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IN HIGH SCHOOL MATHEMATICS.

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A STUDY OF THE EFFECTIVENESS OF A COMPUTER  
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IN HIGH SCHOOL MATHEMATICS

by  
Franklin Delano Ronan

A dissertation submitted in partial fulfillment  
of the requirements for the degree of  
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Doctoral Committee:

Professor Finley Carpenter, Chairman  
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ABSTRACT

A STUDY OF THE EFFECTIVENESS OF A COMPUTER  
WHEN USED AS A TEACHING AND LEARNING TOOL  
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Franklin Delano Ronan

Chairman: Finley Carpenter

Conducted in the Dearborn Public Schools, Dearborn, Michigan, the study was designed to determine if students who use a computer to learn mathematics attain a higher level of achievement than other students of the same ability level who do not use a computer to learn mathematics.

Students in two algebra-trigonometry classes participated in the study: 14 boys and 12 girls in the experimental group which used a computer as a teaching and learning tool; 14 boys and 11 girls in the control group which did not use a computer. All students were classified as "middle ability," having been empirically placed at that level by mathematics teachers based upon each student's prior achievement in mathematics; but randomly scheduled into the two groups by a computer.

Both the experimental group and control group were taught by the same teacher. Except for instruction and assignments in the experimental group involving the use of the computer terminals and the language called BASIC, the course objectives, methods, techniques, and instructional materials were the same for both groups. The experimental group used the computer in three ways: (1) as a computational tool; (2) as a "teaching" and learning tool; and (3) experimentally. The study took one

19-week semester to complete.

Major findings:

1. After treatment involving (1) algebraic review material and radicals in equations, (2) trigonometric functions and complex numbers, and (3) circular functions and their inverses, there was no significant difference between the mean achievement of students who used a computer during treatment and the mean achievement of students who did not use a computer during treatment.

2. After treatment involving exponential functions and logarithms, students who used a computer during treatment attained a significantly higher level of achievement than students who did not use a computer during treatment.

3. After treatment involving trigonometric identities and formulas, students who did not use a computer during treatment attained a significantly higher level of achievement than students who used a computer during treatment.

4. The group of students which used a computer to learn algebra-trigonometry showed significant growth during treatment in understanding mathematical concepts, development of mathematical skills, and ability to perform mathematical problem-solving.

5. The group of students which did not use a computer to learn algebra-trigonometry during treatment showed significant growth in understanding mathematical concepts and ability to perform mathematical problem-solving, but no significant growth in development of mathematical skills.

6. During the semester's treatment, there was no significant difference in growth between the ability of students who used a computer to apply mathematical concepts and the ability of students who did not



use a computer to apply mathematical concepts.

7. During the semester's treatment, there was a significant difference in growth between the mathematical skills developed by students who used a computer in algebra-trigonometry and the mathematical skills developed by students who did not use a computer in algebra-trigonometry; students who used a computer attained a significantly higher level of achievement than those students who did not use a computer.

8. During the semester's treatment, there was no significant difference in growth between the problem-solving ability of students who used a computer and the problem-solving ability of students who did not use a computer.

9. After the semester's treatment, there was a significant difference between the logic and reasoning ability of students who used a computer in algebra-trigonometry and students who did not use a computer in algebra-trigonometry; students who used a computer attained a significantly higher level of achievement than those students who did not use a computer.

10. There is widespread need for revision and further development of instructional materials relevant to computer-assisted problem-solving in mathematics.

## ACKNOWLEDGMENTS

Designing and conducting a doctoral study as well as writing the report often requires the cooperation and assistance of many people. The research described in this dissertation is no different. Although limited space does not allow the opportunity to list the name of every person who contributed, I do want to express my sincere appreciation to each individual who helped; especially to:

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## CHAPTER I

### THE PROBLEM

#### Introduction

No longer is "computer" a strange word used by few people. With the role of the computer assuming greater importance in government, business, and education, even elementary school children are beginning to use the word with everyday regularity. Today's youths are repeatedly hearing and seeing references to computers via television and other media. For this reason, students are beginning to become inquisitive about computers; some even fascinated with them.

Because of the growing importance of the computer in our lives, it is becoming the responsibility of educational institutions to introduce students to the use, potential, and limitations of the device. Indeed, the day seems to be rapidly approaching when high school graduates will need knowledge and/or experience of computer techniques to compete for responsible jobs. Hence, in the 1967 report of the President's Science Advisory Committee, it was urged that educational institutions greatly expand computer education.<sup>1</sup> After extensive research and study, the committee concluded:<sup>2</sup>

1. Educational institutions without access to computers will be offering inferior education.
2. Approximately 75 per cent of all U. S. college under-

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<sup>1</sup>Report of the President's Science Advisory Committee, Computers in Higher Education, The White House, Washington, D. C., February, 1967.

<sup>2</sup>Ibid., pp. 4-6.



graduates in 1965 were enrolled in courses in which a computer would have been very useful, yet less than 5 per cent of the students had adequate access to computers.

3. Students in the 1970's will need a basic understanding of computers even if they do not work directly with them.

4. Computers should be universal in high schools as well as colleges.

5. Graduates unable to work with computer problem-solving techniques will be at a disadvantage in many fields.

During the last five years (1966-1970) several school systems have introduced electronic data processing into their vocational curricula. Thus, many students have the opportunity to study the major concepts involved in this relatively new field by using the equipment and/or through computer programming languages which are offered. Recently, electronic data processing has also become the object of study in several academic disciplines; that is, teaching units have been developed which emphasize the impact of the computer on society. Of course, computers have been used in school systems to provide business and administrative services, such as financial reports and student scheduling, for almost a decade. While improvements are still necessary, the services rendered are well established and relatively well understood. The application not well understood, mostly because its content and technology are in the development stage, is the use of a computer to assist instruction.

Like a textbook or movie projector, the computer--when used in an instructional rather than vocational manner--is a tool that can be used when a teacher and/or student feels the need for its assistance.

Thus, computer-assisted instruction (CAI) refers to the use that a person makes of a computer--usually through a terminal device such as a teletypewriter--to help individualize and improve the learning process.

CAI can be utilized in more than one mode. In this respect, it is interesting to note that there is disagreement concerning the number and nature of the "modes." Some people argue that CAI consists of only three: (1) drill and practice, (2) simulation and gaming, and (3) tutorial. They tend to think of CAI as the utilization of a programmed learning machine. Most leaders in the field, however, such as Jerman,<sup>3</sup> Stolurow,<sup>4</sup> Suppes,<sup>5</sup> and Zinn,<sup>6</sup> are beginning to interpret CAI in a broader sense. In addition to the modes which emphasize programmed learning, they include (4) retrieval (or inquiry), (5) problem-solving, and even a few other modes not as well known. As Zinn points out:

"Criteria used for classification according to the various schemes put forth in published writing have not been clearly expressed. However, the underlying assumptions from which the categories have been derived should be more interesting than the total of the classification schemes."<sup>7</sup>

Accordingly, Zinn proposes six dimensions which underlie the "modes":

<sup>3</sup>Max Jerman, "Promising Developments in Computer-Assisted Instruction," Educational Technology, IX (August, 1969), pp. 10-18.

<sup>4</sup>Lawrence M. Stolurow, Computer Assisted Instruction, (Detroit: American Data Processing, Inc., 1968), pp. 11-28.

<sup>5</sup>Max Jerman and Patrick Suppes, "Some Perspectives on Computer-Assisted Instruction," Educational Media, I (July-August, 1969), pp. 4-7.

<sup>6</sup>Karl L. Zinn, "Instructional Uses of Interactive Computer Systems," Datamation, XIV (September, 1968), pp. 22-27.

<sup>7</sup>Karl L. Zinn, "An Evaluative Review of Uses of Computers in Instruction," Project CLUE (Computer Learning Under Evaluation), U. S. Department of Health, Education and Welfare, Project No. 8-0509, I (December, 1970), Appendix D, p. 9.

(1) program or learner control; (2) diagnosis and prescription; (3) variety of functions available to the user; (4) type of interaction; (5) role of the computer for the individual serviced; and (6) "naturalness" of the communication between learner and system. In describing the relationship between these six dimensions and the modes of CAI, Zinn states that the "dimensions can be viewed as defining a space or domain of computer use, and the modes usually mentioned as simple categories are more appropriately described as filling some part of the space."<sup>8</sup>

Although the study described in this dissertation is an investigation of the effectiveness of the problem-solving mode of CAI in mathematics, a brief explanation of the other major modes is included so that the differences can be realized and better understood.

Drill and Practice.<sup>9</sup> The CAI mode most commonly utilized in elementary school programs is that generally known as "drill and practice." This mode is often used in solving quantitative and qualitative problems, such as those in mathematics or in language drill and analysis. The main purpose of drill and practice is development and mastery of a learning task. Utilization of this mode is best for rote practice subject-matter; for example, arithmetic and foreign languages. In order for the student to use this CAI mode, it is necessary for the computer to be programmed to process the specific drill and practice materials selected or developed by the teacher--and which, in the teacher's opinion, the student needs to help meet course objectives. In using the drill and practice mode, it is possible for the computer

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<sup>8</sup>Ibid., p. 10.

<sup>9</sup>Stolurow, op. cit., pp. 14-15.

to accumulate information about each student's performance so that the teacher can have a better understanding of the way in which the student is progressing in particular skill or concept development.

Simulation and Gaming.<sup>10</sup> The purpose of simulation and gaming is to encourage students to develop their own ways, strategies, and/or styles of learning. In simulating an experience, the teacher formulates a model of some real or idealized situation (such as a physics experiment or the management of a company). The complex relationships among the variables which represent the situation are aspects that the student is expected to learn and interpret. In fact, the student is afforded the opportunity to recognize critical conditions and make decisions that either increase the likelihood of particular outcomes, or decrease the likelihood of others. Thus, this mode of CAI provides the learner with a chance to amass a great deal of experience in a short period of time, and allows him the opportunity to practice skills such as problem analysis, inference, and decision-making which resemble those that may be encountered when employed on a job. In contrast to simulation, which attempts to depict a real or idealized situation, gaming is not intended to represent a realistic condition or interpersonal interaction.

Tutorial.<sup>11</sup> Use of the tutorial mode of CAI again necessitates the teacher taking responsibility for the student's instruction. The logic of the process must be formalized in detail by the teacher, often with the help of a computer programmer, then entered into the computer system. Alternative ways for formalizing the logic are necessary so

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<sup>10</sup>Ibid., pp. 16-17.

<sup>11</sup>Ibid., pp. 18-20.

that the process does not imply a rigid pre-determination of instructional sequences. Thus, the teacher can determine the kind of relationships which occur during the instructional experience. Through the use of a terminal, such as a teletypewriter, a student is presented a segment of information which he must analyze, synthesize, and evaluate before responding. When the student does respond through the use of the terminal, he is branched to the next question, tract, concept, or whatever, based upon his previous response.

Retrieval (or Inquiry).<sup>12</sup> When a student seeks to retrieve information from a computer, he asks questions of the system--usually through the use of a computer terminal device. In turn, the computer via the terminal, provides the student with the answer(s) it has stored in its files. Of course, this mode of CAI necessitates that the teacher either gets material into the computer system so that the information is retrievable, or he works with a knowledgeable programmer to accomplish the task for him. Hence, a student can ask for any information and/or its relationship which is in the information bank.

Problem-solving. Using a computer in the problem-solving mode can relieve a student of much of the time-consuming task of massaging information; instead, he can spend his time concentrating upon the procedure or method necessary to solve a problem. Most people agree that little is accomplished if a student spends a great deal of time repeating a learning task he has already mastered, such as addition or subtraction. Of far greater value is the method which one might adopt to treat data according to a series of steps. Hence, this mode provides a student

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<sup>12</sup>Ibid., pp. 15-16.

with additional time to concentrate upon developing processes and procedures.

Most unique in the problem-solving mode of CAI is the opportunity it affords students to "teach" the computer. If it is true that one of the best ways to learn and understand the content of a course is by teaching the subject-matter itself--which most people agree--then this mode of CAI allows students that rare opportunity to learn by teaching. For example, a student can be given a particular mathematics problem, the correct programming of which would help to demonstrate his knowledge and understanding of a mathematical concept. Accordingly, programming a computer is similar to teaching others how to solve a problem. The computer, in a sense, plays the role of a person who the student must teach. An important difference, however, is that the computer does not permit the student to do a poor job of teaching. The student must analyze the problem by recognizing the basic mathematical concepts; then he must follow the necessary sequence to derive the correct answer.<sup>13</sup> To accomplish this task, the student can use various approaches. However, if any logic or arithmetic errors are made, they are readily noted in the form of a diagnostic message to the student on the printout. A sample printout can be found in Appendix A of this report. Richardson describes the experience accordingly:

"The computer serves as a 'mathematics laboratory,' permitting the student to write logical and computational procedures in a suitable algorithmic language, and to demonstrate their operation. In such a learning process, the student does not need to know anything about the internal workings

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<sup>13</sup>Altoona Area School District, "Computer Related Programs for Algebra II," Altoona, Pennsylvania, 1968, p. 1.

of the computer; all that is needed is the algorithmic language. In this way, the computer gives the student a more intimate feeling for mathematics and will influence his approach to problem-solving in general. It also provides the teacher with a means for demonstrating the more difficult mathematical concepts, so that the student may grasp them more easily."<sup>14</sup>

Especially noteworthy in this "teaching" process is the opportunity the computer affords students to be creative, thereby helping the teacher to individualize instruction. The problem-solving process, when applied to computer programming, usually consists of the following steps:<sup>15</sup>

1. Definition of the problem to be solved
2. Analysis of the problem
3. Development of a computer program
4. Input of the program into the computer
5. Execution of the program with test data
6. Interpretation of the trial execution
7. Corrections and revisions of the program
8. Documentation of the problem-solving procedure

Adherence to the above procedure, or one that is similar, provides the student with a basic guideline, yet allows him considerable flexibility and opportunity to be creative. Depending upon the nature of the assignment given by the teacher, a student might deviate from the suggested procedure. For example, steps four and five might be

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<sup>14</sup>Jesse O. Richardson, "Computer-Assisted Mathematics," Computer Research Project H-212, Massachusetts Department of Education, p. 6.

<sup>15</sup>Computer-oriented Mathematics Committee, Computer Facilities for Mathematics Instruction (Washington, D. C.: National Council of Teachers of Mathematics, Inc., 1969), pp. 6-7.

combined; or step seven might not be necessary. When computers become more interactive, even the order of the steps may change.

Some people argue that computer-assisted problem-solving will provide students with the opportunity for a greater appreciation of mathematical concepts, such as limits and convergence, by processing computer programs that perform these functions.<sup>16</sup> Indeed, it is a known fact that, within a few minutes, students can test the convergence properties of a large number of examples and compare the results with the theory they learned in class. Without the use of a computer, the computations alone for this work would take an entire week of class time.

Regardless of the mode of CAI which one might choose to utilize, change and/or modification to the existing curriculum is not necessary to take advantage of the computer. Accordingly, many educators believe that a well-developed, modern curriculum should not necessarily be changed to permit the use of a computer as a teaching or learning tool. They contend that it might be better if a computer were used to help attain those objectives already deemed relevant and/or necessary to meet student needs. Others argue the contrary, saying that the computer is becoming so much a part of "everyday living" that it should be an object of instruction; and therefore included in the stated objectives of curricula.

#### Description of the Study

Little is known of the effect of computer-assisted problem-solving on student learning behavior. The lack of research, which has

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<sup>16</sup>Richardson, op. cit., pp. 6-7.



so often been cited, suggests the need for studies such as the one reported in this discussion.

Identification of the specific problem in this study can be stated in the form of a simple question: What effect on student learning behavior does the use of a computer have when used as a teaching and learning tool in high school algebra-trigonometry?

Students in two algebra-trigonometry classes participated in the investigation--26 students in the experimental group; 25 students in the control group. All students who participated were of "middle ability" level, having been empirically placed at that level by mathematics teachers based upon each student's prior achievement in mathematics; but randomly scheduled into the two groups by a computer.

Both groups of students were taught by the same person, a teacher with considerable experience in mathematics and computer-assisted problem-solving. Except for instruction and assignments in the experimental group relevant to the computer language called BASIC (Beginner's All-purpose Symbolic Instruction Code) and the use of the teletypewriter terminals necessary to communicate with the computer, the course objectives, methods, techniques, and instructional materials were the same for both groups. Thus, the experimental group used a computer as an instructional tool to learn algebra-trigonometry; the control group did not.

It was not economically feasible for students in the experimental group to use a computer owned or leased by the school district. Instead a commercial, time-shared computer system was used. A time-shared system permits many people to communicate simultaneously with the computer, using remote terminals and regular telephone lines on a

dial-up basis. With this system, students in the experimental group were able to sit at remote teletypewriter stations to write, submit, and debug programs. To accomplish this, the user and the computer needed to converse in a computer language which was easily understood by an inexperienced user. Several languages such as APL, BASIC, and CAL (See Chapter II.) have been developed recently for this purpose. Because of its simplicity and easy adaptation to mathematics, BASIC was selected as the language to use with the experimental group in this study. The language was initially designed for people who had no previous knowledge of computers as well as for those more expert.

The study reported in this dissertation took one 19-week semester to complete. Although most utilization of the computer terminals was in the mathematics workroom or the hallway adjacent to the classroom, a telephone line was installed in the classroom to allow the teacher the opportunity to use a single terminal for teaching and demonstration purposes.

The senior high school mathematics curriculum in the Dearborn Public Schools (Dearborn, Michigan), where the study took place, is a well organized, innovative curriculum which is the result of the exhaustive work of many teachers and administrators over a long period of time. Changes in the curriculum are continually being made to incorporate the most up-to-date programs, methods, and techniques when it is felt that such changes will help to better meet the needs of local students. For example, in the case of utilizing a computer to teach mathematics, it was decided to install one computer terminal in each of the school system's three high schools on an experimental basis in September, 1969. However, several of the people involved in the project were somewhat

apprehensive; they were concerned that there would not be enough time during the semester to teach the required material and still utilize the computer as a teaching and learning tool. Thus, a control group/experimental group study was conducted by the writer to answer the point in doubt; and to identify related benefits and problems concerning the use of a computer terminal in an instructional environment. The results of the study showed that students who use a computer to learn mathematics need less time than "other" students for drill problems to reinforce difficult concepts; apparently because the problems are analyzed so thoroughly before developing an algorithm. Hence, in spite of the time used to learn a computer language and utilize appropriate equipment, the "computer group" was able to receive instruction concerning all the required material since less time was needed for the tedious calculations experienced by the "non-computer group." In a similar study, the Altoona (Pennsylvania) Area Public Schools reached the same conclusion.<sup>17</sup> Results such as these encouraged the study reported in this dissertation.

Upon hearing of the pilot program conducted during the 1969 fall semester and the intention of the writer to investigate the effectiveness of the computer as a teaching and learning tool in high school mathematics, a computer time-sharing company volunteered to support the research by installing two additional terminals, free of all financial charges or other commitments, in the high school where the study would be conducted. The offer was graciously accepted. Thus, in conducting the investigation reported in this dissertation, three computer terminals

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<sup>17</sup>Albert E. Hickey, ed., Computer-Assisted Instruction: A Survey of the Literature, Technical Report No. 8, Office of Naval Research Contract (Newburyport, Massachusetts: ENTELEK, Inc., 1968), p. 40.

were utilized; and the curricular strategy adopted called for the identification of those aims and objectives within the existing algebra-trigonometry curriculum of the Dearborn Public Schools where a computer could best be used to improve student learning behavior.

#### Importance of the Study

Interest in pursuing the project was stimulated by the lack of research in the field and the widespread enthusiasm as well as criticism of many people who argue the worth of using a computer as a teaching and learning tool--in other disciplines as well as mathematics. Hence, in the interest of providing the best possible learning environment for pupils, the following null hypotheses were tested in the study:

1. Prior to treatment, there is no significant difference in achievement between the experimental group and control group when measured by test instruments in the following areas: intelligence quotient; reading vocabulary; reading comprehension; mathematical concepts; mathematical skills; mathematical problem-solving.

2. When measured by teacher-constructed post-unit tests that exclude reference to computers, computer languages, computer programming, and other areas of computer knowledge, there is no significant difference in achievement between the experimental group which used a digital, time-shared, electronic computer to learn mathematics and the control group which did not use a computer to learn mathematics.

3. When tested by a teacher-constructed post-treatment instrument measuring ability to understand and apply mathematical concepts, there is no significant difference in achievement between the experimental group which used a digital, time-shared, electronic computer to

learn mathematics and the control group which did not use a computer to learn mathematics.

4. When tested by a standardized post-treatment instrument measuring level of mathematical skills, there is no significant difference in achievement between the experimental group which used a digital, time-shared, electronic computer to learn mathematics and the control group which did not use a computer to learn mathematics.

5. When tested by a standardized post-treatment instrument measuring mathematical problem-solving ability, there is no significant difference in achievement between the experimental group which used a digital, time-shared, electronic computer to learn mathematics and the control group which did not use a computer to learn mathematics.

6. When tested by a teacher-constructed post-treatment instrument measuring logic and reasoning ability, there is no significant difference in achievement between the experimental group which used a digital, time-shared, electronic computer to learn mathematics and the control group which did not use a computer to learn mathematics.

7. When measured by an investigator-constructed opinion instrument, there is no significant difference between the early-treatment opinions of the experimental group relevant to the use of a digital, time-shared, electronic computer to learn mathematics and the group's post-treatment opinions concerning the use of the computer to learn mathematics.

Hence, the major purpose of the study was to determine if students who use a computer to learn mathematics would attain a higher level of achievement than other students of the same ability level who do not use a computer to learn mathematics. Achievement was identified

by administering test instruments in the following areas:

Post-unit

- Algebraic Review Material and Radicals in Equations
- Exponential Functions and Logarithms
- Trigonometric Functions and Complex Numbers
- Trigonometric Identities and Formulas
- Circular Functions and Their Inverses

Post-treatment

- Mathematical Concepts
- Mathematical Skills
- Mathematical Problem-solving
- Logic and Reasoning

Since the opinion of students regarding such experience is also a concern to many educators, it too was included as an important facet in the investigation.

If supportive conclusions are reported from this study and other similar investigations, there is some justification for the cost of computer resources for such teaching and learning. Indeed, there will also be reason to examine alternative means of introducing the technique or process.

Instructional Techniques Used

The instructional materials that a teacher might use to implement a computer as a medium of teaching and learning are limited. The few materials that are available were prepared to meet course objectives other than those of the Dearborn Public Schools where the study took place. Hence, it was necessary to adapt or develop appropriate materials

to meet Dearborn's objectives. These revised and newly developed materials were used by the teacher when utilizing the computer as a teaching and learning tool to teach the BASIC language; to demonstrate the use of the computer terminals; to illustrate the computer's problem-solving capability; and to introduce students to strategies and procedures they might use to "teach" the computer.

Students in the experimental group worked directly with three teletypewriter terminals, two of which were connected through private telephone lines to a large-scale, digital computer located approximately 15 miles away. The students typed numerals, letters, and symbols found on the keyboard to solve mathematical problems and "teach" the computer that which they learned in class. When desired, changes in method or procedure were made immediately by the students with a direct type-in on the keyboard, without lengthy manipulations of punched cards. When on-line, errors in "teaching" were indicated by the computer as soon as they occurred; and the student was assisted in making corrections by the diagnostic comments typed on the printout.<sup>18</sup> Thus, using the BASIC language, students: (1) wrote their problem-solving programs on notebook paper in "long-hand"; (2) punched the program off-line into paper tape via one of the computer terminals; (3) submitted the paper tape on-line to the computer through the tape reader; (4) received a print-out of the program and its problem-solving application; and if necessary, (5) analyzed and debugged the program for another "run" if the computer indicated that a "teaching" mistake had been made.

