whether a portfolio's returns are due to smart investment decisions or a result of excess risk" (Sharpe Ratio). It is calculated as follows:

$$Sharpe\ ratio,\ S = \frac{R - Rf}{\sigma}$$

. In the above equation, R represents the rate given as a return representation of the portfolio or assets that have been invested. The risk-free rate is represented and also the standard deviation. When comparing two investments, a higher Sharpe ratio means the expected return is larger for the same level of risk. For the purposes of this thesis, an average risk free rate of return of 4.3% was used (Market Yield on U.S. Treasury Securities).

Effect of Number of Companies in Index

To determine the eKect of varying the number of equities in the portfolio, the total return and variances of each one factor portfolio were computed. The table below shows the diKerent variance and average returns for a 50, 75 and 100 component fundamental portfolio created using book value as the measure of a company's size.

Number of Components	Variance in Returns	Average Semi-annual Return
50	0.0132	6.7%
75	0.0135	7.1%

100	0.0129	7.1%

Table 4.6. Total Portfolio Return

The data suggests that there is not a big difference between the three portfolios, meaning that the number of assets included is an insignificant factor when building a sub-index with fundamental factors. The graph below depicting portfolio returns over time confirms this hypothesis. The same results can be observed when constructing the index using the other factors.

Construction of Final Fundamental Index

Based on the findings up to this point, the fundamental index to be optimized was constructed based solely on the P/E ratio. Thirty companies were selected using data from the start of 1995. The number of companies to be included in the portfolio was kept to thirty to minimize the computational time required for the optimization while ensuring sufficient diversification. Thirty companies were selected for the purposes of this analysis.

These thirty companies are assumed to be held constantly from 1995-2003 for the entire duration of the analysis. No reconstitution was executed for the fundamental index. The assets were re-weighted at the beginning of each of the three phases by solving an instance of the mean variance optimization at each point.

The relationship between the mean and the variance and their optimization

The mean variance optimization (MVO) seeks to minimize the total risk in a portfolio subject to some arbitrary target return (Markowitz). It is formulated as follows:

Minimize

$$\sum_{i,\ j,1}^{n} x_i x_j \sigma_{i j}$$

s/t

$$\sum_{k=0}^{n} x_i r_i = R$$

$$\sum_{k=0}^{N} X_i = 1$$

From the above formula, x*i* represents the weight of asset i in that portfolio and ri is taken to be the return expected to be generated from asset I and asset j. the tareted return is represented by R.

Covariance is defined as a measure of how two variables change together. A high co-variance shows that when you compare two variable separately they tend to agree by moving toward the same direction. On the other hand, the converse is true with a low covariance.

The above can be presented in a formula as shown below:

$$Cov(x, y) = E[(x - E(x))(y - E(y))]$$

In the mean variance optimization - MVO model, risk is measured as variance of the expected return. It always seeks to show the best way to allocate the best alternatives between the variance and the return expected from the portfolio. The output of the model gives the expected risk level and portfolio weights corresponding to a specific target return.

Results of Mean Variance Optimization

Annual price data for each of the 30 companies became very useful in coming up with the matrices of return and covariance. These are all for the mean variance optimization.

Three time periods were considered for the MVO. Since the objective of the thesis is to determine whether a fundamental index could overcome the performance challenges faced by a cap weighted index during pricing bubbles, the phases before, during and afler the technology bubble were used. The three stages were 1995-1997, 1997-2001, and 2001-2003.

Time Period	Time Return
1995-1997	21%
1997-2001	13%
2001-2003	-23%

Table 4.7. Target return Inputs for MVO

The resulting weights of the MVO for each of the time periods are as follow:

Company	1995-1997	1997-2001	2001-2003
1	0.0%	3.8%	6.2%
2	1.7%	0.0%	0.0%
3	0.0%	0.0%	0.0%
4	0.0%	0.0%	0.0%
5	0.0%	0.0%	0.0%
6	0.2%	0.0%	0.0%
7	0.0%	0.0%	0.0%
8	0.8%	0.0%	0.0%
9	0.3%	0.0%	0.0%
10	0.0%	0.0%	0.0%
11	0.0%	0.0%	0.0%
12	0.0%	0.0%	0.0%
13	0.0%	0.0%	0.0%
14	0.0%	0.0%	0.0%
15	0.0%	0.0%	0.0%
16	0.0%	0.0%	0.0%
17	0.0%	0.0%	0.0%
18	0.0%	0.0%	0.0%
19	0.0%	0.0%	0.0%
20	0.0%	0.0%	0.0%
21	0.0%	0.0%	0.0%
22	0.0%	0.5%	0.6%
23	0.0%	0.0%	21.5%
24	86%	0.0%	6.9%
25	0.0%	0.0%	0.0%
26	0.0%	0.0%	0.0%
27	9.9%	94.1%	0.0%
28	0.7%	0.0%	0.0%
29	0.0%	1.6%	0.0%

30	0.4%	0.0%	0.0%

Table 4.8. MVO Weights for Fundamental Index

Performance of MVO Portfolio

The weightings from the solutions to the mean variance optimizations were used to calculate the expected returns of the fundamental index over the three periods; before, during and afler the tech bubble

Portfolio	Average Annual returns			Sharpe ratio
	1995-1997	1997-2001	2001-2003	
S&P 500	21%	13%	-23%	0.07
Low P/E Index with MVO	28%	35%	4%	0.57

Table 4.9. Performance of MVO Index

The results clearly demonstrate that the mean variance optimized fundamental index outperforms the S&P 500 during all three phases.

5 Conclusions and Recommendations

5.1 Introduction

The main aim of all the investors the world round is to minimize risk and maximize their expected returns. For pension fund managers, this is even more urgent as they are tasked with increasing the contributions of the members. Therefor for an investment portfolio, its management requires a keen consideration when it comes to the selection of the particular assets that maximize the return. They also need to manage well the proportions that will be invested in the assets of choice. The motivation behind this study is to actually explore how the modern portfolio theory can be best used in coming up with ways to optimally allocate assets so as to achieve the best growth for investments. For pension funds this is the single biggest aim of the portfolio/pension managers. The Markowitz theorem for portfolio optimization has been found to be a good tool for maximizing investments as compared to the Index funds.

5.2 Recommendations

So far, the aim off this study has been achieved. As intended, the study has been able to show and demonstrate how useful the Markowitz model for portfolio optimization is and how wide its applications are in the current investment strategies. It informs all optimal assets allocations as needed by investors. Therefore, the findings of this study will come in handy in determining how pension funds will be invested. The resources collected by these funds will be distributed effectively to the various investment strategies that are available for optimal results. this will solely depend on the risk appetite and characteristics of the individual pension schemes.

For index funds, by directly comparing actively managed funds with a fundamental index it becomes obvious that any pricing errors can lead to serious ineKiciencies in the traditional index. Price should therefore no longer be the single indicator of a company's value, considering the various other options available. Consequently, a high market capitalization is not automatically enough to predict a company's success.

5.3 Suggestions for Further Studies

Potential improvements to this research include:

1. Study a larger time window to determine whether the results are consistent, ensuring equal and fair representation from all industries.
2. Evaluate the associated costs of managing a fundamentally weighted portfolio with and without optimizing the asset weights with the Markowitz Model

5.4 Conclusion

For pension funds, one of the most important factors is the return earned by the fund t the end of an investment period. Therefore, how the risk of a particular pension fund ia managed is very important and this risk can demonstrate efficiently how the financial products have been tailored. The Markowitz portfolio theory gives us the mean variance relationship that can be applied to the various investment strategies.

This study therefore is aimed at coming up with the strategies that can be sued to allocate pension fund assets to the investment opportunities that could best give a good yield or return from investment giving keen attention to the desired levels of risk that can be comfortable undertaken to give this maximum return. The results obtained herein have showed a significant improvement when it comes to the allocation of pension assets to those asset classes that can give maximum yield. This has been a useful tool in allocation of assets to different investment opportunities.

This thesis has eKectively confirmed that moving from the traditional capitalization weighted index fund to fundamentally constructed index can lead to significant improvements in a portfolio's returns.

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