Utilization of the computer was not limited to assignments

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<sup>18</sup>Richardson, op. cit., pp. 5-6.

that required "teaching" the computer. Problems were also assigned which allowed students the opportunity to use the computer as a computational tool. In addition, time was provided before and after school hours and during the lunch and study periods so that students could "experiment" with the computer.

Except for instructional demonstrations, most student programs were punched into paper tape in the mathematics workroom or the hallway adjacent to the classroom. Use of the terminal devices in the classroom was too noisy and disruptive for other pupils not using the terminals. Hence, each student was allowed to leave the classroom to punch his program. After the program was punched, the pupil returned to the classroom so that another student could leave to work at one of the three terminals. When all students had punched their programs, the same rotating procedure was used to submit programs on-line to the computer. When working on-line, students were allowed to respond immediately to diagnostic messages in attempting to debug their strategies rather than wait to do so off-line. On those days when the rotating procedure was utilized, the teacher used the opportunity to work individually with other students remaining in class. Since the entire process might have taken the experimental group several days to complete, students were encouraged to use the terminals before and after the regular school day as well as during the lunch and study periods; thus it was not necessary to use "too much" classroom time.

#### Limitations of the Study

Like most educational research, the study described in this report is not without limitations. For example, although the commercial



market is finally beginning to show some evidence of the development of computer-assisted ~~problem-solving~~ materials, the selection is still quite limited and narrow in scope. Consequently, the instructional materials adapted and devised specifically for this study reflect this widespread need. Too, several existing materials lack thorough development and refinement; some even need major revision. Equally important, many of these materials must still be tested under rigid conditions for validity and reliability.

Because "above average" students participated in the investigation, the results of the study might be meaningful to more people if the participants had possessed "average" intelligence--if they generally achieved in accordance with "average" levels. However, students with average academic competency seldom elect courses like algebra-trigonometry.

The experimental sample included a total of 51 students. Involvement of more participants would have allowed statistical analysis relevant to socio-economic background, grade level, gender, instructional materials and strategies, group size, motivation, and computer language(s).

Only three computer terminals were used in the study because the investigator believes that three terminals, in terms of cost, is a realistic number for a school to possess at this point in time. However, what effect might more terminals, or fewer, have upon the development of student learning behavior?

## CHAPTER II

### REVIEW OF THE RELATED LITERATURE AND STATE-OF-THE-ART

Computers have been utilized in educational institutions for more than a decade, yet their use to assist instruction is still considered experimental. Although the literature describes several ways or modes that the computer can be used to assist instruction, the study described in this report involves only one: the problem-solving mode. Hence, review of the related literature in this chapter is limited to the mode of computer-assisted instruction (CAI) utilized in this study.

#### The Problem-Solving Mode of CAI

Of the several modes of CAI which one might use, the one most widespread in mathematics is that described as computer-assisted problem-solving. The popularity of this mode of CAI appears to be a natural out-growth of the way that the computer was first used by the scientific community. Hatfield describes the scene accordingly:

" . . . to accept, process, and print output to a program which makes use of the machine's capability to store and access large data bases, make quantitative decisions, and perform detailed computations, all at very great speeds."<sup>1</sup>

Utilization of the problem-solving mode provides the user with the opportunity to maintain control of the computer as an instructional tool. Unlike other modes of CAI, the computer is not used to generate pre-programmed material or lessons; rather, it is the task of the user to write the program. To do so, he must separate his problem

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<sup>1</sup>Larry L. Hatfield, "Computers in Mathematics Instruction," A Paper Presented to the National Council of Teachers of Mathematics Annual Meeting, April, 1969, p.3.

into simple elements. As a result of this exercise, the user has the opportunity to develop generalized problem-solving skills--or sharpen his algorithmic thinking. Through the computer terminal, the learner literally uses the computer as a tool, an extension of himself, the same way that he would use a slide-rule.<sup>2</sup> If the learner is on-line with the computer, he receives an immediate reply to his logic in the form of a diagnostic message. When such feedback is in the form of hard copy, the message(s) can be used as an object of instruction with the teacher. Thus, by using the computer in the problem-solving mode of CAI, the user is relieved of much of the time-consuming task of repeating skills already mastered, such as addition and subtraction; instead, he can use the time to concentrate upon developing the procedure or method necessary to solve a problem. Even more important, the user is afforded an opportunity to "teach" the computer by writing programs to solve mathematical or scientific problems.

#### Importance of Computer Time-Sharing

The advent of computer time-sharing during the 1960's greatly enhanced the use of computer-assisted problem-solving. A time-shared computer allows many people to communicate simultaneously with the computer, using remote terminals and regular telephone lines on a dial-up basis. With this system, users can sit at remote teletypewriters or video stations to develop, run, and debug programs while on-line to the computer. The computer facility itself may be just a few yards from the terminal stations; or it may be as far away as several thousand miles.

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<sup>2</sup>Dudley L. Post, ed., The Use of Computers in Secondary School Mathematics (Newburyport, Massachusetts: ENTELEK Incorporated, 1970), p. 128.

The concept of computer time-sharing is based on the principle that there is enough capacity in a computer system for multiple users, as long as each terminal station is active only a small fraction of the time. Spencer describes the process:

"Each user of a time-sharing system has the illusion that he is the only person using the system. Each user can run his program on-line as he would with a conventional computer. How does a time-sharing computer system take care of several users simultaneously? Each one has control over the computer for a specified quantum of time. The computer picks up orders from one user, works on his problem, say for 1/20 of a second, and stores the partial answer. It then moves to the next user, receives his orders, works on the second problem for 1/20 of a second, and moves to the third user, etc. When a problem is completed, the answer is printed on the user's console. The computer system accomplishes this work so fast that the user feels the system is working for him full-time."<sup>3</sup>

The first general-purpose time-shared computer system, called the Compatible Time-Sharing System (CTSS), was developed for an International Business Machines' (IBM) 709 computer in 1961 at the Massachusetts Institute of Technology Computation Center. A year later, the company of Bolt Beranek and Newman (BBN) developed a time-sharing system for Digital Equipment Corporation's PDP-1 computer.<sup>4</sup>

By 1963 the computer time-sharing field began to mushroom. RAND Corporation in California announced the JOHNNIAC Open Shop System (JOSS); the Department of Electrical Engineering at Massachusetts Institute of Technology (MIT) developed a three-terminal time-sharing

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<sup>3</sup>Donald D. Spencer, A Guide to BASIC Programming: A Time-Sharing Language (Reading, Massachusetts: Addison-Wesley Publishing Company, 1970), p. 1.

<sup>4</sup>Ibid., p. 3.

system on a PDP-1 computer; Adams Associates, Incorporated, a software company, announced an eight-terminal system using a PDP-4 computer; and System Development Corporation (SDC) introduced a system that could accommodate 20 to 39 terminals, using an IBM Q-32 computer. By late 1963, Project Multiple Access Computer (MAC), sometimes called the granddaddy of all large time-sharing systems, was demonstrated on an IBM 7094 computer system. Originally servicing 160 terminals at various locations on the MIT campus, the project now utilizes a General Electric (GE) 645 computer which can accommodate 500 terminals, 150 to 200 simultaneously on-line.<sup>5</sup>

Since Project MAC was initiated in late 1963, many universities, computer manufacturers, and companies have implemented large time-sharing facilities and services which utilize other computer systems. A few of the better known time-sharing systems today are:

- Burroughs B5500 System
- Control Data Corporation 3300 System
- Digital Equipment Corporation PDP-8/i
- Digital Equipment Corporation PDP-10
- General Electric 235 System
- General Electric 420 System
- General Electric 635 System
- Hewlett-Packard 2000A System
- Honeywell 1648 System
- IBM 360-50 System
- IBM 360-67 System

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<sup>5</sup>Ibid., pp. 3-6.

IBM 7044 System

RCA Spectra 70/46 System

Scientific Data Systems 940/945 Systems

Scientific Data Systems Sigma 5/7 Systems

Univac 494 System

Univac 1108 System

### Importance of Problem-Solving Language

Since most time-sharing services are used to solve scientific and mathematical problems, they characteristically employ a problem-solving computer language. Such languages as APL, BASIC, or CAL make the problem-solving mode of instruction easy to implement on a time-shared computer system.

When selecting a computer language, several factors should be considered; these include: (1) the instructional strategies to be used; (2) the subject-matter to be taught; (3) the ability of students; (4) the ease of learning the language; (5) the ease of using the language; (6) the equipment needed in the computer configuration to implement the language; (7) the adequacy of its record keeping; and (8) the accumulated experience from other schools using the language.<sup>6</sup>

Most problem-solving languages are characterized by an active on-line interaction between the user and the computer; and most evaluate each typed statement immediately after it has been completed. All problem-solving languages must include statements to perform standard computations and commands, such as: (1) invert a matrix; (2) raise to an exponent; (3) take a square root; (4) represent common functions

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<sup>6</sup>Post, op. cit., pp. 16, 40.

(log, sine, etc.); and (5) the ability to create and use new functions.<sup>7</sup>

Problem-solving languages are suitable for both individual problem-solving and classroom demonstration applications. The following are some of the most popular problem-solving languages which are used with time-sharing systems:

"APL (A Programming Language) is a problem-solving language that operates on a variety of IBM computers including the 1130, 1500, and 360 systems. The selectric typewriter is the usual terminal used. APL originated as a mathematical notation system in a book, A Programming Language, written by K. E. Iverson and has since become known as the "Iverson notation." It is a very concise notation that makes use of special keyboard characters to represent complex mathematical operations with very few keystrokes. APL has the disadvantage of departing significantly from the standard high school mathematical notation found in textbooks. For example, no hierarchy of arithmetic operations is observed. However, it has been successfully taught at the high school level, and it has many supporters who rate it highly.

"BASIC is a language developed at Dartmouth College for the GE-265 TSS (time-sharing system), and it has since been made operational on many other computers, including the GE-600 series, SDS 940 TSS, HP-2000 TSS, IBM 360 system, DEC PDP/8, and other types. It also is normally used with a teletype terminal. BASIC is the most widely available of all the problem-solving languages, primarily because of the success of commercial time-sharing services that offer it. The number of special projects involving the use of BASIC and the abundance of related course material attest to its popularity.

"JOSS (JOHNNIAC Open-Shop System) is the forerunner of several special problem-solving languages including CAL, FOCAL, and TELCOMP. JOSS was developed at the RAND Corporation on the JOHNNIAC, one of the first digital computers (now in the Smithsonian Institution), and has since been made to operate on an IBM 360. Its use has been primarily as an aid to scientists, with very little application at the high school level. However, several of the more recent languages that closely resemble JOSS have been successfully used in high school and below, suggesting the importance of

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<sup>7</sup>Post, op. cit., pp. 52-53.

this pioneer effort. JOSS provides a range of capabilities from simple calculation to the most complex kinds of programming. Most users acknowledge that JOSS and JOSS-like languages are somewhat more difficult to learn than BASIC, but in turn claim more freedom in their ability to program complex problems.

"CAL is a problem-solving language developed at the University of California for the SDS 940 computer. Like most other problem-solving languages, it operates through a Teletype. CAL is similar to JOSS.

"FOCAL is another JOSS-like problem-solving language which was developed by Digital Equipment Corporation for their PDP/5 and PDP/8 line of computers. This language has been quite popular, especially in high schools, where the relatively low cost of the PDP/8S and PDP/8L computers have made it possible for many schools to purchase one or more. Like BASIC, course material at the high school level has been written for FOCAL.

"TELCOMP, too, is a JOSS-like language. It was developed by Bolt Beranek and Newman for their time-sharing service. TELCOMP has been used experimentally at several grade levels and operates on a DEC PDP/7-PDP/8 system.

"QUIKTRAN is a problem-solving version of the FORTRAN language, originally developed by IBM as their entry in the time-sharing market. However, it appears that IBM's other problem-solving languages, APL and BASIC (CALL-360), have displaced QUIKTRAN, so its future is not clear."<sup>8</sup>

Because of its simplicity and flexibility, BASIC (Beginner's ALL-purpose Symbolic Instruction Code) was selected as the language to use with the experimental group in this study. BASIC was initially designed for people who had no previous knowledge of computers, as well as for those more expert.

The BASIC language was developed at Dartmouth College by J. G. Kemeny. His objective was to invent a computer language so simple

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<sup>8</sup>Post, op. cit., pp. 53-55.



that "any" student could use a computer facility with a minimum of instruction in programming. With BASIC, Dartmouth students actually applied the power of a computer to real problems after only two hours of instruction, skipping the internal mechanics of the machine.<sup>9</sup>

There are, of course, many general-purpose languages. For example, FORTRAN and COBOL are widely used in business and industry; they are, however, more difficult to learn than BASIC, and do not significantly increase the problem-solving capability of the novice programmer. In fact, many advanced programmers prefer the simplicity of BASIC.

#### Computer-Assisted Problem-Solving in Mathematics

Although computer-assisted problem-solving is still relatively new to secondary school education, the National Council of Teachers of Mathematics (NCTM) indicated its interest in using computers in mathematics instruction as early as 1963. The Council published two important booklets that year: Computer-Oriented Mathematics, An Introduction for Teachers;<sup>10</sup> and Report of the Conference on Computer-Oriented Mathematics and the Secondary School.<sup>11</sup> Since these early publications, other articles and booklets on the topic have been published by the Council indicating a continued interest in the use of computers in mathematics. For example,

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<sup>9</sup> John G. Kemeny and Thomas E. Kurtz, "Dartmouth Time-Sharing," Science, CLXII (October 11, 1968), p. 228.

<sup>10</sup> Computer-oriented Mathematics Committee, Computer-Oriented Mathematics, An Introduction for Teachers (Washington, D. C.: National Council of Teachers of Mathematics, Inc., 1963).

<sup>11</sup> Computer-oriented Mathematics Committee, Report of the Conference on Computer-Oriented Mathematics and the Secondary School (Washington, D. C.: National Council of Teachers of Mathematics, Inc., 1963).

the article "Computers for School Mathematics"<sup>12</sup> was published in The Mathematics Teacher in May, 1965, stating important guidelines for those pursuing the use of computers in mathematics instruction. Within the following four years, three additional publications provided further assistance to those interested in the field: Computer Facilities for Mathematics Instruction;<sup>13</sup> An Introduction to an Algorithmic Language (BASIC);<sup>14</sup> and Computer-Assisted Instruction and the Teaching of Mathematics.<sup>15</sup>

By 1969, an entire issue of The Arithmetic Teacher was devoted to "CAI," and a new department entitled, Computer-Oriented Mathematics, was introduced in The Mathematics Teacher. In the meantime, other groups also demonstrated their interest in the field:

"... one can find analogous activities being conducted by the Committee on Undergraduate Programs in Mathematics of the Mathematical Association of America by the National Science Teachers Association and by the School Mathematics Study Group (MSG) and the Minnemast curriculum projects. The Center for Research in College Instruction of Science and Mathematics (CRICISAM) Project at Florida State University is developing a computer oriented calculus curriculum."<sup>16</sup>

Because utilization of the computer as an instructional tool

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<sup>12</sup>"Computers for School Mathematics," The Mathematics Teacher, LVIII (May, 1965).

<sup>13</sup>Computer-oriented Mathematics Committee, Computer Facilities for Mathematics Instruction (Washington, D. C.: National Council of Teachers of Mathematics, Inc., 1969).

<sup>14</sup>Computer-oriented Mathematics Committee, An Introduction to an Algorithmic Language (BASIC) (Washington, D. C.: National Council of Teachers of Mathematics, Inc., 1968).

<sup>15</sup>Ralph T. Heimer, ed., Computer-Assisted Instruction and the Teaching of Mathematics (Washington, D. C.: National Council of Teachers of Mathematics, Inc., 1968).

<sup>16</sup>Hatfield, op. cit., p. 9.

is still experimental, many contentions and conjectures have been made concerning its value to instruction and learning. For example, Kemeny argues that students can use the computer to more effectively learn those principles taught theoretically in class:

"I feel that the right attitude is to teach them the algorithms in principle and then the right way to do the algorithm in practice is to program it for a computer. Thus the computer is being used in such a way as to force the student to explain the given algorithm to a computer. If a student succeeds in this, he will have a depth of understanding of the problem which will be much greater than anything he has previously experienced."<sup>17</sup>

Hoffman believes that students have a greater understanding of solutions to problems when the problems are programmed for solution by the computer. He contends that the computer helps the teacher to attain objectives which were very difficult to attain without the computer:

"In the experience which I have had along this line, it has been quite clear that students acquire astonishingly high insight into the mathematical problems which they have programmed for solution on a digital computer. It is easy to see why this should be so, as a computer program must take into account all possible cases of a general nature . . . To generalize, by using the computer the student is forced to acquire insight into the general algorithm, the conditions under which it applies, and the general class of problems to which a given procedure can be applied as well as to those special cases to which the general solution is not applicable. This is clearly what we have always been trying to teach mathematics students, and contact with the computer makes it easier to accomplish the desired ends."<sup>18</sup>

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<sup>17</sup> John Kemeny, "The Role of Computers and Their Applications in the Teaching of Mathematics," Needed Research in Mathematics Education (New York: Columbia Teachers College Press, 1966), p. 113.

<sup>18</sup> Walter Hoffman, "Computer Mathematics--Secondary School or Higher Education?," Report of the Conference on Computer Oriented Mathematics and the Secondary School (Washington, D. C.: National Council of Teachers of Mathematics, 1963), p. 20.

Strachey, one of the pioneers of the first electric computer, asserts that the use of computers in an educational environment allows teachers the opportunity to put greater emphasis on the problem-solving process. He believes that students benefit greatly from having algorithms diagnosed immediately; and especially the opportunity to correct their mistakes "on-the-spot:"

"In the early days of computer programming-- say 15 years ago--mathematicians used to think that by taking sufficient care they would be able to write programs that were correct. Greatly to their surprise and chagrin, they found that this was not the case and that with rare exceptions the programs as written contained numerous errors . . . . The chief impact of this state of affairs is psychological . . . . The trouble, I think, is that so many educational processes put a high premium on getting the correct answer the first time. If you give the wrong answer to an examination question, you lose your mark and that is the end of the matter. If you make a mistake in writing your program--or, indeed, in many other situations in life outside a classroom--it is by no means a catastrophe; you do, however, have to find your error and put it right. Maybe it would be better if more academic teaching adopted this attitude also."<sup>19</sup>

The opinions of Kemeny, Hoffman, and Strachey suggest that computer-assisted problem-solving enhances student learning behavior. They are not alone in this belief. Meserve,<sup>20</sup> Dorn,<sup>21</sup> and others have made similar contentions. It now becomes a challenge to those conducting serious research on the topic to cite factual conclusions concerning

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<sup>19</sup>Christopher Strachey, "Systems Analysis and Programming," Scientific American, CCXV (September, 1966), pp. 112-118.

<sup>20</sup>Bruce E. Meserve, "Evolution in College Mathematics," The Continuing Revolution in Mathematics (Washington, D. C.: National Council of Teachers of Mathematics, 1968), p. 113.

<sup>21</sup>"The Computer: A Mathematics Laboratory," Journal of Educational Data Processing, Special Issue (Spring, 1963), p. 79.

such theories. With computer-assisted problem-solving becoming an integral part of many instructional programs, such research should soon be forthcoming. In a recent survey commissioned by the National Science Foundation and conducted by American Institutes for Research (AIR), 12.9 percent of more than 23,000 secondary schools queried were found to use computers for instructional programs.<sup>22</sup> Of these, most are using the problem-solving mode--with Dartmouth College and the Altoona (Pennsylvania) Area School District perhaps the most noted.

Dartmouth College embarked upon computer-assisted problem-solving in 1964 when it acquired a General Electric-265 computer system with the help of General Electric (GE) and the National Science Foundation.<sup>23</sup> By the fall of that year, a team of men led by Professors John Kemeny and Thomas Kurtz had increased the number of terminals interfaced to the system to 20; and had written and introduced the problem-solving language called BASIC to Dartmouth students. Today, Dartmouth's Computation Center is the focal point of a complex time-sharing system servicing several hundred terminals, including 25 high schools in the New England area.<sup>24</sup>

The first K-12 school district in the country to operate its own full-scale, computer time-sharing system was the Altoona Area Public Schools. Started in 1964 with the aid of a federal grant, the system has now been expanded to serve 4,000 students in 16 different schools

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<sup>22</sup>Charles A. Darby, Jr., Arthur L. Korotkin, and Tania Romashko, Survey of Computing Activities in Secondary Schools, National Science Foundation, Grant No. NSF-C584 (Washington, D. C.: American Institutes for Research, October, 1970), p. 9.

<sup>23</sup>Peter Carr, "Time-Sharing Offers a New Tool for General Teaching," Computerworld, IV (March 25, 1970), p. S/2.

<sup>24</sup>Ibid.

across five central Pennsylvania counties. During a visit to Altoona in 1969, Dr. Thomas Heslep, superintendent of the school district, told this writer that the computer is integrated as an instructional tool into the school system's mathematics and science curricula. He stated that there have been no drastic changes, however, in course content or teaching personnel. Instead, instruction in the BASIC language is incorporated directly into the mathematics and science curricula. Beginning with grade eight, students learn just enough programming each year to assist them in solving problems given at that particular mathematics or science level. When in high school, the Formula Translation (ForTran) language is also taught. In solving their problems, Altoona students do most of their work outside the classroom. They are required to make an analysis of their problem, draw a flow chart, and write a documented program in BASIC or ForTran. The program is then punched into paper tape at one of the remote teletypewriter terminals and transmitted to the computer.

When the computer was first installed in the Altoona Area Public Schools in 1964, some of the teachers complained that there was not enough time to teach all the required material and still introduce a computer problem-solving language. However, through actual experience with a test group, it was found that less time was necessary for drill work to reinforce difficult concepts; presumably because students had to analyze their problems completely before developing an algorithm. Hence, more material was covered since less time was necessary for tedious calculations. In addition, problems that were once bypassed because of their complexity and lengthiness, were now tackled with success. In one of the few studies investigating the effectiveness of

computer-assisted problem-solving, Altona found that experimental groups which were taught the BASIC language and computer usage for algebra problem-solving attained a higher level of learning than control groups which did not experience such instruction--especially in arithmetic reasoning.

A project somewhat similar to that of the Altona Area School District was conducted by the Massachusetts Board of Education from 1965 through 1967. The design of Project H-212, as it was called, made use of the computer as the basis for a laboratory approach to the presentation of mathematics. Classroom instruction was augmented by student experiments in devising and testing mathematical algorithms on the computer. Richardson lists the following questions which were identified for investigation in grades 6 through 12 during the project:

1. How can a time-shared computer be programmed as a useful tool for teaching mathematics?
2. How can classroom teachers be taught the necessary techniques to enable them to use this tool successfully?
3. How can multiple-user computer facilities be developed on an economically feasible basis for school use?
4. How can the mathematics curriculum best be augmented to make effective use of the computer as a tool for classroom instruction?<sup>25</sup>

Students participating in Project H-212 were taught to write computer programs in the problem-solving language called TELCOMP. However, the primary goal of the project was not to impart facility

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<sup>25</sup> Jesse O. Richardson, Teaching Mathematics Through the Use of a Time-Shared Computer, U. S. Department of Health, Education and Welfare, Report No. CRP-H-212 (Washington, D. C.: Office of Education, Bureau of Research, 1968), p. 1.

in programming for its own sake, but to exploit it for the presentation of mathematical ideas through classroom instruction and individual student laboratory work. Project H-212 sought to show that teaching concepts related to computing, programming, and information processing could be used to facilitate and enhance the presentation of standard school mathematical curricular material--including arithmetic, algebra, and elementary calculus.<sup>26</sup> As a result of the project, the following was concluded:

1. It is possible to construct programming languages of great expressive power, yet so simple to learn that they can be effectively taught to elementary school children.

2. Children are easily motivated to write programs at computer consoles. This kind of mathematical activity is immensely enjoyable to children generally, including those not in the top levels of mathematical ability.

3. Programming facilitates the acquisition of rigorous thinking and expression. Children impose the need for precision on themselves through attempting to make the computer understand and perform their algorithms.

4. A series of key mathematical concepts, such as variable, equation, function, and algorithm, can be presented with exceptional clarity in the context of programming.

5. The use of a programming language effectively provides a working vocabulary, an experimental approach, and a set of experiences for discussing mathematics. Mathematical discussion among high school

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<sup>26</sup>Ibid.



students, relatively rare in the conventional classroom, was commonplace in this laboratory setting.

6. Computers and programming languages can be readily used in either of two ways in the mathematics classroom:

- by individual students for independent study on extra-curricular problems or special projects.
- as a laboratory facility to supplement regular classroom lecture and discussion work.

7. A third way of using computers and programming that might have radical implications for the presentation of mathematics was uncovered--the concept of teaching programming languages as a conceptual and operational framework for the teaching of mathematics.<sup>27</sup>

Today's successor to Project H-212 is Project LOCAL (Laboratory Program for Computer-Assisted Learning). Initiated in June, 1967, to improve secondary school instruction by using the computer as a teaching aid, Project LOCAL is a cooperative endeavor. Five towns in Massachusetts participated in the project: Lexington, Natick, Needham, Wellesley, and Westwood. Originally funded under Title III of the Elementary and Secondary Education Act (ESEA), LOCAL is now chartered in Massachusetts as a tax-exempt educational corporation. The project's five PDP-8 computers, which can accommodate over 3,500 students, provide services to more than 15 school systems in the Boston Metropolitan area. In addition, 200 teachers from the area have been trained in the techniques of computer-assisted problem-solving.<sup>28</sup> The computers are used for two

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<sup>27</sup> Ibid., p. 2.

<sup>28</sup> Robert N. Havan, "Project LOCAL," (unpublished paper, Westwood, Massachusetts, 1970), p. 1.

main purposes:

1. To improve the achievement of mathematics and science curriculum objectives, especially in the areas of problem-solving skills and attitude.
2. To teach to the widest possible segment of the pupil population, a basic understanding of the computer in its role as an important element of modern society.<sup>29</sup>

#### Related Empirical Research

Two of the most comprehensive research studies relevant to computer-assisted problem-solving are those of Kieren<sup>30</sup> and Hatfield.<sup>31</sup> Both studies stemmed from the two-year Computer-Assisted Mathematics Project, better known as CAMP, conducted at the University of Minnesota High School. Financed through a grant from the General Electric Foundation, CAMP was directed by David Johnson. The project was originated as a development and research program at grades 7, 9, and 11 to identify appropriate material in the existing mathematics program that might be more effectively studied through designing computer programs. The grant supported a single teletypewriter terminal and related time-sharing contracts over two years of work.<sup>32</sup>

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<sup>29</sup>Ibid.

<sup>30</sup>Thomas E. Kieren, "The Computer as a Teaching Aid for Eleventh Grade Mathematics: A Comparison Study," (unpublished Ph.D. dissertation, University of Minnesota, 1968).

<sup>31</sup>Larry L. Hatfield, "Computer-Assisted Mathematics: An Investigation of the Effectiveness of the Computer Used as a Tool to Learn Mathematics," (unpublished Ph.D. dissertation, University of Minnesota, 1969).

<sup>32</sup>Kieren, op. cit.

Kieren's two-year study involved eleventh grade mathematics students in the ~~CAMP~~ project: 36 students during the first year; 45 students in the second year. All participants were assigned either to the computer group or the non-computer group for the study. The main difference in treatment between the two groups was that experimental participants learned much of their mathematics by writing BASIC programs which involved the problems, concepts, and skills from the regular mathematics course. In contrast, the control group did not use the computer or study any related material. During both years of the study, the control group and experimental group were taught by the same teacher using the same text material--the Secondary Mathematics Study Group (SMG) Intermediate Mathematics. Various measures of mathematical achievement were obtained during each of the two experimental years. Methods of variance and covariance analysis were applied to these measures to test hypotheses of no differences in group means after instruction. In addition, the proportions of correct responses were examined for 348 test items used during the second year. Kieren reported one rejection out of eight of the null hypotheses of no treatment effects during the first year. This rejection favored the mean of the computer class on the standardized Contemporary Mathematics Test, Advanced Level. No significant difference of treatment by previous achievement level interaction were found for the eight tests.<sup>33</sup>

"During the second year, the achievement of the computer class was found to be significantly higher according to the means for the Unit Test on Quadratic Functions when the analysis involved the pretreatment STEP 2B and Unit Test on Functions scores as covariates. Using the same eleven tests and the analysis of variance procedure, Kieren rejected the null

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<sup>33</sup>Kieren, op. cit.

hypothesis of no differences due to treatment for the means of the Unit Test on Trigonometry and the COOP Trigonometry in favor of the regular class. As in the first year, the test of interaction revealed no significant differences. However, an inspection of the cell means suggests that the computer seemed to be relatively more effective for students of average previous achievement."<sup>34</sup>

Kieren's study revealed that the null hypothesis of no difference in the proportion of students correctly responding to a test item was rejected for 43 of the 348 items included. A thorough study of the items indicated that the computer had little positive effect on simple skills such as computation with complex numbers and geometrical treatments of trigonometry. In contrast, the computer "seems to make its strongest contributions in the areas of complex skills, organization of data and drawing conclusions therefrom, and the study of infinite processes."<sup>35</sup>

In another two-year study similar to Kieren's, Hatfield worked with seventh grade students. The design and instructional procedures were analogous to those in Kieren's study. However, revisions of the supplementary materials and constructed tests were sufficiently extensive to require that each year be treated as a separate experiment. For purposes of analysis, each treatment was blocked into three levels according to previous achievement on the STEP Mathematics 3A test. Analysis of variance was used to compare the main effects resulting from treatments and differential effects of treatments across previous achievement levels. To identify the particular contributions of each treatment, proportions

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<sup>34</sup> Hatfield, "Computers in Mathematics Instruction," op. cit., p. 17.

<sup>35</sup> Kieren, op. cit., pp. 127-128.

of students responding correctly to a test item were compared.<sup>36</sup> Hatfield describes the results of the study:

"During Year 1, the effect due to treatment as measured by group means was significant for only one (Numeration Systems) of the eleven criterion tests. This difference favored the noncomputer treatment with the greatest difference in cell means occurring at the low previous achievement level. During this initial unit, the computer students also learned the BASIC programming procedures which seemed to interfere with the concurrent study of numeration systems. The equality of item proportions was tested for 266 items. The proportions of correct responses in the control group was significantly greater on 22 items while on 19 items the computer group was favored. On 6 of the 8 significant "skill" items, the control group was favored.

"During Year 2, the means analysis of treatment effect revealed significance on one (Elementary Number Theory) of the six unit tests and two (Contemporary Mathematics Test and Thought Problems) of the six post-treatment tests. These significant differences all favored the computer treatment. Comparisons of cell means on these three tests revealed that the High and Average previous achievement computer groups were especially favored. The number theory unit was recognized as a particularly relevant setting for the use of the computer. The emphasis of this unit was on exploration and inquiry with problems involving many laborious calculations. The orientation was to use the computer as a laboratory tool to explore a number of interesting number theory settings. The proportions for 327 items revealed that the computer group scored significantly better on 25 items while 13 items favored the control group. The computer group was significantly favored on 12 of the 16 "problem" items and 10 of the 16 "concept" items."<sup>37</sup>

The results of Kieren and Hatfield's studies do not support computer-assisted problem-solving as the optimal instructional approach in all settings. Nevertheless, the studies do provide evidence that

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<sup>36</sup> Hatfield, "Computers in Mathematics Instruction," op. cit., p. 18.

<sup>37</sup> Hatfield, "Computers in Mathematics Instruction," op. cit., pp. 18-19.

seventh and eleventh grade students can learn to program the computer to learn mathematics. Furthermore, in several particular settings, the computer-approach did result in significantly improved performance.

#### Cost of Computer-Assisted Problem-Solving

One of the major obstacles to implementation of computer-assisted problem-solving in education is the high cost of computer time-sharing. When considering such instruction, three distinct "hardware" costs should be considered: (1) the computer; (2) the terminal; and (3) the telephone.<sup>38</sup>

The cost of the computer can be either a variable charge, whereby the user need pay only for what he uses; or a flat rate for the month, offering the user unlimited service of the computer for one price. Utilization of the variable charge method involves a separate, distinct expense for each of the following:

1. Terminal Connect Time--amount of time in minutes that the terminal is on-line to the computer, regardless of whether the computer is manipulating information or not.
2. Central Processing Unit (CPU) Time--amount of time in seconds that the computer is in use manipulating or calculating information.
3. Storage--volume of information that is saved in auxiliary files of the computer for future reference.

In most cases, charges for terminal connect time range from \$3 to \$25 per hour; CPU time ranges from \$.02 to \$.08 per second; and storage ranges from \$1 to \$3 per unit or program. Specific costs depend upon the vendor, the user, and the volume of use. Fortunately,

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<sup>38</sup>Post, op. cit., pp. 69-83.

educational users often receive a discount, sometimes as high as 50 percent from commercial rates. Depending upon the volume of use, some vendors also offer a "sliding scale" charge to customers; Table 1 is an example.

TABLE 1  
A SAMPLE OF COMPUTER TIME-SHARING "SLIDING SCALE" RATES

DAYTIME RATES			
For Connect Time between 7:00 A.M. and 6:00 P.M. Monday through Friday			
IF CONNECT TIME IS	CONNECT	CPU	STORAGE UNITS
0-25 hours, your rate is	\$10.00 per hour	\$.05 per second	\$2.00 per unit
25-50 hours, your rate is	8.00 per hour	.04 per second	1.50 per unit
over 50 hours, your rate is	6.50 per hour	.03 per second	1.00 per unit
NIGHT TIME RATES			
For Connect Time between 6:00 P.M. and 7:00 A.M. and All Day Saturday and Sunday			
	CONNECT	CPU	STORAGE UNITS
For all usage during nighttime and Saturdays and Sundays	\$4.00 per hour	\$.03 per second	Based on day-time rate

In contrast to the variable charge method, many vendors also offer a "flat rate." Utilization of this method of payment allows the user unlimited connect time, unlimited CPU time, and sometimes even unlimited storage--all at a fixed rate per month. Careful scrutiny of specific needs helps to determine which of the two methods of payment is

most economical.

In addition to the cost of computer time, a second expense to computer time-sharing users is the cost of the terminal or console device --often a teletypewriter or a visual display unit. When using computer time-sharing, such a device handles the input and the output, sending data into the computer and printing or displaying the computer's output.

The most popular of the computer terminal devices used in education is the Automatic Send Receive (ASR), Model 33 teletypewriter with a paper-tape punch/reader. The paper-tape punch/reader is an attachment to the Model 33 console which allows the user to punch input (e.g. programs) while the terminal is off-line; that is, without connecting the terminal to the computer:

"A program is typed at the terminal exactly as if it were being sent to the computer and is reproduced at the same time in the form of a string of punches in a length of paper tape. When the tape is fed back through the tape reader on the terminal, the Teletype will print out the identical program, so typographical and programming errors can be corrected without the computer being accessed. Not only is computer time saved by sending only 'precorrected' programs, but programs which are read in by tape instead of by manual typing are sent at the rate of 10 characters per second, which is considerably faster than most students type. This increased input speed saves enough computer time to more than offset the cost of the tape punch/reader attachment."<sup>39</sup>

The cost of the ASR-33 without the paper-tape punch/reader depends upon the vendor and competition, usually ranging from a low of approximately \$1,000. Without the attachment, the terminal is called a Keyboard Send Receive (KSR)-33.

More expensive than the ASR-33 is the ASR Model 35, a larger, quieter, and more durable terminal. Like the ASR-33, the Model 35 can

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<sup>39</sup>Post, op. cit., pp. 79-80.



be purchased with or without the paper-tape option.

The newest entry into the ASR series is the Model 37 which operates at 15 characters per second instead of the usual 10 characters per second. However, because of the low cost and relatively few complaints, the ASR-33 remains the most popular terminal used in education today; especially the mobile version. Having wheels on the unit allows teachers the opportunity to easily take the terminal into the classroom, provided that the room is equipped with a telephone and a 110-volt electrical outlet.

Regardless of the model or the options, all three ASR models can be leased as well as purchased. Leasing rates begin at approximately \$50 per month for an off-line, Model 33 terminal; \$85 for an on-line, Model 33 terminal.

Although the ASR is the most popular computer terminal utilized by educators, there are many other "hard copy" terminals available. American Data Systems, DATEL Corporation, International Business Machines (IBM), and several other vendors market such terminals. Many of these units use the major components of the IBM, Model 2741 Selectric Typewriter, modifying it for use as a computer terminal.

Far more expensive than the hard copy terminals which provide paper output, are the visual display terminals having a television-like screen. Activated by the computer, output on the screen is printed silently in standard letter and number characters--or graphic displays. Some display units are even capable of output showing slopes, intercepts, parabolas, and drawings. Although most input is generated via a typewriter keyboard, some units also accept graphic input from an electronic pointer called a light pen. Thus, patterns of light can be traced

directly onto the cathode ray tube (CRT) screen. Speed, quiet, and visual display are three of the main advantages of the CRT terminals. However, recent development of the non-impact printer has increased the demand for "hard copy" terminals because they too are quiet and fast. In addition, visual display terminals are two to five times more expensive than the hard copy terminals.

A third hardware expense, one which is often overlooked when considering computer time-sharing, is that of the telephone. Indeed, if it were not for the transmission provided by the telephone company, remote computer time-sharing would not be possible today. It is the telephone line that provides the link between the terminal device and the computer. To access the computer, the user need only dial the telephone number of the computer on the terminal data phone; then type the appropriate code numbers on the typewriter keyboard of the terminal.

Because a teletypewriter terminal needs only a voice-grade transmission line the same as a telephone, the terminal can be wired into an ordinary telephone jack. Thus, just as telephone transmission is charged on the basis of distance and time, so is data transmission. If the data transmission via the terminal and/or computer requires a long-distance phone call, the telephone expense can be substantial. However, special telephone-line arrangements can sometimes be made with the local telephone company to reduce the high cost. Much depends upon the telephone company's local facilities and services.

#### Computer-Assisted Problem-Solving and Educational Theory

Of great importance to educators is the potential of computer-assisted problem-solving to support curriculum development; and in a

variety of ways. Computer-assisted problem-solving does not appear to be restricted in its application to any particular academic discipline. Instead, the breadth of potential application may be as great as that to which books, films, and tape recordings have been applied. Hopefully, current and future research will help to identify the most beneficial role of this new mode of instruction. Even more important, computer-assisted problem-solving may permit the meaningful development of learning principles in the field of psychology; and a more direct application to the prescriptive field of instructional development.<sup>40</sup> If these two fields can be brought closer together, most educators agree that it will contribute measurably to instructional development and curriculum design.

As a pedagogical tool, computer-assisted problem-solving is student oriented and initiated in contrast to the other modes of CAI which emphasize "canned" dialogue. Students have the opportunity to make decisions relative to the strategies they develop by limiting the computer output to the particular information or variables being examined. Indeed, formal knowledge is only part of the instructional process. Heuristic knowledge,<sup>41</sup> concerned with the art of solving problems, is also important. Such knowledge is emphasized when students write programs to solve problems. As Hatfield points out:

"The design of computer algorithms seems ideal for experiencing such heuristic precepts as 'formulate a plan,' 'find a related problem,' 'observe special

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<sup>40</sup>Robert J. Seidel, "Computers in Education: The Copernican Revolution in Education Systems," Computers and Automation, XVIII (March, 1969), p. 24.

<sup>41</sup>George Polya, How To Solve It (Garden City, New York: Doubleday Anchor Book, 1957).

cases,' or 'simplify the conditions.'"<sup>42</sup>

Since computer-assisted problem-solving emphasizes concept learning, discovery, and the opportunity for heuristic knowledge, it is not surprising that proponents of the mode subscribe to the ideas of Jerome Bruner. A look at Bruner's four essentials to develop a theory of instruction<sup>43</sup> illustrates the compatibility of computer-assisted problem-solving with his cognitive approach to learning:

1. The experiences which most effectively implant in the individual a predisposition toward learning.
2. The ways of structuring the knowledge so that it can be most readily grasped by the learner.
3. An optimal sequence to present the materials to be learned.
4. The form and pacing of rewards and punishments in the process of learning and teaching.

Bruner contends that a theory of instruction must be prescriptive rather than descriptive; that it must set forth rules concerning the most effective way of achieving knowledge or skill. He asserts that the predisposition to explore alternatives is important to achieve learning; and that instruction must facilitate and regulate the exploration of alternatives on the part of the learner. In describing such search behavior, Bruner believes that there are three aspects to the exploration of alternatives: activation, maintenance, and direction. Stated more definitively, activating exploration of alternatives necessitates uncertainty

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<sup>42</sup>Hatfield, "Computer-Assisted Mathematics: An Investigation of the Effectiveness of the Computer Used as a Tool to Learn Mathematics," op. cit., p. 14.

<sup>43</sup>Jerome S. Bruner, Toward a Theory of Instruction (Cambridge, Massachusetts: Harvard University Press, 1967), pp. 39-72.

and curiosity; maintenance of exploration requires that the benefits from exploring alternatives exceed the risks incurred; and the appropriate direction of exploration depends upon a sense of the goal of a task and a knowledge of the relevance of tested alternatives to the achievement of that goal.

Bruner's ideas concerning the form and pacing of reinforcement are of particular interest to those involved in computer-assisted problem-solving:

"Learning depends upon knowledge of results at a time when and at a place where the knowledge can be used for correction. . . Knowledge of results should come at that point in a problem-solving episode when the person is comparing the results of his try-out with some criterion of what he seeks to achieve. . . Knowing is a process, not a product."<sup>44</sup>

Nevertheless, one of the aspects of computer-assisted problem-solving which has been most controversial is the immediate feedback to students inherent in the on-line process. There is conflicting evidence regarding such quick response each time a student makes an error. Suppes depicts the conflict:

"It is not clear to what extent students should be forced to seek the right answer, and indeed whether this search should take place more in what has come to be called a discovery or inductive mode, as opposed to more classical modes of instruction which consist of giving a rule followed by examples and then exercises or problems that exemplify the rule. A particularly troublesome issue that has come to the fore in recent research is the question whether different kinds of reinforcement and different sorts of reinforcement schedules should be given to children with different basic personalities."<sup>45</sup>

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<sup>44</sup>Ibid., pp. 50-51, 72.

<sup>45</sup>Patrick Suppes, "On Using Computers to Individualize Instruction," The Computer in American Education (New York: John Wiley and Sons, Inc., 1967), p. 21.

Closely related to the personality differences cited by Suppes is the dispute concerning different cognitive styles; for example, whether children are impulsive or reflective in their approach to problems. Perhaps the primary difficulty with the research on cognitive styles, as it relates to computer-assisted problem-solving, is that the research is primarily at an empirical level. As Suppes points out:

"The reach of theory is as yet very short, and it is not at all clear how the empirical demonstration of different cognitive styles can help us to design highly individualized curriculum materials adapted to these different styles. An even more fundamental question of educational philosophy asks how much the society wants to accentuate these differences in style by catering to them with individualized techniques of teaching."<sup>46</sup>

Without a doubt, the most persistent problem in education is the translation of theory into practice. This problem is particularly acute in relating psychological theory to specific instructional procedures. Computer-assisted problem-solving requires that the teacher commit himself to a theory of teaching. Equally important, however, is the need to evaluate the theory. Thus, some people visualize computer-assisted problem-solving as a potential catalyst for translating instructional theory into practice. Hence, one of the most critical applications for computer-assisted problem-solving may be as a school laboratory to learn more about learning; perhaps ultimately, to build a theory of instruction. Because there are tremendous new advantages of storage, retrieval, access, interaction, and complete attentiveness to the needs and desires of the individual student, there may even be the potential to develop a new model of education.<sup>47</sup>

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<sup>46</sup> Ibid., p. 22.

<sup>47</sup> Seidel, op. cit., p. 25.

To further support the value of computer-assisted problem-solving, proponents cite Jean Piaget's often-discussed theory of intellectual development.<sup>48</sup> According to Piaget, the development of intelligence means the development of logical thinking. He emphasizes the importance of activity and discovery, stating that an operation is the essence of knowledge; that it is an interiorized action which modifies the object of knowledge. Accordingly, he identifies three important characteristics inherent in an operation: (1) always an interiorized action; (2) always reversible; (3) never isolated, always linked to other operations. Piaget points out that "ordering things in a series" is an example illustrative of an operation; thus, one might conclude that he would classify the act of writing a computer program as an operation which helps to develop logical thinking.

Not only do those interested in computer-assisted problem-solving refer to Piaget's theory relative to the cognitive domain of behavior, but also to his ideas concerning the affective domain:

"There is close parallel between the development of affectivity and that of the intellectual functions, since these are two indissociable aspects of every action. In all behavior the motives and energizing dynamisms reveal affectivity, while the techniques and adjustment of the means employed constitute the cognitive sensorimotor or rational aspect. There is never a purely intellectual action, and numerous emotions, interests, values, impressions of harmony, etc., intervene, for example, in the solving of a mathematical problem. Likewise, there is never a purely affective act, e.g., love presupposes comprehension. Always and everywhere, in object-related behavior as well as in interpersonal behavior, both elements are involved because the one presupposes the other."<sup>49</sup>

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<sup>48</sup>Jean Piaget, "Development and Learning," Journal of Research in Science Teaching, II (1964), pp. 176-185.

<sup>49</sup>Frank E. Williams, "Models for Encouraging Creativity in the Classroom by Integrating Cognitive-Affective Behaviors," Educational Technology, IX (December, 1969), p. 9.

The work of Robert Gagne is also cited often by those involved in computer-assisted problem-solving; especially Gagne's contention that instruction must be designed to teach the student the capability of doing something, not of "knowing" something:

"The notion of performance objectives is important because it emphasizes the doing. What is being taught is an intellectual skill, not recallable verbal information. To use other terms familiar to curriculum designers, the primary purpose of instruction is process not content. . . the purpose of education is to teach students to think. . . the notion that computers can be used to provide practice for the student in solving problems which simulate real situations is a very appealing one."<sup>50</sup>

#### Impact of Computer Time-Sharing on Society

It is a matter of recorded history that any social group which crosses the threshold of technological change will vacillate between optimism and pessimism. For example, some educators contend that "computer-assisted instruction is destined to have an impact on society of a magnitude comparable to that of the automobile." In contrast, other educators have warned that the role of the computer in education is extremely limited. They argue: "Teaching is a highly personalized process; no computer can encourage Johnny by giving him a friendly pat on the head."<sup>51</sup>

In an attempt to achieve objectivity relative to such debate, one must ultimately ask if the principles commonly proclaimed by psychologists pertaining to human learning are accepted. If they are not, then there seems to be no end to the debate. If however, as it appears,

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<sup>50</sup> Robert Gagne, "Educational Technology as a Technique," Educational Technology, VIII (November 15, 1968), pp. 10-12.

<sup>51</sup> The Computer in Education, An IDEA Occasional Paper (Dayton, Ohio: Institute for Development of Educational Activities, Inc., 1970), p. 8.



the principles are accepted, then many elements in any deliberately created learning situation permit computerization.<sup>52</sup> Obviously, such an assumption does not identify the specific elements in a learning environment which can be computerized; nor the manner in which such computerization can be implemented to achieve desirable ends:

"It does not deny the fact that, as in the case of management and any other utilization of computer systems, the ultimate control and design must be vested in men. Likewise, it does not preclude the possibility that parts of some deliberately created learning experiences must remain under the direct control of persons.

"More than any of the other educational technologies, the computer focuses on the urgent problem of developing a specific combination of people and things to create an efficient and effective learning environment. . . this theorizing leaves many questions unanswered about what types of computer hardware and software may be necessary for educators to accomplish particular kinds of learning that may now be possible. Moreover, it ignores cost."<sup>53</sup>

No doubt, the current popularity of computer-assisted problem-solving in education is due primarily to the success of computer time-sharing. The development of the time-shared technique has allowed many users to interact with a single computer in a seemingly simultaneous manner. Even more amazing, a user is now able to gain access to the power of the computer while he is in a remote location--merely by utilizing a terminal device; it is necessary only that the data terminal be connected to the computer via a telephone line. Indeed, the terminal may be located anywhere that there is a telephone and a 110-volt electrical outlet.

Many proponents of computer time-sharing contend that it is

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<sup>52</sup> Ibid.

<sup>53</sup> Ibid.

quite possible during the next decade that the use of time-sharing will be as common as the use of telephones. They argue that people in education and industry will become more familiar with the benefits of computer time-sharing, as well as dependent upon it, and that the day will come when people will have a data terminal in their homes for quick and easy access to computer services.

Undoubtedly, the power of computer time-sharing systems will become much greater in the years ahead. One central processor will be able to service many more terminals than the number currently operating from a single system. Such growth will result merely as a matter of economics for the time-sharing industry. More important for the user, however, there is a strong likelihood that improved terminal devices will be available.

Perhaps the single most important use of computer time-sharing in the future will be the development of data bases for management information systems. Until recently, one of the main obstacles to the development of information systems was the difficulty of gathering the timely information for a common data base for many remote sites. The capability of a computer time-sharing system is ideally suited for this task. Information can be entered from a number of different terminal locations into one central computer. It can then be used by other remote locations as well as the place of input. However, the capability of numerous terminal locations accessing various files creates a major problem--the possibility of invasion of privacy. To resolve this dilemma, greater emphasis will have to be given to the development of discriminating codes so that confidential files will be accessible only to certain people. At this point in time, there is no apparent foolproof solution to this explosive and

delicate problem. Alas, it appears that computers are beginning to invade every phase of American life:

"Much more than you realize, the lives of ordinary people are being controlled by computers. These mechanical wizards do astonishing things, and experts say the computer age is only dawning. It is estimated sales will triple in just seven years."<sup>54</sup>

Of the \$21.5 billion worth of computers used in 1969, approximately \$1 billion was spent for time-sharing systems. As indicated in Table 2, it is expected that the investment will grow to \$5 billion by the mid-1970's.<sup>55</sup>

TABLE 2  
COMPUTER SYSTEMS IN USE IN THE UNITED STATES

YEAR	NUMBER OF COMPUTERS	DOLLAR VALUE
1955	244	\$177 million
1969	53,500	\$21.5 billion
1975	128,000	\$54 billion (est.)

In addition to threatening one's privacy, another important problem relevant to the use of computers is recognized in the question: "Who's in control, computers or people?" The structure of society is already so complex that it is difficult unraveling the control mechanisms to determine where the dangers exist. Thus, it is anticipated that the problem will get more complex and difficult to combat as computers assume more and more service functions. As Hamming points out:

<sup>54</sup>"How Computers are Changing Your Life," U. S. News and World Report, LXVII (November 10, 1969), p. 96.

<sup>55</sup>Ibid.

"When we program computers to interact with humans, will we, in the interest of machine efficiency decrease the options available to humans, or will we, at the expense of machine efficiency, increase the options available to humans? . . . Each programmer can help make the decision--all he has to do is look at his program and ask the simple question: Am I decreasing or increasing the number of options available to those who will have to interact with this system compared to what they had before? If I am decreasing them, then it's bad. . . . If we are going to end up with a liveable society, we are going to have to insist that those who program computers understand their social responsibilities.<sup>56</sup>

It is expected that the computer will have an important and increasingly greater impact on the way that individuals live. Indeed, a significant portion of the impact will be a direct result of time-sharing systems. More and more, it is expected that computers will monitor bank accounts, prepare payrolls, help to sell reservation tickets (for shows, sporting events, airlines, etc.), feed livestock, design cars, plan cities, combat crime, aid in medical diagnosis and prognosis, guide spacecraft, fight pollution, assist teachers, etc. A survey of 250 computer experts from 22 countries summarizes the prediction that all major industries will be controlled by computers by the end of the twentieth century.

"The impact on the labor force will be immense. Computers controlling manufacturing processes will replace the man-and-machine production lines of today. Computer-controlled manufacturing functions, industrial robots and numerically controlled lathes are the forebears of the larger automated facilities of tomorrow.

"Experts estimate that as much as 50 percent of today's labor force will be dislocated by advancing computer technology. Most will not be out of jobs. New opportunities will be opened for making decisions that end up on tapes in the machines. Working hours may be shorter, overtime eliminated."<sup>57</sup>

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<sup>56</sup>Richard W. Hamming, "Computers and Society," AEDS Monitor, VIII (July-August, 1969), pp. 8-9.

<sup>57</sup>"How Computers are Changing Your Life," op. cit., p. 97.

Nevertheless, educators do not foresee the day that computers will replace teachers in the classrooms. Instead, it is expected that computers will be used to assist and/or help manage instruction and learning. Hence, the direct effect of the computer may be less crucial than the change it will force in thinking patterns. Illogical thinking is incompatible with computer utilization. Such rigor in thinking will inevitably spread beyond the areas of computer applications. According to Dr. Bruce Gilchrist, executive director of the American Federation of Information Processing Societies: "I expect the computer will be almost a universal tool for educated professionals who handle either numbers or data."<sup>58</sup>

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<sup>58</sup>"How Computers Are Changing Your Life," op. cit., p. 98.

## CHAPTER III

### SETTING, PROCEDURES, AND DESIGN OF THE STUDY

No experimental study in education has every been designed in which the variables could be absolutely controlled in a practical setting. To accomplish such is impossible. The best that can be done in conducting an experimental investigation in a public school setting is to come as close as possible to the "ideal" design. Such was the goal of the study reported in this dissertation.

#### Description of the Community Setting

The community of Dearborn, Michigan, was selected as the locale for the experimental study because the Dearborn Public School System is intensely interested in pursuing instruction relevant to computer-assisted problem-solving; and also because the writer has been asked to direct the school system's effort in this endeavor.

Dearborn is a city of approximately 115,000 people, almost all of whom are caucasian. The community is a western suburb of Detroit, automotive capital of the world. A rather unique feature of Dearborn's geography is its internal division into east and west ends, often called East Dearborn and West Dearborn. The division is a result of the consolidation of two smaller communities in 1929: Fordson and Dearborn. Most of the land between the east and west section of the city is owned by the Ford Motor Company. Dearborn has been the central headquarters for the company's world-wide operations since Henry Ford Sr. moved into the area and bought massive plots of real estate more than a half century ago. After extensive research and planning during recent years, the

company is now ready to "break soil" in this area by building residential apartments, a cultural center, a shopping complex, an industrial park, and a "high rise" hotel.

Most of Dearborn's current shopping area is centered along the city's main street, Michigan Avenue, which travels east and west. If one were to visit Dearborn, it is likely he would drive along this main thoroughfare which further divides the city into "North of Michigan Avenue" and "South of Michigan Avenue."

According to Dearborn's Master Plan, 69.8 percent of the land in Dearborn is currently developed; 30.2 percent is vacant.<sup>1</sup> The city's 25.2 square miles of land is zoned into five major classifications, as identified in Table 3.<sup>2</sup>

Together with the gigantic Rouge Plant of the Ford Motor Company which occupies 1,200 acres of land,<sup>3</sup> most of the city's industry is located in East Dearborn. In contrast, West Dearborn is primarily residential, having resulted from the major building boom after World War II. Because of the large labor force needed to operate the Ford Motor Company and other industry in the city, a majority of the people employed in Dearborn do not live in the city.<sup>4</sup>

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<sup>1</sup>The League of Women Voters of Dearborn, Dearborn and Its Government, a handbook for citizens and students prepared by the League of Women Voters of Dearborn (Dearborn: The League of Women Voters of Dearborn, 1964), p. 22.

<sup>2</sup>Ibid.

<sup>3</sup>Ibid.

<sup>4</sup>Ibid.

TABLE 3  
ZONING SECTIONS OF THE CITY OF DEARBORN, MICHIGAN

ZONE	PERCENT OF LAND
Residential . . . . .	34.9%
Commercial . . . . .	4.6%
Industrial . . . . .	21.2%
Streets and Alleys . . . . .	24.2%
Parks and Playgrounds . . . . .	15.1%
TOTAL . . . . .	100.0%

#### Overview of the Educational Setting

Consistent with many other school districts throughout the United States, the Dearborn School System moved through a period of rapid growth following World War II. At the present time, however, Dearborn's school population appears rather stable. Of course, this status may change when the large area of vacant real estate owned by the Ford Motor Company between East Dearborn and West Dearborn is developed for residential occupancy.

The Dearborn School System is a kindergarten through community college school district. Approximately 21,500 students are enrolled in the kindergarten through twelfth grade (K-12) schools; about 12,000 students attend the community college.<sup>5</sup> To meet the instructional needs

<sup>5</sup>"The Schools," a pamphlet for employees and citizens prepared by the School Relations Office of the Dearborn Public Schools (Dearborn: The Dearborn Public Schools, 1969), p. 1.



of the students, 1,167 teachers are employed in the K-12 program; 187 full-time and approximately 260 part-time instructors at the community college.<sup>6</sup>

The Dearborn Board of Education operates 31 school buildings in the K-12 program: 25 elementary schools, 9 junior high schools, and 3 senior high schools. Six of the buildings house both elementary and junior high students. Since 1965, all K-12 buildings operate on an annual promotion basis.

The curriculum of the K-12 program of the Dearborn Public Schools has seven main objectives:<sup>7</sup>

1. To develop intellectual and emotional sensitivity to the moral, spiritual, and aesthetic values of life in a democratic society.
2. To promote a student's understanding and appreciation of himself as a human being.
3. To give a student an ability to read, write, speak, listen; and an ability to use numbers, appreciate the arts, and express himself and understand others.
4. To foster critical and creative thinking for a student in meeting the problems of life.
5. To give a student skills in living effectively with others.
6. To create in a student an understanding as well as an appreciation of the physical world.
7. To create in a student an understanding and appreciation

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<sup>6</sup> Ibid.

<sup>7</sup> The League of Women Voters of Dearborn, Dearborn Schools, a handbook for citizens and students prepared by The League of Women Voters of Dearborn (Dearborn: The League of Women Voters of Dearborn, 1965), p. 31.

of the social world.

Because the Henry Ford Community College is a part of the Dearborn Public School System, any Dearborn resident who has graduated from high school is permitted to enroll on a preferential basis. Nevertheless, only about one-third of the student body is made-up of residents.<sup>8</sup> Non-residents are accepted, but grade requirements and tuition are higher for them.

Most of the part-time students at the community college are enrolled in the employment-related technical programs, while most of the full-time students are working toward a four-year degree.<sup>9</sup> Full-time students usually take their first two years of course-work at the community college; then transfer to another college to obtain a degree. Most of these students attend state universities in or near the Detroit metropolitan area.

The curriculum of the Henry Ford Community College was designed to meet the educational needs of students beyond those already provided in a kindergarten through twelfth grade program:<sup>10</sup>

1. To provide the first and second years of work in the liberal arts and pre-professional fields for those students who wish to transfer to higher educational institutions.
2. To provide one to two year technical programs of business and industrial education for those students who expect to terminate their formal education by preparing for employment at the semi-professional level.

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<sup>8</sup>Ibid., pp. 45-46.

<sup>9</sup>Ibid., p. 24.

<sup>10</sup>Ibid.

3. To provide a program of two additional years of general education for the social, cultural, and personal development of students who wish to continue beyond senior high school.
4. To provide single courses or combinations of courses for adults of the community in those fields in which there is sufficient interest and demand to warrant the organization of classes.
5. To provide counseling and guidance services to all individuals who desire further education and training.
6. To provide educational services to organizations and individuals of the community, including: speakers, resource personnel or material, organization of special institutes or programs, reading lists, and educational counseling and testing.

#### Selection of the School and Teacher

All three public high schools in Dearborn are comprehensive schools, offering a broad variety of courses that include both vocational and academic subjects. All students, regardless of their program, take the same required subjects to be graduated. Differences between college preparatory and occupational programs are a result of elective courses chosen by students.

Dearborn's three public high schools include one with a unique curricular approach; a second with a heavy enrollment of college preparatory students; and a third with a balanced, diversified student body. It is the latter school that was selected for the study: Fordson High School. Table 4 identifies Fordson's enrollment at the time of the study.

Although the socio-economic status of families in the Fordson High School attendance area is quite diversified, the area as-a-whole is generally regarded as "lower-middle," with a strong inclination toward

employment in manufacturing.<sup>11</sup> Examination of the cumulative records of students who participated in the study indicated that members of the experimental group and control group appeared quite typical when compared to the total enrollment of the school.

TABLE 4  
FORDSON HIGH SCHOOL ENROLLMENT, 1/30/70

Grade	Number of Students
10 . . . . .	771
11 . . . . .	651
12 . . . . .	659
Total . . . . .	2,081

Located in East Dearborn, the selection of Dearborn's oldest public high school was based primarily on the diversified student body and the experience and competency of one of the school's mathematics teachers, George Gullen. Besides being one of Dearborn's finest teachers, Gullen's mathematics experience includes considerable background in computer-assisted problem-solving. Having such qualifications made him a natural choice to participate in the study.

#### Selection of the Subject-Matter and Student Participants

After careful review, algebra-trigonometry was selected as the subject-matter for the experimental study. The selection of algebra-trigonometry was based upon the volume of relevant course materials

<sup>11</sup>The League of Women Voters of Dearborn, Dearborn and Its Government, op. cit., pp. 26-27.

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involving computer-assisted problem-solving that has been developed by Gullen and others. In spite of this, it was necessary to develop additional materials before and during the study.

Because all students are not required to take algebra-trigonometry in the Dearborn School System, many potential participants in the study were eliminated from consideration through the normal process of selecting elective courses. Algebra-trigonometry is usually offered to students in the Dearborn School System during the second semester of grade 11. Of the 651 eleventh grade students at Fordson High School during the 1970 spring semester, 122 elected algebra-trigonometry. Based upon previous achievement in mathematics, 102 of the 122 students were classified as "middle ability" level; the other 20 students were classified as "accelerated" or "high ability" level. Because there were not enough "accelerated" algebra-trigonometry students to establish both a control group and experimental group--and because there is greater interest in the effect of computer-assisted problem-solving on "middle-ability" students--it was decided to conduct the study with the latter group. Hence, through computer scheduling, all 102 of the "middle ability" students were randomly scheduled into four groups, two of which were to be assigned to Gullen; the other two groups to another teacher. To assure careful grouping, the records of all 102 students were reviewed. Special attention was given to each student's past achievement in mathematics, intelligence quotient, and social factors that might bias the study. After careful screening, participants in only two of the four groups were considered matched for a control group-experimental group study. These two groups were assigned to Gullen. Students in the other two groups were not randomly assigned.

Course Objectives and Course Content

The senior high school mathematics curriculum in the Dearborn School System is a well organized, innovative curriculum that is the result of the exhaustive work of many teachers and administrators over a long period of time. Changes in curriculum are continually being made to incorporate the most up-to-date programs, methods, and techniques-- when it is felt that such changes help to better meet the needs of local students. Currently, many educators in Dearborn as well as elsewhere believe that a well-developed, modern curriculum should not necessarily be changed to permit the use of a computer as a teaching and/or learning tool. They argue that it is better if the computer is used to help attain those objectives already deemed relevant and/or necessary to meet student needs. Hence, in this study, the computer was used to help attain the current algebra-trigonometry objectives. Moreover, evaluation of the computer's effectiveness was based upon student achievement relative to those objectives; that is:

1. To understand algebra as a study of the structure of the systems of real and complex numbers.
2. To recognize the techniques of algebra and trigonometry as reflections of the structure of the systems of real and complex numbers.
3. To acquire facility in applying algebraic and trigonometric concepts and skills.
4. To perceive the role of deductive and inductive reasoning in algebra and trigonometry.
5. To appreciate the need for precision of language.
6. To comprehend the function concept and its importance in

mathematics.

To help attain the stated objectives, the algebra-trigonometry course consists of five major teaching units. Unit One is a brief review of the previous semester's work in algebra, with special emphasis on irrational numbers. This review leads to an introduction of radicals in equations. Unit Two continues to emphasize algebraic subject-matter; then a gradual shift in emphasis is made to trigonometry in Units Three, Four, and Five. The following outline identifies the content of the course in more detail:

### Course Outline

#### Algebra-Trigonometry

#### Unit One: Algebraic Review Material and Radicals in Equations

- Review
- Using Radicals to Solve Quadratic Equations
- Relations between Roots and Coefficients of a  
Quadratic Equation
- The Nature of the Roots of a Quadratic Equation
- Solving Quadratic Inequalities
- Irrational Equations

#### Unit Two: Exponential Functions and Logarithms

- From Exponents to Logarithms
  - Rational Numbers as Exponents
  - Real Numbers as Exponents
  - Exponential and Logarithmic Functions
- Using Logarithms
  - Common Logarithms
  - Interpolation



Products and Quotients

Powers and Roots

Combined Operations

Using Logarithms to Solve Equations

Unit Three: Trigonometric Functions and Complex Numbers

-- Coordinates and Trigonometry

Rays, Angles, and Points

Sine and Cosine Functions

The Trigonometric Functions

Special Angles

-- Evaluating and Applying Trigonometric Functions

Using Tables

Logarithms of the Value of Trigonometric Functions

Reference Angles

-- Vectors

Adding Vectors

Resolving Vectors

-- Working with Complex Numbers

Complex Numbers

Multiplying Pure Imaginary Numbers

Complex Numbers and Quadratic Equations

Unit Four: Trigonometric Identities and Formulas

-- Identities Involving One Angle

The Fundamental Identities

Proving Identities

-- Identities Involving Two Angles

The Cosine of the Difference of Two Angles

Functions of Sums and Difference of Angles

Double- and Half-Angle Identities

Sum and Product Identities

-- Triangle Applications

The Law of Cosines

The Law of Sines

Solving Triangles

Areas of Triangles

Unit Five: The Circular Functions and Their Inverses

-- Variation and Graphs

Measuring Arcs and Angles

The Circular Functions

Graphs of Cosine and Sine Functions

Graphs of Other Circular Functions

-- Inverse Functions and Graphs

Inverse Values

The Inverse Circular Functions

#### Design of the Study

The primary intent of the study reported in this dissertation was to determine if "middle ability" students who use a computer to learn algebra-trigonometry will attain a higher level of achievement in the subject-matter than other students of the same ability level who do not use the computer to learn algebra-trigonometry. Table 5 identifies the number of pupils, male and female, in both of the groups which participated.

Except for instruction and assignments in the experimental group relevant to the computer language called BASIC and the three ter-

terminal devices used to communicate with the computer, the course objectives, methods, techniques, and instructional materials were the same for both groups. Thus, the teacher and experimental group used the computer as an instructional tool in the algebra-trigonometry course; the teacher and control group did not.

TABLE 5  
NUMBER OF STUDENT PARTICIPANTS IN THE STUDY BY GENDER

Gender	Control Group	Experimental Group	Totals
Male	14	14	28
Female	11	12	23
Total	25	26	51

Since the computer was to be used in the study as an instructional tool rather than as an object of instruction, the teacher used the terminals whenever he felt that they would be beneficial in achieving course objectives. However, students in the experimental group were free and encouraged to use the terminals not only when direct assignments were made concerning the use of the computer, but also when other applicable problems were assigned. Hence, utilization of the computer as a teaching and/or learning tool in this study was entirely at the discretion of the teacher and/or students in the experimental group.

When computer terminals were first installed in the Dearborn Public Schools in September, 1969, several people were concerned that there was not enough time during the semester to teach the required material and still utilize the computer as a teaching and learning tool.

Thus, a study was conducted to answer the point in doubt--and to identify related benefits and problems concerning the use of a computer terminal in an instructional environment. The results of the study, as well as one conducted earlier in the Altoona (Pennsylvania) Area Public Schools, show that students who use a computer to learn mathematics need less time to practice "drill problems" than other students learning mathematics by the "traditional" methods.

Table 6 identifies the number of days spent by both the control group and the experimental group with each of the five instructional units in the algebra-trigonometry course. It should be noted that most of the time necessary to teach the main statements of the BASIC language was spent during the first unit: Algebraic Review Material and Radicals in Equations. Thereafter, the experimental group generally needed fewer days than the control group in each instructional unit.

To identify the amount of time that the teacher and each student used the computer, all on-line work was monitored. Each time that the user worked on-line, the person identified himself with the first three letters of his last name. The computer was programmed to log: (1) the user's identification letters; (2) the date when the person used the computer; (3) the amount of terminal-connect time which the person used, in minutes; (4) the amount of computer time which the person used, in seconds; and (5) the number of units stored by the user, if any. A sample of the log may be found in Appendix B.

Although the class periods at Fordson High School are all 55 minutes in length, it was not possible to fix the period of the day that both groups in the study were to meet. The control group met during the school's first lunch period each day; the experimental group met immediately

after lunch each day. Since most empirical evidence relative to students meeting before and after lunch is still inconclusive, it is assumed that this variable is not relevant in the study.

TABLE 6  
SEQUENCE AND DURATION OF INSTRUCTIONAL UNITS

Unit	Control Group Number of Days	Experimental Group Number of Days
One: Review; Radicals in Equations	16	22
Two: Exponential Functions and Logarithms	17	16
Three: Trigonometric Functions and Complex Numbers	24	22
Four: Trigonometric Identities and Formulas	17	17
Five: The Circular Functions and Their Inverses	16	13
Total	90	90

To minimize curiosity and forms of contamination between students in the two groups, all participants were told that they were a part of a study to help improve the algebra-trigonometry curriculum for future students.

#### Instructional Materials, Methods, and Techniques

Attainment of the course objectives in algebra-trigonometry in the Dearborn Public Schools requires a diversity of instructional materials. Nevertheless, the content of the course follows very closely that which is found in the middle portion of the book, Modern Algebra and Trigonometry,

Structure and Method, Book Two.<sup>12</sup>

Typical of many mathematics classrooms today, the teacher in the study relied heavily upon the chalkboard, wall-graphs, slide-rule, three-dimensional models, and especially the overhead projector--both in the control group and in the experimental group. Numerous transparencies relevant to each of the five instructional units were prepared and utilized. In addition, three computer terminals were used in the experimental group, two of which were on-line.

Because the instructional materials which are available to implement the computer as a medium of teaching and learning were developed to meet objectives other than those of the Dearborn Public Schools, it was necessary to revise those materials and develop others to meet the objectives of the algebra-trigonometry curriculum. Several recent publications as well as other related materials were helpful:

1. A Guide to BASIC Programming: A Time-Sharing Language<sup>13</sup>
2. Computer Assisted Mathematics Program, Intermediate Mathematics.<sup>14</sup>
3. Computer Oriented Mathematics, An Introduction for Teachers<sup>15</sup>

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<sup>12</sup>Mary P. Dolciani, Simon L. Berman, and William Wooton, Modern Algebra and Trigonometry, Structure and Method, Book Two (Boston, Massachusetts: Houghton Mifflin Company, 1965).

<sup>13</sup>Donald D. Spencer, A Guide to BASIC Programming: A Time-Sharing Language (Reading, Massachusetts: Addison-Welsey Publishing Company, 1970).

<sup>14</sup>Dale E. LaFrenz, Computer Assisted Mathematics Program, Intermediate Mathematics (Atlanta, Georgia: Scott, Foresman and Company, 1969).

<sup>15</sup>Computer-oriented Mathematics Committee, Computer-Oriented Mathematics, An Introduction for Teachers (Washington, D. C.: National Council of Teachers of Mathematics, Inc., 1963).

4. "Computer Programs for Junior and Senior Mathematics Courses"<sup>16</sup>
5. GCMP Computers in the Schools, A Basic Primer Teacher Guide<sup>17</sup>
6. Problem-Solving with the Computer<sup>18</sup>

The revised and newly developed materials were used in the experimental group for one or more of the following reasons:

1. To teach the BASIC language.
2. To demonstrate the use of the computer terminal.
3. To illustrate the computer's problem-solving capability.
4. To assign problems.
5. To introduce students to strategies and procedures they might use to "teach" the computer.

Although large-group instruction dominated most of the classroom activity, considerable time was spent by the teacher working individually with students; accordingly, the discovery method of instruction was most often stressed.

Except for instruction in the experimental group relevant to the computer terminals and the BASIC language, considerable effort was devoted to keep the instructional strategies alike in both participating groups. Since the use of the computer with the experimental group repre-

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<sup>16</sup>"Computer Programs for Junior and Senior Mathematics Courses," unpublished computer workshop materials, University of Detroit, 1969.

<sup>17</sup>Greater Cleveland Mathematics Program Staff, GCMP Computer in the Schools (Cleveland, Ohio: Educational Research Council of America, 1969).

<sup>18</sup>Erwin R. Sage, Problem-Solving with the Computer (Newburyport, Massachusetts: ENTELEK Inc., 1969).

sented a manipulation of the instructional variable, the intent was to make it the paramount variable between the two groups. Accordingly, five days were devoted early in the semester introducing the use of the computer terminals and teaching the main statements of the BASIC language to the experimental group. As the study continued, additional time was spent teaching or demonstrating specific features of the language, presenting assignments, or allowing students the opportunity to use the terminals. Most utilization of the computer, however, was beyond the normal instructional period of 55 minutes--usually before school, during a lunch period or study period, or after the regular school day. Because the students who used the computer to learn mathematics needed time to learn the BASIC language and use the computer, but less time to practice drill problems, both groups completed all the required subject-matter necessary to attain the stated course objectives.

Generally, students in the experimental group used the computer in three ways: (1) as a computational tool; (2) as a "teaching" and learning tool; and (3) experimentally.

As a computational tool, students sometimes used the computer to do work which would have required a week or more of manual computation. For example, when discussing limits and convergence, students in the experimental group were able to use the computer to test a large number of examples and compare the results with the theory they learned in class.

More often, students in the experimental group were assigned problems, the correct programming of which helped to demonstrate their understanding of a concept introduced by the teacher. Fulfillment of such assignments required that each student develop a problem-solving



strategy before writing a computer program. In a sense, the student was put in the position of "teaching" the computer. Hence, the student had to understand the problem; analyze it for basic mathematical concepts; then develop a logical sequence or algorithm for submission to the computer. Contrary to classroom teachers, however, the student was not permitted to do a poor job of "teaching." If any logic or arithmetic errors were made, they were readily noted in the form of a diagnostic message when the program was submitted to the computer.

Students in the experimental group worked directly with three teletypewriter terminals, two of which were connected through private telephone lines to a large-scale, digital computer located approximately 15 miles away. The students typed numerals, letters, and symbols found on the keyboard to do computational work, solve mathematical problems, and otherwise "teach" the computer that which they learned in class. When using the computer as a medium of instruction, students in the experimental group used the following procedure:

1. The problem-solving program was written in "long-hand" on notebook paper.
2. The program was punched into paper tape via one of the three computer terminals.
3. The paper tape was submitted on-line to the computer through the tape reader. Hence, a printout of the program and a copy of its problem-solving application was received via the terminal typewriter.
4. When an error or diagnostic message was received via the terminal, the program was analyzed and debugged on-line, if possible; otherwise the program was debugged at the student's leisure and resubmitted to the computer afterward.

Except for instructional demonstrations, most student programs were punched into paper tape in the mathematics workroom or the hallway adjacent to the classroom. Use of the terminals in the classroom was too noisy and disruptive for other students not using the terminals. Accordingly, each student was allowed to leave the classroom occasionally to punch his program into paper tape. After punching the program, he returned to the classroom so that another student could take his place at one of the three terminals. When all students had punched their programs, the same procedure was used to allow students the opportunity to submit their programs on-line to the computer--and/or debug them--which was most often the case. When working on-line, students were allowed to respond immediately to the diagnostic messages in attempting to debug their strategies; although some students chose occasionally not to do so. On those days when this procedure was being used, the teacher often utilized the opportunity to work individually with the students remaining in the classroom. Since the entire process would have taken the experimental group several days to complete, students were encouraged to use the terminals at times other than during the regular meeting period. Thus, it was not necessary to use "too much" regular classroom time.

#### Evaluative Instruments Used in the Study

Evaluative instruments were administered to both the control group and the experimental group at the beginning and the end of the semester, as well as intermittently. They were of two general types: (1) test instruments that measured the achievement of a student; and (2) instruments that identified student opinions.

Test Instruments. Since the computer was used in the study as an instructional tool to help attain current course objectives, none

of the test instruments which were used contained any reference to computers, computer programming, or other related areas of computer knowledge. The primary rationale for selection of the test instruments, as well as the time to administer them, was identification and measurement of student achievement. Each test instrument was selected or designed for one of the following reasons:

1. To assess the capabilities of each student before the experimental treatment began (pre-treatment tests).
2. To measure the proficiency of each student relative to a specific instructional unit (post-unit tests).
3. To assess the level of achievement attained by each student after completion of the semester's coursework (post-treatment tests).

Table 7 identifies the manner in which the test instruments were used, the name of the instrument, the area tested, the instrument's reliability coefficient, and lastly, the instrument's standard error.

Pre-treatment tests were administered to all students participating in the study to assess their capabilities before treatment began. Results of these tests were statistically analyzed to determine if the two groups were significantly different. Since the achievement scores showed that the groups did not differ significantly at that point in time, it seemed reasonable to assume that whatever significant differences in mathematics might occur between the groups afterward would be a result of the different instructional treatments. If the pre-treatment test results of the two groups had differed significantly, it would have been necessary to postpone the investigation; or apply appropriate statistical analysis, if possible. Thus postponement of the study depended upon the number of differences and their level of significance.

TABLE 7

## EVALUATIVE TEST INSTRUMENTS USED IN THE STUDY

Use	Instrument	Form	Area Tested	Reliability Coefficient	Std. Error
Pre-treatment					
	Otis-Gamma Quick Scoring Mental Ability Test	Bm	Intelligence	.91	3.0
	Nelson-Denny Reading Test	A	Vocabulary Comprehension	.93 .81	3.28 5.48
	Survey of Trigonometric Concepts	A	Concepts	.79	4.24
	Cooperative Mathematics Test, Algebra II	A	Skills	.84	3.79
	Cooperative Mathematics Test, Trigonometry	A	Problem-solving	.80	4.50
Post-unit					
	Review; and Radicals in Equations	-	Achievement	-	-
	Exponential Functions and Logarithms	-	Achievement	-	-
	Trigonometric Functions and Complex Numbers	-	Achievement	-	-
	Trigonometric Identities and Formulas	-	Achievement	-	-
	Circular Functions and Their Inverses	-	Achievement	-	-
Post-treatment					
	Survey of Trigonometric Concepts	B	Concepts	.83	3.91
	Cooperative Mathematics Test, Algebra II	B	Skills	.89	3.54
	Cooperative Mathematics Test, Trigonometry	B	Problem-solving	.83	4.06
	Test of Logic and Reasoning	-	Logic and Reasoning	.81	4.01

To measure the intelligence quotient of each student participating in the study, the Otis-Gamma Quick Scoring Mental Ability Test, Form Bm, was administered. The manual of directions to administer the Gamma Test states that "the instrument's main purpose is to measure mental ability, thinking power, or the degree of maturity of the mind."<sup>19</sup> However, users of the test are cautioned that it is possible only to measure the effect mental ability has had in enabling the pupil to acquire certain knowledge and mental skill. To obtain a measure of a pupil's brightness comparable to an intelligence quotient obtained on the Binet Scale, the student's score for the Gamma Test is compared with the norm for his age. Accordingly, a measure found in this manner "is not a quotient, but it is called an 'IQ' because it has the same significance as an IQ."<sup>20</sup> Nevertheless, "Gamma IQ's" tend to be somewhat less variable than ordinary IQ's; that is, they tend to be somewhat nearer to 100. To compute the reliability coefficient of the Otis-Gamma Test, the coefficient of correlation between odd and even items was used. To correct the coefficients of correlation between the half tests, the Spearman-Brown formula was applied to obtain the corresponding coefficient for the two full-length tests given under the same circumstances. Accordingly, Form Bm with 80 objective test items yielded a reliability coefficient of .91 at the eleventh grade level; with a standard error of 3.0 points. When tested for validity, a median value of +.61 was attained, indicating that the items have real validity in a mental ability test.<sup>21</sup>

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<sup>19</sup> Arthur S. Otis, Manual of Directions for Gamma Test (New York: Harcourt, Brace and World, Inc., 1954), p. 1.

<sup>20</sup> Ibid., p. 5.

<sup>21</sup> Ibid.

To assure the investigator that no participating student had a serious vocabulary and/or comprehension deficiency, the Nelson-Denny Reading Test, Form A, was administered to both groups prior to treatment. Although none of the subject-matter in the algebra-trigonometry curriculum requires extensive reading, a serious vocabulary or comprehension deficiency could be a handicap to any student participating in the study. Because the volume of reading demanded of students in the algebra-trigonometry curriculum in the Dearborn Public Schools is negligible, reading rate was not considered relevant. Using the equivalent-forms method to compute the test's reliability coefficient, the vocabulary portion of the instrument with 100 objective test items showed a coefficient of .93 at the eleventh grade level; while the comprehension section with 36 objective test items yielded a coefficient of .81. The standard error of both tests is recorded as 3.28 points and 5.48 points respectively. When evaluated for validity, the vocabulary portion showed a mean index of 47.5; the comprehension section a mean index of 44.6.<sup>22</sup>

Measurement of each student's understanding of mathematical concepts necessitated the design and administration of a teacher-constructed instrument, Survey of Trigonometric Concepts. Development of this test became mandatory when it was determined that no standardized instrument adequately tested mathematical concepts in accordance with the objectives of the algebra-trigonometry curriculum. A copy of Form A and Form B of the instrument, each with 35 objective test items, is included in Appendix C. Form A was used as a pre-treatment test; Form B as a post-treatment test. The reliability coefficient of both forms was

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<sup>22</sup>James I. Brown, Examiner's Manual: The Nelson-Denny Reading Test (Boston: Houghton Mifflin Company, 1960), pp. 26-27.

established by using the alternate or parallel forms method. Accordingly, the correlation between Form A and Form B was taken as a measure of the self-correlation of the test. Under these conditions, the reliability coefficient became an index of the equivalence of the two forms of the test. To utilize this method, Form A and Form B of the instrument were administered alternately to 106 students at the eleventh grade level who did not participate in the study. An interval of three weeks was allowed between administrations of the two forms. Results of the method yielded a reliability coefficient of .79 for Form A and .83 for Form B; with a standard error of 4.24 points and 4.01 points respectively.

The instrument used to measure mathematical skills is one of a battery of standardized tests that is widely known and accepted, the Cooperative Mathematics Test, Algebra II. Form A was administered as a pre-treatment test in the study; Form B as a post-treatment test. Both forms contain 40 objective items. According to the Cooperative Mathematics Handbook, the reliabilities reported for the forms are measures of internal consistency, computed by using the Kuder-Richardson Formula 20. Form A rendered a reliability coefficient for grades 10-12 of .84; and a standard error of 3.79. Form B yielded a reliability coefficient of .89; with a standard error of 3.54.<sup>23</sup> When describing the validity of the test, the handbook states:

"The Cooperative Mathematics Tests are measures of developed abilities, and thus their content validity is of primary importance. Content validity is best insured by entrusting test construction to persons well-qualified to judge the relationship of test content to teaching objectives. . . It is recommended that each test user

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<sup>23</sup>Cooperative Test Division, Cooperative Mathematics Tests Handbook (Princeton, New Jersey: Educational Testing Service, 1964), pp. 62-63.

make an individual judgment of content validity with respect to his own course content and educational aims.<sup>24</sup>

To assess each student's problem-solving ability, the Cooperative Mathematics Test, Trigonometry, was administered to both groups. Form A was used to measure pre-treatment achievement; Form B to measure post-treatment achievement. Both forms contain 40 objective items. Like the Cooperative Mathematics Test, Algebra II, the reliability coefficient for the trigonometry instrument is a measure of internal consistency, calculated by using the Kuder-Richardson Formula 20. The Cooperative Mathematics Handbook shows a reliability coefficient for grades 10-12 of .80 for Form A; .83 for Form B. The standard error of the two forms is 4.50 and 4.06 respectively. Like the Cooperative Mathematics Test, Algebra II, the same vague statement describing validity is applied.<sup>25</sup>

To measure student achievement relative to each of the five instructional units in the course, a teacher-constructed test was administered at the conclusion of each unit. The tests were designed in accordance with the objectives of the algebra-trigonometry curriculum; and all were reviewed and approved by four mathematics teachers as being pertinent and thorough in the coverage of the particular instructional unit which was being tested. All items on the tests required the application of one or more mathematical concepts, mathematical skills, and/or the development of mathematical problem-solving strategies. A copy of each post-unit test can be found in Appendix C.

All students participating in the study were also administered

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<sup>24</sup>Ibid., p. 62.

<sup>25</sup>Ibid., pp. 62-63.



a teacher-made post-treatment instrument to test logic and reasoning ability. This test was designed to focus attention in an area not emphasized by the other three post-treatment tests. Nevertheless, like the other test instruments, it contained no reference to computers, computer languages, computer programming, or other related areas of computer knowledge. The instrument was developed and administered to provide insight concerning one of the possible "by-products" of using a computer as an instructional tool. A copy of the test can be found in Appendix C. To determine the instrument's reliability, the often-used test-retest method was applied. Accordingly, repetition in administering a test determines the agreement between two sets of scores. Hence, the test was administered and re-administered to the same group of students; and the correlation computed between the first and second set of scores. Given a time interval of two months between the first and second administration of the test to offset--in part at least--memory, practice, and other carry-over effects, the retest coefficient is said to result in a close estimate of the stability of the test scores. Accordingly, utilization of the test-retest method resulted in a reliability coefficient of .81; with a standard error of 4.01.

Opinion Instruments. In addition to the test instruments, opinion instruments were designed and administered during the study to identify:

1. Student opinion relative to using a computer as an instructional tool (Instrument A).
2. Student opinion relative to not using a computer as an instructional tool (Instrument B).

Table 8 identifies the opinion instruments used in the study, the approximate week they were filled out, and the subject(s) to whom they were administered. A copy of the instruments can be found in Appendix D.

TABLE 8  
OPINION INSTRUMENTS USED IN THE STUDY

Instrument	Week Administered	Subject(s)
Student Opinion Inventory A	5	Experimental Group
Student Opinion Inventory A	19	Experimental Group
Student Opinion Inventory B	19	Control Group

Instrument A was developed to identify the opinion of students in the experimental group relevant to using the computer as an instructional tool during the study. The instrument was administered shortly after the students began using the computer; and again after a semester's experience with it. Participants were asked to respond to six statements:

1. I like mathematics.
2. I like using a computer to learn mathematics.
3. Using a computer helps me to understand mathematics.
4. Using a computer has increased my interest in mathematics.
5. Using a computer has improved the opportunity for me to be creative in mathematics.

6. In addition to mathematics, I would like the opportunity to use a computer to learn subject-matter in other courses (for example,

science or social studies).

Each statement included three possible responses: agree, don't know, disagree. Students were asked to circle the response which, at the time, best described their opinion.

Administered only once during the semester, Instrument B was designed to identify the opinion of students in the control group relative to not using a computer during the study. Accordingly, the results were studied to determine if these students resented not using a computer as an instructional tool to learn mathematics. Participants were asked to respond to six statements:

1. I like mathematics.
2. I would have liked to have had the opportunity to use a computer to learn mathematics this semester.
3. Using a computer would have helped me to better understand mathematics.
4. Students who used a computer to learn mathematics this semester had an advantage over me.
5. If I take an additional mathematics course I would like to use the computer in the course to learn mathematics.
6. I would like the opportunity to use a computer to learn subject-matter in courses other than mathematics (for example, science or social studies).

Like Instrument A, each statement of Instrument B included three possible responses: agree, don't know, and disagree. Each student was asked to circle the response which best described his opinion.

## CHAPTER IV

### PRESENTATION AND ANALYSIS OF DATA

When reporting statistical data relevant to an important study, extreme care must be taken to insure accuracy in scoring, recording, calculating, and analyzing. Each of these phases affords numerous opportunities to error. To help avoid such mistakes in this study, evaluative scores on test instruments were checked several times; and a computer was used to perform all calculations through a minimum of two separate computer programs.

#### Pre-treatment Testing

Before introducing the computer as an instructional tool to the experimental group, all students who participated in the study were administered pre-treatment tests to measure intelligence quotient, reading vocabulary, reading comprehension, understanding of mathematical concepts, level of mathematical skills, and mathematical problem-solving ability. Although the experimental group and control group were selected because they were considered to be "alike," pre-treatment tests were administered to reaffirm the competency level of the students in both groups; and to use the results as statistical data to analyze pre-treatment/post-treatment test scores.

To test the difference between the experimental group and control group, a two-tailed t-test of significance was applied to the respective mean scores of the groups. Utilization of a t-test in such instances assumes an equality of variance between the two random samples of the population. Hence, before applying a t-test of significance, it

was necessary to test the experimental group and control group for homogeneity of variance. When samples are small and uncorrelated, an F-test can be used to determine equality of variance by dividing the larger variance of the two groups by the smaller variance, then evaluating the resulting F ratio in terms of appropriate degrees of freedom (df). The number of degrees of freedom is the number of free variables in the problem or in the distribution of the random variables connected with it. For each restriction imposed upon the original observation, such as in the estimation of a population value from the sample, the number of degrees of freedom is reduced by one.

In those instances where homogeneity of variance did not exist between the experimental group and control group, application of a t-test of significance was performed by the method developed by Cochran and Cox.<sup>1</sup> The sampling distribution for the Cochran-Cox method does not assume nor necessitate homogeneity of variance between two samples of a population. Instead, the variance of each mean is calculated separately; then a criterion  $t$  is obtained by computing a weighted mean of the two  $t$ -values for the two samples, the weights being the two variances of the respective means. The observed value of  $t$  is then compared with the weighted value of  $t$  to judge significance. If the observed value of  $t$  is less than the criterion  $t$ , the null hypothesis of unequal means is accepted; however, if the observed value of  $t$  is greater than the criterion  $t$ , the null hypothesis of unequal means is rejected.

As indicated by the writer prior to initiating the investigation, the five percent level of confidence was used to determine signi-

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<sup>1</sup>W. G. Cochran and G. M. Cox, Experimental Designs (2nd ed; New York: Wiley, 1957), p. 101.

ficant difference in this study. Hence, the two samples are considered truly different whenever the calculated F ratio or value of  $t$  shows that the obtained difference between the experimental group and control group would be expected to occur not more than five times in 100 by chance if the two samples were in fact alike. However, if the statistical analysis indicates that the difference between the two samples might have appeared by chance more than five times out of 100, the null hypothesis is accepted.

Intelligence Quotient. Innate intelligence can be an important factor in determining a student's academic success. To measure the intelligence quotient of each student who participated in the study, the Otis-Gamma Quick-Scoring Mental Ability Test was administered to both groups. As indicated in Table 9, the mean of both groups was considerably higher than that attributed to a so-called "average student" who is generally considered to have an intelligence quotient of 100. The mean of the experimental group was 117.15; the mean of the control group was 118.12. Hence, the difference between the means of the two groups was .97. With a variance of 60.7 for the experimental group compared to 52.36 for the control group, the value of the F ratio was calculated as 1.16. Since the expected value of the F ratio between the two groups at the five percent level of confidence is 1.97, no significant difference existed in the variances of the experimental group and control group when tested for intelligence quotient.

Because the variances of the two groups were not significantly different, a  $t$ -test was applied to measure the difference between the means of the experimental group and control group. The expected value of  $t$  between the two groups at the five percent level of confidence is 2.01; thus the calculated  $t$  of .46 indicated that no significant

difference existed between the means of the experimental group and control group when tested for intelligence quotient; and the null hypothesis of no significant difference was accepted.

TABLE 9

## ANALYSIS OF GROUP RESULTS FOR PRE-TREATMENT TEST OF INTELLIGENCE QUOTIENT

Group	N	Mean	Variance
Experimental	26	117.15	60.7
Control	25	118.12	52.36
Calculated F =	1.16	Calculated t =	.46
Expected F <sub>.05</sub> =	1.97	Expected t <sub>.05</sub> =	2.01
Difference:	Insignificant	Difference:	Insignificant
Hypothesis:	Accepted	Null Hypothesis:	Accepted

Reading Vocabulary and Reading Comprehension. Although the algebra-trigonometry curriculum in the Dearborn Public Schools does not necessitate that students read a great deal, it is possible that a serious vocabulary and/or comprehension deficiency could hinder a student's achievement in the course. To identify participants with a severe handicap in these two areas, the Nelson-Denny Reading Test, Form A, was administered.

When participants were tested for reading vocabulary, the results showed a mean difference between the two groups of 3.88 in favor of the control group, as shown in Table 10. With a variance of 48.52 for the experimental group and 101.56 for the control group, the calculated value of the F ratio of 2.09 indicated a significant difference

in the variances of the two groups at the five percent level of confidence. Hence, the Cochran-Cox method was used to measure the difference between the sample means. Accordingly, the value of the criterion  $t$  was computed as .32 while the value of the observed  $t$  was calculated as 1.59. Since the criterion  $t$  was less than the observed  $t$ , the null hypothesis of no significant difference between the two groups was rejected in favor of the experimental group. However, individual test scores showed that no student in either group had a serious vocabulary deficiency--which was the main purpose in administering the test. The lowest raw score recorded by a participating student was 19. According to the Grade Equivalent Norm Table in the examiner's manual of the Nelson-Denny Reading test, a raw score of 19 is equated at the 9.9 grade level.

TABLE 10

## ANALYSIS OF GROUP RESULTS FOR PRE-TREATMENT TEST OF READING VOCABULARY

Group	N	Mean	Variance
Experimental	26	31.96	48.52
Control	25	35.84	101.56
Calculated F =	2.09	Observed $t$ =	1.59
Expected F <sub>.05</sub> =	1.96	Criterion $t$ <sub>.05</sub> =	.32
Difference:	Significant	Difference:	Significant
Hypothesis:	Rejected	Null hypothesis:	Rejected

When tested for reading comprehension, a mean difference of 1.52 in favor of control students separated the two groups. As shown in Table 11, analysis of the experimental group's test results showed a



variance of 125.12 compared to 155.43 for the control group. Accordingly, the value of the calculated F ratio of 1.24 showed that no significant difference existed between the variances of the two groups at the five percent level of confidence when tested for reading comprehension. However, the lowest raw score recorded by a participating student was 16. In the examiner's manual, a score of 16 is equated at the 7.0 grade level, indicating a possibility of a fairly serious comprehension deficiency for an eleventh grade student. However, after analysis of this participant's other pre-treatment test scores as well as reference to his cumulative school record, it was concluded that the student did not possess a reading comprehension deficiency that would hinder his achievement in algebra-trigonometry.

TABLE 11

## ANALYSIS OF GROUP RESULTS FOR PRE-TREATMENT TEST OF READING COMPREHENSION

Group	N	Mean	Variance
Experimental	26	44	125.12
Control	25	45.52	155.43
Calculated F =	1.24	Calculated t =	.46
Expected F <sub>.05</sub> =	1.96	Expected t <sub>.05</sub> =	2.01
Difference:	Insignificant	Difference:	Insignificant
Hypothesis:	Accepted	Null hypothesis:	Accepted

Since the variances of the experimental group and control group were not significantly different, a t-test was applied to measure the difference between the means of the two groups. Calculation of the t

resulted in a value of .46 compared to an expected value of 2.01 at the five percent level of confidence. Therefore it was concluded that no significant difference existed between the experimental group and control group when tested for reading comprehension.

Mathematical Concepts. The ability of students to understand and apply mathematical concepts was one of the major points of interest in the study. To measure the ability of participants in this area at the outset of the investigation as well as their growth during treatment, a teacher-constructed test was designed and administered. Development of a teacher-made instrument became necessary when it was concluded that no standardized test adequately measured this facet of mathematics at the eleventh grade level. Thus a test called Survey of Trigonometric Concepts, Form A and Form B, was developed. As shown in Table 12, calculation of the mean score for each of the two groups on Form A of the test resulted in a difference of .3 in favor of the experimental group. In addition, the variance of the experimental group showed a value of 14.65 compared to 8.46 for the control group. Computation of the F ratio yielded a value of 1.73, thus indicating that no significant difference existed at the five percent level of confidence between the variances of the two groups.

With no significant difference evident in the variances of the two groups, a t-test was applied. Accordingly, the calculated t resulted in a value of .31. Since the expected value of t at the five percent level of confidence is 2.01, no significant difference existed in the means of the two groups when tested for understanding and ability to apply mathematical concepts prior to treatment.

Mathematical Skills. Another major area of interest in the

TABLE 12

## ANALYSIS OF GROUP RESULTS FOR PRE-TREATMENT TEST OF MATHEMATICAL CONCEPTS

Group	N	Mean	Variance
Experimental	26	24.58	14.65
Control	25	24.28	8.46
Calculated F =	1.73	Calculated t =	.31
Expected F <sub>.05</sub> =	1.97	Expected t <sub>.05</sub> =	2.01
Difference:	Insignificant	Difference:	Insignificant
Hypothesis:	Accepted	Null hypothesis:	Accepted

study was the development of mathematical skills by students. Pre-treatment testing showed that the experimental group achieved a mean score of 22.73; the control group 22.52. Thus, results of administering the Cooperative Mathematics Test, Algebra II, Form A, indicated a difference favoring the experimental group of .21 separating the group means. As summarized in Table 13, the variance of the experimental group was 20.92 compared to 21.26 for the control group. With a calculated F ratio of 1.02, no significant difference existed in the variances of the two groups at the five percent level of confidence.

Application of a t-test of significance resulted in a t-value of .16. The expected value of t at the five percent level of confidence is 2.01. Hence, no significant difference existed between the means of the experimental group and control group when tested for ability to apply mathematical skills prior to treatment.

Mathematical Problem-solving. To measure the mathematical

TABLE 13

## ANALYSIS OF GROUP RESULTS FOR PRE-TREATMENT TEST OF MATHEMATICAL SKILLS

Group	N	Mean	Variance
Experimental	26	22.73	20.92
Control	25	22.52	21.26
Calculated F =	1.02	Calculated t =	.16
Expected F <sub>.05</sub> =	1.96	Expected t <sub>.05</sub> =	2.01
Difference:	Insignificant	Difference:	Insignificant
Hypothesis:	Accepted	Null hypothesis:	Accepted

problem-solving ability of students participating in the study, the Cooperative Mathematics Test, Trigonometry, Form A, was administered to both groups. Table 14 shows that the pre-treatment results of testing yielded a mean difference of .66 in favor of the control group; with a variance of 8.34 for the experimental group compared to 7.97 for the control group. Calculation of the F ratio yielded a value of 1.05. Since the expected F ratio between the two groups at the five percent level of confidence is 1.97, there was no significant difference between the variances of the two groups when tested for problem-solving ability.

When a t-test was applied to determine the level of significance between the means of the two groups, a value of .82 was derived. Because the expected value of t at the five percent level of confidence is 2.01, it was concluded that no significant difference existed between the two groups when tested for mathematical problem-solving ability prior to treatment.

TABLE 14  
ANALYSIS OF GROUP RESULTS FOR PRE-TREATMENT TEST  
OF MATHEMATICAL PROBLEM-SOLVING

Group	N	Mean	Variance
Experimental	26	8.5	8.34
Control	25	9.16	7.97
Calculated F =	1.05	Calculated t =	.82
Expected F <sub>.05</sub> =	1.97	Expected t <sub>.05</sub> =	2.01
Difference:	Insignificant	Difference:	Insignificant
Hypothesis:	Accepted	Null hypothesis:	Accepted

#### Post-unit Testing

Rather than limit the statistical analysis in the study to pre-treatment/post-treatment test data, interim or unit tests were administered at the conclusion of each of the five major units. Although these tests were teacher-made instruments, all were reviewed and approved by four mathematics teachers as being in accordance with the course objectives, as well as pertinent and thorough in the coverage of the particular instructional unit which was being tested. All items in the tests required the application of one or more mathematical concepts, mathematical skills, and/or the development of mathematical problem-solving strategies. To test the difference in achievement between the two groups, a one-tailed t-test of significance was applied to the post-unit mean scores.

Unit One. When administered the post-unit test which included

algebraic review material and problems involving radicals in equations, the experimental group achieved a mean score of 15.04 compared to 15.12 for the control group. Thus, a difference of .08 favoring the control group separated the two means. As indicated in Table 15, the variance of the experimental group was computed as 9.72 while the variance of the control group was computed as 7.16. With a calculated F ratio of 1.28, no significant difference existed between the variances of the two groups at the five percent level of confidence after instruction regarding review material and radicals in equations.

TABLE 15  
ANALYSIS OF GROUP RESULTS FOR POST-UNIT TEST  
OF REVIEW MATERIAL AND RADICALS IN EQUATIONS

Group	N	Mean	Variance
Experimental	26	15.04	9.72
Control	25	15.12	7.61
Calculated F =	1.28	Calculated t =	.39
Expected F <sub>.05</sub> =	1.97	Expected t <sub>.05</sub> =	1.68
Difference:	Insignificant	Difference:	Insignificant
Hypothesis:	Accepted	Null hypothesis:	Accepted

Since there was equality of variance between the experimental group and control group, a t-test was applied to measure the level of significance between the means of the two groups. Application of the t-test resulted in a calculated value of .39 in contrast to the expected value of 1.68 at the five percent level of confidence. Hence, it was

concluded that no significant difference existed between the means of the two groups when tested after treatment for understanding of review material and radicals in equations; and the null hypothesis of no significant difference was accepted.

Unit Two. Analysis of post-unit test results for the instructional unit, Exponential Functions and Logarithms, showed a mean difference between the two groups of 2.42 in favor of the experimental group. The mean of the experimental group was 29.56; the mean of the control group was 27.14. Computation of each group's variance resulted in a value of 17.57 and 36.53 respectively. As shown in Table 16, the calculated value of the F ratio was 2.08. Thus, the results of post-unit testing for the unit revealed a significant difference between the variances of the two groups at the five percent level of confidence; and the Cochran-Cox method which does not require homogeneity of variance

TABLE 16  
ANALYSIS OF GROUP RESULTS FOR POST-UNIT TEST  
OF EXPONENTIAL FUNCTIONS AND LOGARITHMS

Group	N	Mean	Variance
Experimental	26	29.56	17.57
Control	25	27.14	36.53
Calculated F =	2.08	Observed t =	1.67
Expected F <sub>.05</sub> =	1.96	Criterion t <sub>.05</sub> =	.31
Difference:	Significant	Difference:	Significant
Hypothesis:	Rejected	Null hypothesis:	Rejected

was used to apply a t-test. Accordingly, the value of the criterion  $t$  was computed as .31 compared to the value of the observed  $t$  which was calculated as 1.67. Since the criterion  $t$  was less than the observed  $t$ , it was concluded that a significant difference existed between the means of the two groups when tested for understanding of exponential functions and logarithms; and the null hypothesis of no significant difference was rejected in favor of the experimental group.

Unit Three. Post-unit test results for the unit, Trigonometric Functions and Complex Numbers, showed a group mean of 23.54 for the experimental group; 21.32 for the control group. The difference between the means was 2.22. As summarized in Table 17, the variance of the experimental group was 24.66 compared to 33.56 for the control group; hence, the calculated value of the F ratio was 1.36. Since the expected value of the F ratio is 1.96 at the five percent level of confidence,

TABLE 17  
ANALYSIS OF GROUP RESULTS FOR POST-UNIT TEST  
OF TRIGONOMETRIC FUNCTIONS AND COMPLEX NUMBERS

Group	N	Mean	Variance
Experimental	26	23.54	24.66
Control	25	21.32	33.56
Calculated F =	1.36	Calculated t =	1.47
Expected F <sub>.05</sub> =	1.96	Expected t <sub>.05</sub> =	1.67
Difference:	Insignificant	Difference:	Insignificant
Hypothesis:	Accepted	Null Hypothesis:	Accepted



it was concluded that no significant difference existed between the variances of the two groups when tested for understanding of trigonometric functions and complex numbers.

Application of a t-test to measure significant difference between the means of the experimental group and control group revealed a value of 1.47. Because the expected value of t at the five percent level of confidence is 1.67, the null hypothesis of no significant difference between the means of the two groups was accepted when students were tested for understanding of trigonometric functions and complex numbers.

Unit Four. After completion of the unit, Trigonometric Identities and Formulas, analysis of post-unit test scores showed a mean difference between the two groups of 1.57 in favor of control group students. Table 18 shows that the experimental group attained a mean score of 27.06 while the control group achieved a mean of 28.56. Computation of each group's variance revealed a value of 52.17 for the experimental group; 19.05 for the control group. Application of an F-test of significance resulted in a ratio of 2.74, indicating a significant difference in the variances of the two groups at the five percent level of confidence. Thus, to measure the difference between the means of the two groups, the Cochran-Cox method was used to apply a t-test. Utilization of the Cochran-Cox method, which does not require equality of variance, resulted in a value of .72 for the criterion t compared to a value of .89 for the observed t. Since the criterion t was less than the observed t, a significant difference existed between the means of the two groups when tested for understanding of trigonometric identities and formulas; and the null hypothesis of no significant difference was rejected in favor of the control group.

TABLE 18

ANALYSIS OF GROUP RESULTS FOR POST-UNIT TEST  
OF TRIGONOMETRIC IDENTITIES AND FORMULAS

Group	N	Mean	Variance
Experimental	26	27.06	52.17
Control	25	28.56	19.05
Calculated F =	2.74	Observed t =	.89
Expected F <sub>.05</sub> =	1.97	Criterion t <sub>.05</sub> =	.72
Difference:	Significant	Difference:	Significant
Hypothesis:	Rejected	Null hypothesis:	Rejected

Unit Five. The final post-unit test in the study was administered to measure each participating student's understanding of circular functions and their inverses. Table 19 summarizes the analysis of the scores achieved by the students. The experimental group's mean score was 6.71 compared to 7.5 for the control group; hence, the difference between the group means was .79. Computation of each group's variance resulted in a value of 9.3 for the experimental group; 5.79 for the control group. Application of an F-test of significance yielded a ratio of 1.61 compared to the expected F ratio of 1.97. Therefore, no significant difference existed between the variances of the two groups at the five percent level of confidence when tested for understanding of circular functions and their inverses. Applying a t-test to measure the difference between the means of the two groups resulted in a value of 1.02. Since the expected value of t at the five percent level of significance is 1.67, no significant difference existed between the means of

the experimental group and control group when tested for understanding of circular functions and their inverses; and the null hypothesis of no significant difference was accepted.

TABLE 19  
ANALYSIS OF GROUP RESULTS FOR POST-UNIT TEST  
OF CIRCULAR FUNCTIONS AND THEIR INVERSES

Group	N	Mean	Variance
Experimental	26	6.71	9.3
Control	25	7.5	5.79
Calculated F =	1.61	Calculated t =	1.02
Expected F <sub>.05</sub> =	1.97	Expected t <sub>.05</sub> =	1.67
Difference:	Insignificant	Difference:	Insignificant
Hypothesis:	Accepted	Null hypothesis:	Accepted

#### Post-treatment Testing

To help assess the effect of the full semester's treatment on students who participated in the study, four post-treatment tests were utilized. Three of the instruments were administered in a different form during pre-treatment testing to measure each participant's understanding of mathematical concepts, development of mathematical skills, and mathematical problem-solving ability. To test the null hypotheses of no significant difference between the means of the two groups after treatment, each student's pre-treatment test score was subtracted from his corresponding post-treatment score; and the result was then used to compute a mean for each of the three tests.

To identify the difference between the corresponding means of the experimental group and control group, a two-tailed t-test of significance was applied. Utilization of a t-test in this case assumes homogeneity of variance between the two random samples of the population. Therefore, before applying a t-test, it was necessary to test the two groups for equality of variance. Accordingly, an F-test of significance was used.

The fourth and final post-treatment test administered to students during the study was designed to determine if students who wrote computer algorithms showed growth in an area not given a great deal of emphasis in the other three post-treatment tests; that is, in the development of logic and reasoning ability. Hence, the fourth test instrument was developed and administered to provide insight concerning a possible "by-product" of using a computer as an instructional tool. The test contained no reference to computers, computer programming, or other related areas of computer knowledge.

Mathematical Concepts. Post-treatment test results for the instrument, Survey of Trigonometric Concepts, Form B, showed that the experimental group achieved a mean score of 26.81 compared to 26.2 for the control group. When each participant's corresponding pre-treatment score was subtracted from his post-treatment score, the result showed that students in the experimental group had attained a mean 2.24 points higher after treatment than before treatment; and that students in the control group had achieved a mean 1.92 points higher after treatment than before treatment.

A one-way analysis of variance was performed with both samples to identify the margin of growth attained during the semester's treatment. Table 20 shows that the calculated value of F for the experimental

group was 4.43 compared to the expected value of 4.26 at the five percent level of confidence. Thus, a significant difference existed between pre-treatment and post-treatment achievement of the experimental group when tested for understanding of mathematical concepts.

TABLE 20  
ANALYSIS OF EXPERIMENTAL GROUP'S PRE-TREATMENT/POST-TREATMENT  
SCORES FOR THE TEST OF MATHEMATICAL CONCEPTS

Source of Variation	df	Sum of Squares	Mean Square	F	Difference
Within samples	24	350.64	14.61		
Between samples	1	64.7	64.7	4.43	Significant
Total	25	415.34			

Correspondingly, the calculated F ratio of the control group was 4.57. Since the expected value of F at the five percent level of confidence is 4.28, a significant difference also existed between the pre-treatment and post-treatment achievement of the control group when tested for understanding of mathematical concepts. Table 21 summarizes the results.

To determine whether a significant difference in growth occurred between the two groups during treatment, the arithmetic difference between pre-treatment and post-treatment scores was subjected to an F-test. The expected value of F at the five percent level of confidence is 1.97. As shown in Table 22, the calculated F in this instance resulted in a value of 1.91; thus indicating homogeneity of variance. Hence, a t-test was applied to determine the significant difference between the means of

TABLE 21

ANALYSIS OF CONTROL GROUP'S PRE-TREATMENT/POST-TREATMENT  
SCORES FOR THE TEST OF MATHEMATICAL CONCEPTS

Source of Variation	df	Sum of Squares	Mean Square	F	Difference
Within samples	23	177.33	7.71		
Between samples	1	35.28	35.28	4.57	Significant
Total	24	212.61			

TABLE 22

SUMMARY ANALYSIS OF THE DIFFERENCE BETWEEN PRE-TREATMENT/POST-TREATMENT  
SCORES FOR THE TEST OF MATHEMATICAL CONCEPTS

Group	N	Mean	Variance
Experimental	26	1.85	14.94
Control	25	1.92	7.83
Calculated F =	1.91	Calculated t =	.08
Expected F <sub>.05</sub> =	1.97	Expected t <sub>.05</sub> =	1.67
Difference:	Insignificant	Difference:	Insignificant
Hypothesis:	Accepted	Null hypothesis:	Accepted

the two groups. Accordingly, a value of .08 was derived. Since the expected value of t at the five percent level of confidence is 1.67, it was concluded that the growth of the two groups did not differ significantly during treatment when students were tested for their under-

standing of mathematical concepts; and the null hypothesis of no significant difference was accepted.

Mathematical Skills. To identify each participating student's level of mathematical skills after treatment, Form B of the Cooperative Mathematics Test, Algebra II, was administered. Computation of the mean score for each group showed that students in the experimental group achieved a value of 27; and that students in the control group attained a value of 24.16. Subtraction of each student's corresponding pre-treatment score from his post-treatment score indicated that the experimental group achieved a mean 4.15 points higher after treatment than it did before treatment; and that the control group attained a mean 1.64 points higher after treatment than before treatment.

To measure the development of each group's mathematical skills during treatment, a one-way analysis of variance was performed. As cited in Table 23, the calculated value of F for the experimental group resulted in a ratio of 14.43 whereas the expected value of F at the five percent

TABLE 23

ANALYSIS OF EXPERIMENTAL GROUP'S PRE-TREATMENT/POST-TREATMENT  
SCORES FOR THE TEST OF MATHEMATICAL SKILLS.

Source of Variation	df	Sum of Squares	Mean Square	F	Difference
Within samples	24	394.08	16.42		
Between samples	1	236.94	236.94	14.43	Significant
Total	25	631.02			

level of confidence is 4.26. Accordingly, a significant difference existed between pre-treatment and post-treatment achievement for the experimental group when tested for development of mathematical skills; that is, the group showed a significant margin of growth during the study. In contrast the calculated value of F for the control group was 1.5 compared to the expected value of 4.28. As shown in Table 24, there was no significant difference between the pre-treatment and post-treatment achievement of the control group when tested for development of mathematical skills.

TABLE 24  
ANALYSIS OF CONTROL GROUP'S PRE-TREATMENT/POST-TREATMENT  
SCORES FOR THE TEST OF MATHEMATICAL SKILLS

Source of Variation	df	Sum of Squares	Mean Square	F	Difference
Within samples	23	516.35	22.45		
Between samples	1	33.62	33.62	1.5	Insignificant
Total	24	549.97			

To determine whether a significant difference in growth occurred between the experimental group and control group during treatment, the arithmetic difference between each group's pre-treatment and post-treatment levels of achievement was subjected to an F-test. Calculation of the F ratio resulted in a value of 2.04. Since the expected value of F at the five percent level of confidence is 1.97, a significant difference existed between the variances of the two groups; as shown in Table 25.



TABLE 25

SUMMARY ANALYSIS OF THE DIFFERENCE BETWEEN PRE-TREATMENT/POST-TREATMENT  
SCORES FOR THE TEST OF MATHEMATICAL SKILLS

Group	N	Mean	Variance
Experimental	26	4.15	11.98
Control	25	1.64	24.41
Calculated F =	2.04	Observed t =	2.11
Expected F <sub>.05</sub> =	1.97	Criterion t <sub>.05</sub> =	.32
Difference:	Significant	Difference:	Significant
Hypothesis:	Rejected	Null hypothesis:	Rejected

Accordingly, the Cochran-Cox method of applying a t-test was used to measure the level of significance between the means of the experimental group and control group. Utilization of the Cochran-Cox method yielded a value of .34 for the criterion t compared to a value of 2.11 for the observed t. Since the value of the criterion t was less than the observed t, it was concluded that a significant difference in growth occurred between the two groups during treatment; and the null hypothesis of no significant difference in the development of mathematical skills was rejected in favor of the experimental group.

Mathematical Problem-solving. The Cooperative Mathematics Test, Trigonometry, Form B, was used to test each participating student's problem-solving ability after treatment. Calculation of the experimental group's mean score resulted in a value of 13.19 compared to 14.32 for the control group. Subtraction of each participant's pre-treatment score

from his corresponding post-treatment score showed that the experimental group attained a mean 4.69 greater after treatment than it did before treatment; and that the control group achieved a mean 5.16 greater after treatment than before treatment.

Measurement of each group's development of mathematical problem-solving ability during treatment was accomplished through a one-way analysis of variance test. As summarized in Table 26, the calculated F ratio for the experimental group yielded a value of 17.36. Since the

TABLE 26

ANALYSIS OF EXPERIMENTAL GROUP'S PRE-TREATMENT/POST-TREATMENT  
SCORES FOR THE TEST OF MATHEMATICAL PROBLEM-SOLVING

Source of Variation	df	Sum of Squares	Mean Square	F	Difference
Within samples	24	395.76	16.49		
Between samples	1	286.23	286.23	17.36	Significant
Total	25	681.99			

expected value of F at the five percent level of confidence is 4.26, a significant difference existed between the pre-treatment and post-treatment scores achieved by the experimental group; that is, significant growth was achieved by the group during treatment when tested for mathematical problem-solving ability. Similarly, the control group also showed significant growth during the semester's treatment. Calculation of the control group's F ratio resulted in a value of 31.03 compared to the expected value of 4.28 as shown in Table 27.

TABLE 27

ANALYSIS OF THE CONTROL GROUP'S PRE-TREATMENT/POST-TREATMENT  
SCORES FOR THE TEST OF MATHEMATICAL PROBLEM-SOLVING

Source of Variation	df	Sum of Squares	Mean Square	F	Difference
Within samples	23	246.79	10.73		
Between samples	1	332.82	332.82	31.03	Significant
Total	24	579.61			

To test the difference between the variances of the experimental group and control group, the arithmetic difference between each group's pre-treatment and post-treatment level of achievement was subjected to an F-test. Calculation of the F ratio resulted in a value of 3.21. Since the expected value of F at the five percent level of confidence is 1.97, a significant difference existed between the variances of the two groups. Accordingly, the Cochran-Cox method of applying a t-test was used to determine the level of significance between the means of the experimental group and control group. As indicated in Table 28, the value of the criterion t was computed as .76 while the value of the observed t was computed as .34. Since the criterion t was greater than the observed t, it was concluded that the growth of the experimental group and control group did not differ significantly during treatment when tested for mathematical problem-solving ability. Hence the null hypothesis of no significant difference was accepted.

Logic and Reasoning. The final post-treatment test administered to student-participants was designed to measure the level of achieve-

TABLE 28

SUMMARY ANALYSIS OF THE DIFFERENCE BETWEEN PRE-TREATMENT/POST-TREATMENT  
SCORES FOR THE TEST OF MATHEMATICAL PROBLEM-SOLVING

Group	N	Mean	Variance
Experimental	26	4.69	35.74
Control	25	5.16	11.14
Calculated F =	3.21	Observed t =	.34
Expected F <sub>.05</sub> =	1.97	Criterion t <sub>.05</sub> =	.76
Difference:	Significant	Difference:	Insignificant
Hypothesis:	Rejected	Null hypothesis:	Accepted

ment in an area not emphasized in the other three post-treatment tests; that is, the development of logic and reasoning ability. The results of administering the instrument, Test of Logic and Reasoning, are summarized in Table 29. The experimental group achieved a mean score of 31.96 compared to 28.04 for the control group. Hence, a difference of 3.92 existed between the means of the two groups. With a variance of 7.08 for the experimental group and 32.04 for the control group, the calculated value of the F ratio was 4.53. Since the expected value of F at the five percent level of confidence is 1.97, a significant difference existed in the variances of the two groups. Accordingly, the Cochran-Cox method was used to measure the difference between the sample means. Utilization of the Cochran-Cox method resulted in a criterion t of .17 compared to 3.19 for the observed t. Thus, a significant difference existed between the means of the experimental group and control group

when tested for logic and reasoning ability; and the null hypothesis of no significant difference was rejected in favor of the experimental group.

Table 29

## ANALYSIS OF GROUP RESULTS FOR POST-TREATMENT TEST OF LOGIC AND REASONING

Group	N	Mean	Variance
Experimental	26	31.96	7.08
Control	25	28.04	32.04
Calculated F =	4.53	Observed t =	3.19
Expected F <sub>.05</sub> =	1.97	Criterion t <sub>.05</sub> =	.17
Difference:	Significant	Difference:	Significant
Hypothesis:	Rejected	Null hypothesis:	Rejected

Student Opinions

Whether student learning behavior is affected by the use of a computer when it is utilized as a teaching and/or learning tool is of great importance to many people, especially in education. However, the opinion of students relevant to the computer as an instructional tool is also a major concern. To gain some insight regarding such opinion, survey instruments were designed and administered to all students who participated in the study. A copy of the instrument administered to the experimental group, as well as the one administered to the control group, can be found in Appendix D.

Opinions of Students in the Experimental Group. Instrument A was developed to identify the opinion of students in the experimental

group. The instrument was administered twice during the study: shortly after students began using the computer as an instructional tool; and again after treatment. Participants were asked to respond to six statements. Each statement included three possible responses: agree; don't know; disagree. Students were asked to circle the response which, at the time, best described their opinion. Table 30 summarizes the responses to each statement.

To statistically analyze the results, responses were weighted accordingly:

Agree	+ 1 point
Don't know	0
Disagree	- 1 point

A one-way analysis of variance was then performed to the early-treatment/post-treatment scores for each of the six statements. As identified by the investigator prior to initiating the study, the five percent level of confidence was used to measure significant difference. Accordingly, a calculated F ratio greater than 4.04 indicated a significant difference in the post-treatment opinion of students in the experimental group when compared to their early-treatment opinion.

Some educators who use a computer as an instructional tool contend that students like courses in which a computer is used more than other courses in which a computer is not used. However, the calculated value of the F ratio relevant to the statement, "I like mathematics," indicated that students in the experimental group did not change their opinion significantly after having used the computer in algebra-trigonometry. The calculated F ratio of 1.48 was considerably below the expected value of 4.04. Thus, the null hypothesis of no significant difference between

TABLE 30

EARLY-TREATMENT AND POST-TREATMENT OPINIONS OF STUDENTS IN THE  
EXPERIMENTAL GROUP REGARDING THE USE OF A COMPUTER IN MATHEMATICS

Statement	Early-treatment	Post-treatment
1. I like mathematics.		
Agree	19	21
Don't know	4	5
Disagree	3	0
2. I like using a computer to learn mathematics.		
Agree	12	23
Don't know	12	0
Disagree	2	3
3. Using a computer helps me to understand mathematics.		
Agree	11	15
Don't know	10	8
Disagree	5	3
4. Using a computer has increased my interest in mathematics.		
Agree	10	13
Don't know	7	6
Disagree	9	7
5. In addition to mathematics, I would like the opportunity to use a computer to learn subject-matter in other courses (e.g. social studies or science).		
Agree	12	13
Don't know	7	7
Disagree	7	6
6. I have a better understanding of the computer because of the opportunity I have had to use a computer to learn mathematics.		
Agree	19	25
Don't know	6	1
Disagree	1	0

early-treatment and post-treatment opinion was accepted, as shown in Table 31.

TABLE 31

## "I LIKE MATHEMATICS":

AN ANALYSIS OF EARLY-TREATMENT/POST-TREATMENT OPINION  
OF STUDENTS IN THE EXPERIMENTAL GROUP

Source of Variation	df	Sum of Squares	Mean Square	F	Difference	Null Hypothesis
Within groups	49	15.68	.32			
Between groups	1	.48	.48	1.48	Insignificant	Accepted
Total	50	16.16				

Early-treatment response to the statement, "I like using a computer to learn mathematics," showed that two students did not like using a computer to learn mathematics, while 12 others "didn't know." After treatment, all students except three indicated that they liked using a computer to learn mathematics. When these results were subjected to a one-way analysis of variance, an F ratio of 4.63 was computed. Since the expected value of the F ratio at the five percent level of confidence is 4.04, a significant difference existed between early-treatment and post-treatment opinion. As shown in Table 32, the null hypothesis of no significant difference was rejected.

Statement three of the opinion instrument sought to determine if the students who used a computer as an instructional tool believed that the computer helped them to understand mathematics. Although 15 students agreed with the statement after treatment compared to 11 students earlier, no significant difference existed between early-



TABLE 32

"I LIKE USING A COMPUTER TO LEARN MATHEMATICS":  
AN ANALYSIS OF EARLY-TREATMENT/POST-TREATMENT OPINION  
OF STUDENTS IN THE EXPERIMENTAL STUDY

Source of Variation	df	Sum of Squares	Mean Square	F	Difference	Null Hypothesis
Within groups	49	20.58	.42			
Between groups	1	1.92	1.92	4.63	Significant	Rejected
Total	50	22.50				

treatment and post-treatment opinion. Table 33 shows that the F ratio between early-treatment and post-treatment opinion resulted in a value of .05. Since the expected value of the F ratio at the five percent level of confidence is 4.04, the null hypothesis of no significant difference was accepted.

TABLE 33

"USING A COMPUTER HELPS ME TO UNDERSTAND MATHEMATICS":  
AN ANALYSIS OF EARLY-TREATMENT/POST-TREATMENT OPINION  
OF STUDENTS IN THE EXPERIMENTAL GROUP

Source of Variation	df	Sum of Squares	Mean Square	F	Difference	Null Hypothesis
Within groups	49	26.46	.54			
Between groups	1	.69	.69	1.28	Insignificant	Accepted
Total	50	27.15				

Proponents supporting the use of a computer as an instructional tool often contend that utilization of a computer in a course increases student interest in the subject-matter. The opinion of students in the experimental group did not support this contention. No significant difference was found between early-treatment and post-treatment opinion when students were asked to respond to the statement: "Using a computer has increased my interest in mathematics." Table 34 shows that the calculated value of the F ratio was .1 compared to the expected value of 4.04. Thus, no significant difference existed between early-treatment and post-treatment opinion; and the null hypothesis of no significant difference was accepted.

TABLE 34

"USING A COMPUTER HAS INCREASED MY INTEREST IN MATHEMATICS":  
AN ANALYSIS OF EARLY-TREATMENT/POST-TREATMENT OPINION  
OF STUDENTS IN THE EXPERIMENTAL GROUP

Source of Variation	df	Sum of Squares	Mean Square	F	Difference	Null Hypothesis
Within groups	49	36.75	.75			
Between groups	1	.48	.48	.64	Insignificant	Accepted
Total	50	37.23				

During early-treatment, 19 of the 26 students in the experimental group replied affirmatively to the statement: "In addition to mathematics, I would like the opportunity to use a computer to learn subject-matter in other courses (for example, social studies and science)." After treatment, 25 of the 26 students indicated an affirmative reply.

Accordingly, an analysis of early-treatment and post-treatment opinion resulted in an F ratio of 1.03, as shown in Table 35. The expected value of F at the five percent level of confidence is 4.04; thus, no significant difference existed between the early-treatment and post-treatment opinion of students in the experimental group.

TABLE 35

"IN ADDITION TO MATHEMATICS, I WOULD LIKE THE OPPORTUNITY TO USE A COMPUTER TO LEARN SUBJECT-MATTER IN OTHER COURSES":  
AN ANALYSIS OF EARLY-TREATMENT/POST-TREATMENT OPINION  
OF STUDENTS IN THE EXPERIMENTAL GROUP

Source of Variation	df	Sum of Squares	Mean Square	F	Difference	Null Hypothesis
Within groups	49	32.83	.67			
Between groups	1	.69	.69	7.81	Insignificant	Accepted
Total	50	33.52				

Shortly after treatment began, 19 students in the experimental group indicated they they had a better understanding of the computer because of the opportunity afforded them to use the device in mathematics. After treatment, all students in the experimental group except one agreed to the statement: "I have a better understanding of the computer because of the opportunity I have had to use a computer to learn mathematics." A one-way analysis of variance between early-treatment and post-treatment results revealed an F ratio of 7.81 compared to the expected ratio of 4.04 at the five percent level of confidence. Thus, a significant difference existed between the early-treatment

and post-treatment opinion of students in the experimental group, as cited in in Table 36, and the null hypothesis was rejected.

TABLE 36

"I HAVE A BETTER UNDERSTANDING OF THE COMPUTER BECAUSE OF THE OPPORTUNITY I HAVE HAD TO USE A COMPUTER TO LEARN MATHEMATICS":  
AN ANALYSIS OF EARLY-TREATMENT/POST-TREATMENT OPINION  
OF STUDENTS IN THE EXPERIMENTAL GROUP

Source of Variation	df	Sum of Squares	Mean Square	F	Difference	Null Hypothesis
Within groups	49	12.25	.25			
Between groups	1	1.92	1.92	7.81	Significant	Rejected
Total	50	14.17				

Opinions of Students in the Control Group. What are the opinions of students who could not use a computer as an instructional tool although it was available to other students in another section of the same course? To answer this question, students in the control group were administered Opinion Instrument B after treatment, consisting of six statements. Each statement included three possible responses: agree; don't know; disagree. Students were asked to circle the response which, at the time, best described their opinion. Table 37 summarizes the responses to each statement.

Like students in the experimental group, most students in the control group liked mathematics; 20 of the 25 students indicated accordingly. In addition, 17 of the students in the control group replied that they would liked to have had the opportunity to use a computer to

TABLE 37

POST-TREATMENT OPINIONS OF STUDENTS IN THE CONTROL GROUP  
RELEVANT TO NOT USING A COMPUTER IN MATHEMATICS.

Statement	Post-treatment
1. I like mathematics.	
Agree	20
Don't know	4
Disagree	1
2. I would have liked to have had the opportunity to use a computer to learn mathematics this semester.	
Agree	17
Don't know	5
Disagree	3
3. Using a computer would have helped me to better understand mathematics.	
Agree	6
Don't know	12
Disagree	7
4. Students who used a computer to learn mathematics this semester had an advantage over me.	
Agree	7
Don't know	11
Disagree	7
5. If I take an additional mathematics course, I would like to use a computer in the course to learn mathematics.	
Agree	17
Don't know	4
Disagree	4
6. I would like the opportunity to use a computer to learn subject-matter in courses other than mathematics (e.g. social studies or science).	
Agree	17
Don't know	5
Disagree	3

learn mathematics; while only three responded negatively. Nevertheless, only six of the students expressed the opinion that using a computer would have helped them to better understand mathematics; 12 didn't know; seven disagreed. Similarly, in replying to the statement, "Students who used a computer to learn mathematics this semester had an advantage over me," seven agreed; 11 didn't know; and seven disagreed.

A rather sizeable number, 17 of the 25 students in the control group, expressed an affirmative reply to the statement: "If I take an additional mathematics course I would like to use the computer in the course to learn mathematics." Only four students indicated disagreement; while four others responded that they "didn't know." Almost identically, 17 students replied that they would like the opportunity to use a computer to learn subject-matter in courses other than mathematics; five indicated that they "didn't know"; only three responded negatively.

## CHAPTER V

### SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

#### Purpose of the Study

The major purpose of this study was to determine if students who use a computer to learn mathematics attain a higher level of achievement than other students of the same ability level who do not use a computer to learn mathematics. Since the opinion of students regarding such experience is also an important concern to many educators, questionnaires too were administered during the investigation. If supportive conclusions can be reported from this study and other similar investigations, there is justification for the cost of computer resources for such teaching and learning. Indeed, there would also be reason to examine alternative means of introducing the technique or process.

Interest in pursuing the project was stimulated by the lack of empirical research regarding computer-assisted problem-solving, as well as the widespread enthusiasm of many people who support the use of computers in the development of student learning behavior.

#### Research Procedures

Students in two algebra-trigonometry classes participated in the project; 14 boys and 12 girls in the experimental group which used a computer as a teaching and learning tool; 14 boys and 11 girls in the control group which did not use a computer. All students were of "middle ability" level, having been empirically placed at that level by mathematics teachers based upon each student's prior achievement in mathematics, but randomly scheduled into the two groups by a computer.

Both the experimental group and control group were taught by

the same teacher, a person with considerable experience in mathematics and computer-assisted problem-solving. Except for instruction and assignments in the experimental group involving the use of the computer terminals and the language called BASIC, the course objectives, methods, techniques, and instructional materials were the same for both groups. Thus, the experimental group used a computer as an instructional tool to learn algebra-trigonometry; the control group did not.

Students in the experimental group worked directly with three teletypewriter terminals, two of which were connected through private telephone lines to a large-scale, digital computer located approximately 15 miles away. The students typed numerals, letters, and symbols found on the keyboard to do computational work, solve mathematical problems, and otherwise "teach" the computer that which they learned in class.

The study took one 19-week semester to complete. Most utilization of the computer terminals was in the mathematics workroom or the hallway adjacent to the classroom so that the students using the terminals would not disturb the other members remaining in the room. However, a telephone line was installed in the classroom to allow the teacher the opportunity to use a terminal for teaching and demonstration purposes.

Since the computer was utilized in the study as an instructional tool rather than as an object of instruction, the teacher used the computer whenever he considered it beneficial in achieving the course objectives. Students in the experimental group were free and encouraged to use the computer terminals not only when direct assignments were made concerning the use of the computer, but also when other applicable problems were assigned. Utilization of the computer as a teaching and/or



learning tool during the study was entirely at the discretion of the teacher and/or students in the experimental group. Accordingly, the terminals were generally used in three ways: (1) as a computational tool; (2) as a "teaching" and learning tool; and (3) experimentally.

Hence, except for instruction in the experimental group relevant to the computer terminals and the BASIC language, considerable effort was devoted to keep the instructional strategies alike in both participating groups. Since the use of the computer with the experimental group represented a manipulation of the instructional variable, the intent was to make it the paramount variable in the instruction and learning of the two groups. Accordingly, one off-line and two on-line computer terminals were available to students in the experimental group during the study; and the curricular strategy adopted called for the identification of those aims and objectives within the existing algebra-trigonometry curriculum of the Dearborn Public Schools where a computer could best be used to improve student learning behavior.

Evaluative instruments were administered to both the experiment group and control group before and after treatment, as well as intermittently. The instruments were of two major types: (1) test instruments that measured the achievement of individual students; and (2) opinion instruments.

None of the instruments which were used to measure achievement contained any reference to computers, computer languages, or other related areas of computer knowledge. The primary rationale for selection of the test instruments, as well as the time to administer them, was identification and measurement of student achievement. Pre-treatment tests were administered to all participating students to assess intelli-

gence level, reading ability, and mathematical achievement before the experimental treatments began. To measure student achievement relative to each of the five instructional units in the course, a teacher-constructed test was administered at the conclusion of each unit. Finally, to help assess the effect of the semester's treatment, four post-treatment tests were utilized. Three of the instruments were administered in a different form during pre-treatment testing to measure each participant's understanding of mathematical concepts, development of mathematical skills, and performance of mathematical problem-solving. The fourth post-treatment test was administered to determine if the students who wrote computer algorithms showed growth in an area not given a great deal of emphasis in the other three post-treatment tests, that is, in the development of logic and reasoning ability.

To identify the opinion of students relevant to the use of a computer as an instructional tool--both in the experimental group as well as in the control group--appropriate opinion instruments were designed and administered to all participants.

#### Statistical Procedures

To measure the difference in achievement between the experimental group and control group before treatment, a two-tailed t-test of significance was applied. Utilization of a t-test in such instances where the difference between group means are measured assumes homogeneity of variance between the two random samples of the population. Hence, before applying the t-test, an F test was performed to determine if homogeneity of variance existed between the two groups. In those instances where equality of variance did not exist between the groups, application of a t-test of significance was performed by the method developed by

Cochran and Cox. The Cochran-Cox method does not assume nor necessitate homogeneity of variance between two samples of a population.

To measure the differences in growth between the experimental group and control groups after treatments, a one-tailed t-test was applied.

#### Results of Testing the Null Hypotheses

Pre-treatment testing of intelligence quotient, reading comprehension, mathematical concepts, mathematical skills, and mathematical problem-solving showed that no significant difference existed between the experimental group and control group at the five percent level of confidence. Only when tested for reading vocabulary did a significant difference exist between the two groups. However, this group difference was not considered important because no student in either group possessed a vocabulary deficiency below the 9.9 grade level. Hence, with the exception of reading vocabulary, the null hypothesis depicting the similarity of the two groups prior to treatment was accepted:

There is no significant difference in readiness between the experimental group and control group when measured by test instruments in the following areas prior to treatment: intelligence quotient, reading vocabulary, reading comprehension, mathematical concepts, mathematical skills, and mathematical problem-solving.

Interim or post-unit tests which included (1) review material and radicals in equations, (2) trigonometric functions and complex numbers, and (3) circular functions and their inverses, showed no significant difference between the two groups at the five percent level of confidence. Accordingly, the null hypothesis depicting the similarity of the experimental group and control group for these three units was accepted:

When evaluated by teacher-constructed post-unit tests that excluded reference to computers, computer languages, computer programming, and other areas of computer knowledge, there is no significant difference in achievement between the experimental group which used a digital, time-shared, electronic computer to learn mathematics and the control group which did not use the computer to learn mathematics.

When the two groups were administered the post-unit test of exponential functions and logarithms, however, a significant difference was found to exist in favor of the experimental group. In contrast, when students were administered the post-unit test of trigonometric identities and formulas, a significant difference was found to exist in favor of the control group. Thus, the null hypothesis depicting the similarity of the two groups for these two units after treatment was rejected.

When the groups were tested at the end of the semester's treatment for understanding of mathematical concepts and mastery of mathematical problem-solving, no significant difference in growth was found between the experimental group and control group. Hence, the following null hypotheses were accepted:

When tested by a teacher-constructed post-treatment instrument measuring ability to understand and apply mathematical concepts, there is no significant difference in achievement between the experimental group which used a digital, time-shared, electronic computer to learn mathematics and the control group which did not use the computer to learn mathematics.

When tested by a standardized post-treatment instrument measuring mathematical problem-solving ability, there is no significant difference in achievement between the experimental group which used a digital, time-shared, electronic computer to learn mathematics and the control group which did not use the computer to learn mathematics.

When tested for mastery of mathematical skills, a significant difference in growth was found to have occurred during treatment in

favor of the experimental group. Accordingly, the following null hypothesis was rejected:

When tested by a standardized post-treatment instrument measuring level of mathematical skills, there is no significant difference in achievement between the experimental group which used a digital, time-shared, electronic computer to learn mathematics and the control group which did not use the computer to learn mathematics.

When tested after treatment for ability to use logic and reasoning, a significant difference was found to exist again between the two groups in favor of the "computer group." Thus, the following null hypothesis was rejected:

When tested by a teacher-constructed post-treatment instrument measuring logic and reasoning ability, there is no significant difference in achievement between the experimental group which used a digital, time-shared, electronic computer to learn mathematics and the control group which did not use the computer to learn mathematics.

Finally, no significant difference existed between the early-treatment and post-treatment opinion of students in the experimental group when asked to respond to the following statements:

- I like mathematics.
- Using a computer helps me to understand mathematics.
- Using a computer has increased my interest in mathematics.
- In addition to mathematics, I would like the opportunity to use a computer to learn subject-matter in other courses. (for example, social studies or science).

Accordingly, the null hypothesis was accepted:

When measured by an investigator-constructed opinion instrument, there is no significant difference between the early-treatment opinions of the experimental group relevant to the use of a digital, time-shared, electronic computer to learn mathematics and the group's post-treatment opinions concerning the use of the computer to learn mathematics.

However, a significant difference was found to exist between the early-treatment and post-treatment opinion of students in the experimental group when asked to respond to the statements:

- I like using a computer to learn mathematics.
- I have a better understanding of the computer because of the opportunity I have had to use a computer to learn mathematics.

Regarding these two statements, the null hypothesis of no significant difference between the early-treatment and post-treatment opinion of students in the experimental group was affirmatively rejected.

#### Conclusions

As a result of the investigation and the inherent statistical analysis, several conclusions were derived:

1. Prior to treatment, there was no significant difference in readiness between the experimental group and control group in the following areas: intelligence quotient, reading comprehension, mathematical concepts, mathematical skills, and mathematical problem-solving.
2. No individual student who participated in the study possessed a serious reading deficiency that would hinder his achievement in mathematics.
3. After unit treatment involving algebraic review material and radicals in equations, there was no significant difference between the mean achievement of students who used a computer during treatment and the mean achievement of students who did not use a computer during treatment.
4. After unit treatment involving exponential functions and logarithms, there was a significant difference between the mean achievement

of students who used a computer during treatment and the mean achievement of students who did not use a computer during treatment; students who used a computer attained a significantly higher level of achievement than students who did not use a computer.

5. After unit treatment involving trigonometric functions and complex numbers, there was no significant difference between the mean achievement of students who used a computer during treatment and the mean achievement of students who did not use a computer during treatment.

6. After unit treatment involving trigonometric identities and formulas, there was a significant difference between the mean achievement of students who used a computer during the treatment and the mean achievement of students who did not use a computer during treatment; students who did not use a computer attained a significantly higher level of achievement than students who used a computer.

7. After unit treatment involving circular functions and their inverses, there was no significant difference between the mean achievement of students who used a computer during treatment and the mean achievement of students who did not use a computer during treatment.

8. The group of students which used a computer to learn algebra-trigonometry showed significant growth during treatment in understanding mathematical concepts, developing mathematical skills, and performing mathematical problem-solving.

9. The group of students which did not use a computer to learn algebra-trigonometry showed significant growth during treatment in understanding mathematical concepts and performing mathematical problem-solving, but no significant growth in developing mathematical skills.

10. During the semester's treatment, there was no significant

difference in growth between the ability of students who used a computer to apply mathematical concepts and the ability of students who did not use a computer to apply mathematical concepts.

11. During the semester's treatment, there was a significant difference in growth between the mathematical skills developed by students who used a computer in algebra-trigonometry and the mathematical skills developed by students who did not use a computer in algebra-trigonometry; students who used a computer attained a significantly higher level of achievement than those students who did not use a computer.

12. During the semester's treatment, there was no significant difference in growth between the problem-solving ability of student who used a computer and the problem-solving ability of students who did not use a computer.

13. After the semester's treatment, there was a significant difference between the logic and reasoning ability of students who used a computer in algebra-trigonometry and students who did not use a computer in algebra-trigonometry; students who used a computer attained a significantly higher level of achievement than those students who did not use a computer.

14. Experiencing the use of a computer as an instructional tool in algebra-trigonometry did not significantly change the opinion of students regarding the following statements:

- I like mathematics.
- Using a computer helps me to understand mathematics.
- Using a computer has increased my interest in mathematics.
- In addition to mathematics, I would like the opportunity



to use a computer to learn subject-matter in other courses (for example, science or social studies).

15. Experiencing the use of a computer as an instructional tool in algebra-trigonometry significantly and affirmatively changed the opinion of students regarding the following statements:

- I like using a computer to learn mathematics.
- I have a better understanding of the computer because of the opportunity I have had to use a computer to learn mathematics.

16. Students in the control group expressed a general sentiment that they were interested in using a computer as an instructional tool; however, they did not believe that a computer would necessarily help them understand the subject-matter of a course.

17. There is a widespread need for revision and further development of instructional materials relevant to computer-assisted problem-solving in mathematics.

#### Implications and Recommendations for Future Study

The statistical evidence obtained through this study provides evidence to support the use of computer-assisted problem-solving in mathematics. However, the writer does not contend or imply that the use of a computer as a teaching and learning tool results in optimal learning. Instead, the results of the investigation indicate that there are certain areas and aspects in mathematics in which the use of a computer as an instructional tool favorably effects student learning behavior; and accordingly, its use is highly desirable. Furthermore, the study indicates that most students seem to enjoy using a computer as an instructional tool.

Nevertheless, it is obvious that a great deal of additional research must still be undertaken to further examine the hypotheses inherent in this study, and to answer questions such as the following:

1. Is there a relationship between the amount of time that a student uses a computer as an instructional tool in a course and his academic achievement in the course?
2. Do students who write computer algorithms and use a computer as an instructional tool achieve at a higher level than students who write computer algorithms which are evaluated by a teacher without being processed through a computer?
3. What effect on student learning behavior does the use of computer-assisted problem-solving have upon "below-average" students?
4. Does the academic performance of males differ from that of females when they use a computer as a problem-solving tool?
5. What effect on student learning behavior does the use of a computer have when used as a teaching and learning tool in science, economics, or other curricular areas?
6. What effect does socio-economic background have upon the academic achievement of students who utilize computer-assisted problem-solving?
7. Is group size an important factor when utilizing computer-assisted problem-solving?
8. Does the use of different programming languages have differing effects upon student learning behavior?
9. Does the utilization of particular computer terminals, such as visual display or "hard copy," result in a difference in student learning behavior?

10. What effect does the number of computer terminals available to students have upon the development of student learning behavior?

11. What degree of retention do students have regarding subject-matter in a course in which computer-assisted problem-solving is utilized?

12. How might a mixture of computer-assisted problem-solving with other "modes" of CAI effect the development of student learning behavior?

13. What are the transfer effects of using computer-assisted problem-solving?

14. Do students who use a computer as an instructional tool obtain beneficial learning experiences that are not identified through the administration of traditional test instruments?

15. Is the computer becoming so important in our society that its utilization should be specifically identified in course objectives?

In conclusion, the study reported in this dissertation attempted to gather evidence concerning the educational value of computer-assisted problem-solving in high school mathematics. Though tentative, the results of the investigation contribute empirical evidence to assist educators in their quest to provide the best possible learning environment for students. The results clearly encourage further exploration of this important instructional strategy.

APPENDIX A

Sample of a Computer Program with Diagnostic Error Messages from Computer

COMPUTER PROGRAM USING BASIC LANGUAGE

EXPLANATION: This program uses the law of cosines,  
 $a^2 = b^2 + c^2 - (2ac) \cos \alpha$ .  
 $\alpha$  is the included angle between sides b and c. Remember  
the measure of the angle must be changed from degrees to  
radians before using the function COS.

```

10 PRINT "SIDE", "SIDE", "INCLUDED ANGLE", "THIRD SIDE"
11 PRINT
20 READ S1,S2,A1
30 LET A = A1 * 3.14159265/180
40 LET S3 = SQR(S1^2 + S2^2 - 2*S1*S2*COS(A))
50 PRINT S1,S2,A1,S3
60 GO TO 20
900 DATA 4,3,90, 2,1,60, 1,1,90, 7.5,7.5,60, 4,3,30, 7,5,45
910 DATA 4,4,35
999 END
RUN

```

COMPUTER RESPONSE

SIDE	SIDE	INCLUDED ANGLE	THIRD SIDE
4	3	90	5.
2	1	60	1.73205
1	1	90	1.41421
7.5	7.5	60	7.5
4	3	30	2.05314
7	5	45	4.95
4	4	35	2.40565

OUT OF DATA IN 20

RAN 0.67 SEC.

READY.

COMPUTER PROGRAM WRITTEN WITH ERRORS

EXPLANATION: This program uses the law of cosines,  
 $a^2 = b^2 + c^2 - (2ac) \cos \alpha$ .  
 $\alpha$  is the included angle between sides b and c. Remember  
 the measure of the angle must be changed from degrees to  
 radians before using the function COS.

```

10 PRINT "SIDE", "SSIDE", INCLUDED ANGLE", "THIRD SIDE
11 READ
20 READ S1,S2,A1
30 LET A = A1*3.14159265///180
40 LET S3 = SQR(S1^2) + S2^2 - S**S1*S2*COS(A))
50 PRINT S1,S2,A1,S3
60 GO TO 20
990 DATA 3,4,90, 2,1,60, 1,1,90, 7.5,7.5,60, 4,3,30, 7,5,45
910 DATA 4,4,35
999 DATA END
END
RUN

```

DIAGNOSTIC MESSAGE

```

ILLEGAL FORMULA      IN 10
ILLEGAL VARIABLE    IN 11
ILLEGAL VARIABLE    IN 20
ILLEGAL FORMULA     IN 30
ILLEGAL FORMULA     IN 40
ILLEGAL CONSTANT    IN 999
NO END INSTRUCTION

```

RAN 0.67

READY.

APPENDIX B

Sample of a Computer Log

## A C T S

## BILLING

Dearborn Board of Education  
 Mr. Frank Ronan  
 4824 Lois  
 Dearborn, Michigan 48126

April 30, 1970  
 Page No. 11  
 Invoice No.

RE: 265 BILLING TERMS: NET 10

USER NUMBER	NUMBER	DATE	TERMINAL TIME IN MINUTES	COMPUTER TIME IN SECONDS	STORED UNITS
Y 60450	MCD	APR 28	6.0	70.1	
Y 60450	MCD	APR 29	9.0	26.0	
Y 60450	MCD	APR 31			
USER TOTALS ---			31.0	106.9	
Y 60450	OLI	APR 21	32.0	24.6	
Y 60450	OLI	APR 27	11.0	11.8	
Y 60450	OLI	APR 28	3.0	25.8	
Y 60450	OLI	APR 30	3.0	27.0	
Y 60450	OLI	APR 31			
USER TOTALS ---			49.0	89.2	
Y 60450	POL	APR 23	2.0	5.1	
Y 60450	POL	APR 27	19.0	16.3	
Y 60450	POL	APR 31			
USER TOTALS ---			21.0	21.4	
Y 60450	RAG	APR 7	2.0	3.6	
Y 60450	RAG	APR 31			
USER TOTALS ---			2.0	3.6	
Y 60450	ROE	APR 22	37.0	92.0	



APPENDIX C  
Teacher-Constructed Test Instruments

## SURVEY OF TRIGONOMETRIC CONCEPTS

FORM A

On the following pages you will find concepts of mathematics and their application in problem form. Some of the questions are of the True - False type while others are in multiple choice form, having only one correct answer. Choose the one correct answer to each question and shade in its part on the separate answer sheet.

Read the following examples.

## TRUE - FALSE

1. The answer to:  $3x + 6 = 0$ , will be a negative number.

## MULTIPLE CHOICE

2. If  $a = 5$ , what is  $5/5 - a$ ? The correct answer is

- (1) 1  
 (2) 5  
 (3) 10  
 (4) 0  
 (5) Not given

## SAMPLE ANSWER SHEET

	T	F			
1.	<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	<u>5</u>
2.	<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	<u>5</u>

Use a regular #2 pencil for marking the answer sheet. If you erase, do so cleanly. Use the scratch paper provided for working the problems. Await the signal to begin. You will have 40 minutes to work on this survey of trigonometric concepts.

SURVEY OF TRIGONOMETRIC CONCEPTS

PART I: TRUE-FALSE

1. The product of two rational numbers can never be irrational.
2. The equation  $a^2 + b = b^2 + a$  is an illustration of the commutative property of addition.
3. If  $a < 0$  and  $b > 0$ , then  $ab < 0$ .
4.  $4^{-2} = + 1/16$
5.  $7^3 = 343$
6. If  $f(x) = x^2 - x + 1$ , then  $f(-1) = f(2)$
7. If  $x = \sqrt[4]{7}$ , then  $4x = 7$

PART II. MULTIPLE CHOICE

8. Given true statements "a R b" and "b R d" where "R is transitive," which one of the following statements is true?
  - (1) b R d and d R b
  - (2) b R a
  - (3) b R a or d R b
  - (4) a R d
  
9. Consider the following equations:
  - I.  $a(b + c) = ac + bc$
  - II.  $xy + w = xy + w$
  - III.  $5d - 2 + 3 = 2d + 1 + 3d$

Which of the equations are identities?

- (1) I only
- (2) II only
- (3) III only
- (4) I and II
- (5) I, II, and III

10. Which one of the following illustrates the fact that subtraction of integers is not commutative?

(1)  $(7 - 2) - 3 \neq 7 - (2 - 3)$

(2)  $4 - 4 = 8 - 8$

(3)  $6 - 15 \neq 15 - 6$

(4)  $4 - 11 = 11 - 4$

(5)  $17 - 13 = 16 - 12$

11. The conjugate of the complex number  $2 - 3i$  is

(1)  $3i - 2$

(2)  $3 - 2i$

(3)  $2 + 3i$

(4)  $2$

(5)  $-3$

12. Simplify:  $i^3 =$

(1)  $-1$

(2)  $+1$

(3)  $-i$

(4)  $+i$

(5) None of these

13.  $x^{\frac{3}{4}}$  means

(1)  $\frac{3}{4} \cdot x$

(2)  $\frac{3x}{4}$

(3)  $\frac{x^3}{4}$

(4)  $\sqrt[4]{x^3}$

(5) None of these

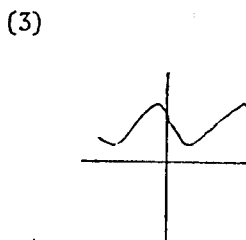
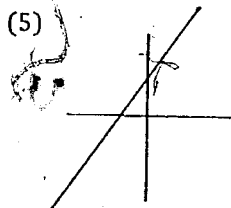
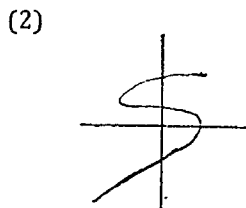
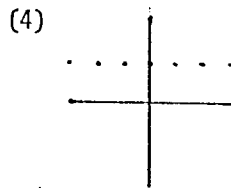
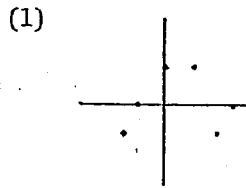
14. A logarithmic equation that means the same as  $125 = 5^3$  is

- (1)  $\log_5 3 = 125$
- (2)  $\log_3 5 = 125$
- (3)  $\log_5 125 = 3$
- (4)  $\log_3 125 = 5$
- (5) None of these

15.  $\text{Arctan } \sqrt{3}$  equals

- (1)  $\pi/2$
- (2)  $\pi/3$
- (3)  $\pi/4$
- (4)  $\pi/6$
- (5)  $\pi$

16. Which of the following is NOT the graph of a function?



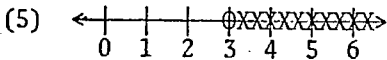
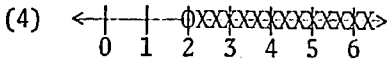
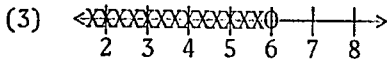
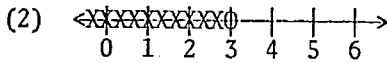
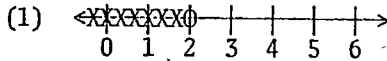
17. The range of the relation  $\{(-2, 1), (-1, 2), (0, 1), (3, 0), (4, -1), (3, -3)\}$  is
- (1)  $\{-2, -1, 0, 3, 4\}$
  - (2)  $\{-3, -1, 0, 1, 2\}$
  - (3) {all integers}
  - (4) {all real numbers}
  - (5)  $\{-3, 4\}$
18. The statement, "b is directly proportional to c and the constant of proportionality is n," is expressed mathematically as
- (1)  $b = nc$
  - (2)  $c = n + b$
  - (3)  $c = mn + b$
  - (4)  $b = c + n$
  - (5)  $b + c = n$
19. The expression  $\frac{x - 3}{x^2 - 4}$  does not represent a real number if
- (1)  $x = 3$
  - (2) x is irrational
  - (3)  $x = 2$
  - (4)  $x = 4$
  - (5)  $x = 0$
20. If x represents 3 and y represents 5, the symbol  $xy$  represents
- (1) 35
  - (2) 15
  - (3) 8
  - (4) 2
  - (5) -2

21. The polynomial  $3x^2 - 4xyz$  is a
- (1) binomial of degree 2
  - (2) trinomial of degree 2
  - (3) binomial of degree 3
  - (4) trinomial of degree 3
  - (5) None of these
22. Which one of the following is NOT equivalent to  $p < g$ ?
- (1)  $p + 3 < g + 3$
  - (2)  $-3p < -3g$
  - (3)  $2p < 2g$
  - (4)  $p - 2 < g - 2$
  - (5) All of them are equivalent
23. Which one of the following belongs to the solution set of the equation  $\frac{3}{t} - t = -2$
- (1) -3
  - (2) -2
  - (3) -1
  - (4) 0
  - (5) +1
24. Which two of the following equations are equivalent?
- |                        |                       |
|------------------------|-----------------------|
| I. $3x - 5 = 3 - 5x$   | II. $3x + 5 = 5x + 3$ |
| III. $5x + 3 = 3x - 5$ | IV. $3x - 5 = 5x - 3$ |
- (1) I and II
  - (2) II and III
  - (3) I and III
  - (4) II and IV
  - (5) I, II, III, and IV

25.  $\{1, 4, 7, 10\} \cup \{4, 6, 8, 10\} =$

- (1)  $\{1, 4, 6, 7, 8, 10\}$
- (2)  $\{4, 10\}$
- (3)  $\{5, 10, 15, 20\}$
- (4)  $\{1, 6, 7, 8\}$
- (5)  $\{50\}$

26. Which of the following is the graph of the set  $\{x: 2x < 6\}$



27. The intersection of sets  $\{a, b, c, d, e\}$  and  $\{a, e, i, o, u\}$  is

- (1)  $\{a, e\}$
- (2)  $\{a, b, c, d, e, i, o, u\}$
- (3)  $\{b, c, d, i, o, u\}$
- (4)  $\{b, c, d\}$
- (5)  $\emptyset$

28. Dave substituted 3 in place of  $x$  and 5 in place of  $y$  in an equation, obtaining the following:  $3 + 3 = 2 + 5 - 1$ . From this he may conclude

- (1) The equation is  $x + 3 = 2 + y - 1$
- (2) The equation is  $3 + x = 2 + y - 1$
- (3) The point  $(3, 5)$  is on the graph of the equation
- (4) The point  $(6, 6)$  is on the graph of the equation
- (5) (No conclusion possible from given information)



29. The point with coordinates  $(0, 2)$  is on the graph of an equation. Which one of the following is definitely not the equation?

- (1)  $x + 2 = y$
- (2)  $(3 - x)^2 = y + 7$
- (3)  $x^2 = y^2 + 6$
- (4)  $x/2 + y/2 = 1$
- (5)  $xy + x = 0$

30. Which one of the following illustrates that subtraction is not an associative operation?

- (1)  $7 - (5 - 2) \neq (7 - 5) - 2$
- (2)  $8 - 3 \neq 3 - 8$
- (3)  $8 - 5 = 5 - 2$
- (4)  $(4 - 3) \cdot 5 \neq 4 \cdot 5 - 3 \cdot 5$
- (5)  $10 - 4 \neq 10 - 6$

31.  $5^x = 32$ , therefore  $x$  must be

- (1) between 2 and 3
- (2) between 3 and 4
- (3) between 5 and 6
- (4) between 6 and 7
- (5) None of these

32. Which one of the following is the converse of the statement "If the wind doesn't blow, the kite won't fly."?

- (1) If the wind blows the kite will fly.
- (2) If the kite will fly, the wind blows.
- (3) If the kite won't fly, the wind won't blow.
- (4) If the wind blows, the kite won't fly.
- (5) None of these

33. Which one of the following equations shows that 5 is a factor of 15?

(1)  $15 = 5 + 10$

(2)  $15 = 5 \cdot 3$

(3)  $5 \cdot 15 = 75$

(4)  $15 - 5 = 10$

(5)  $15 + 5 = 20$

34. Consider the numbers 12,  $-41/9$ ,  $\sqrt{5}$ . Which of them can be represented by points on the number line?

(1) 12 only

(2)  $-41/9$  only

(3) 12 and  $-41/9$

(4) 12,  $-41/9$  and  $\sqrt{5}$

(5) None of them

35. Which one of the following ordered pairs is in the solution set of the system?

$$\begin{aligned} 2x + 3y &= 8 \\ x + y &= 3 \end{aligned}$$

(1) (4, 0)

(2) (4, 1)

(3) (1, 2)

(4) (2, 1)

(5) (5, 0)

## SURVEY OF TRIGONOMETRIC CONCEPTS

FORM B

On the following pages you will find concepts of mathematics and their application in problem form. Some of the questions are of the True - False type while others are in multiple choice form, having only one correct answer. Choose the one correct answer to each question and shade in its part on the separate answer sheet.

Read the following examples.

## TRUE - FALSE

1. The answer to:  $3x + 6 = 0$ , will be a negative number.

## MULTIPLE CHOICE

2. If  $a = 5$ , what is  $5/5 - a$ ? The correct answer is

- (1) 1  
 (2) 5  
 (3) 10  
 (4) 0  
 (5) Not given

## SAMPLE ANSWER SHEET

	T	F			
1.	<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	<u>5</u>
2.	<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	<u>5</u>

Use a regular #2 pencil for marking the answer sheet. If you erase, do so cleanly. Use the scratch paper provided for working the problems. Await the signal to begin. You will have 40 minutes to work on this survey of trigonometric concepts.

SURVEY OF TRIGONOMETRIC CONCEPTSPART I: TRUE-FALSE

1. The sum of two rational numbers is sometimes an irrational number.
2. The equation  $5 + \sqrt{3} = 3 + \sqrt{5}$  is an illustration of the commutative property of addition.
3. If  $a < 0$  and  $b < 0$ , then  $ab > 0$ .
4.  $x^4$  means  $x + x + x + x$ .
5.  $\sqrt[3]{5}$  is a symbol for the number such that  $\sqrt[3]{5} \cdot \sqrt[3]{5} \cdot \sqrt[3]{5} = 5$
6.  $x^3 = 10$ , therefore  $x$  is between 3 and 4.

PART II: MULTIPLE CHOICE

7.  $9^{-2}$  equals

(1)  $\frac{1}{81}$

(2)  $\frac{1}{-81}$

(3) -18

(4) 3

(5) -3

8. If  $f(n) = 3n + 2$ , then  $f(3) =$

(1) 8

(2) 11

(3) 12

(4) 18

(5) 35

9. Due to the transitivity of the subset relation, given  $A \subset B$  and  $B \subset C$ , one can conclude

- (1)  $A \cap B = B \cap C$
- (2)  $A \cap C = \emptyset$
- (3)  $A \subset C$
- (4)  $A = C$
- (5)  $A = B = C$

10. Consider the three equations:
- I.  $2x + 3 = x^2 + 3 - x^2 + 2x$
  - II.  $a + b = b + a$
  - III.  $c^2 - d^2 = c^2 - d^2$

Which of the equations are identities?

- (1) III only
  - (2) II only
  - (3) II and III
  - (4) I, II, and III
  - (5) None are identities
11. Which one of the following illustrates the fact that division of integers is NOT commutative?

- (1)  $(8 \div 4) \div 2 \neq 8 \div (4 \div 2)$
- (2)  $10 \div 5 \neq 5 \div 10$
- (3)  $8 \div 1 \neq 7 \div 1$
- (4)  $6 \div 6 = 3 \div 3$
- (5)  $6 \div 3 = 4 \div 2$

12. The conjugate of the complex number  $5 + 2i$  is

- (1)  $5i + 2$
- (2)  $2i + 5$
- (3)  $5 - 2i$
- (4)  $5$
- (5)  $+ 2$

13. Simplify:  $i^4 =$

- (1) -1
- (2) +1
- (3) -i
- (4) +i
- (5) None of these

14.  $8^{2/3}$  equals

- (1) 4
- (2)  $8 + 2/3$
- (3)  $16/3$
- (4)  $64/3$
- (5)  $8\sqrt[3]{8}$

15. Solve for x:  $\log_9 x = 3$

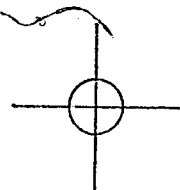
- (1) {3}
- (2) {9}
- (3) {27}
- (4) {729}
- (5) None of these

16. Arctan 1 equals

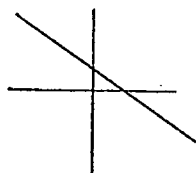
- (1)  $\pi$
- (2)  $\frac{\pi}{2}$
- (3)  $\frac{\pi}{3}$
- (4)  $\frac{\pi}{4}$
- (5)  $\frac{\pi}{6}$

17. Which of the following is NOT the graph of a function?

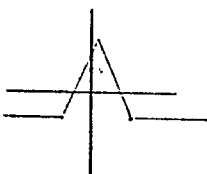
(1)



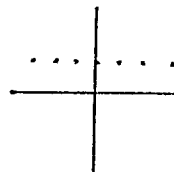
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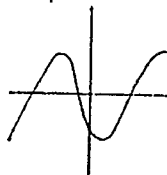
(3)



(4)



(5)



18. The domain of the relation

$\{(-2, 1), (-1, 2), (0, 1), (3, 0), (4, -1), (3, -3)\}$  is

(1)  $\{-2, -1, 0, 3, 4\}$ (2)  $\{-3, -1, 0, 1, 2\}$ 

(3) {all integers}

(4) {all real numbers}

(5) None of these

19. The statement, "x is directly proportional to y and the constant of proportionality is a," is expressed mathematically as

(1)  $x = y + a$ (2)  $x = ay$ (3)  $y = a + x$ (4)  $xy = a$ (5)  $y = ax + b$

20. The expression  $\frac{x^2 - 5}{x^2 - 9}$  does not represent a real number if

(1)  $x = \sqrt{5}$

(2)  $x = 0$

(3)  $x = 3$

(4)  $x = 9$

(5)  $x = \frac{5}{9}$

21. If a represents 6 and b represents 2, the symbol  $ab$  represents

(1) 62

(2) 26

(3) 12

(4) 8

(5) 4

22. The polynomial  $2x^3y^3 - 7x^2y^4$  has degree

(1) 2

(2) 3

(3) 4

(4) 5

(5) 6

23. Which one of the following is NOT equivalent to  $x > y$ ?

(1)  $5x > 5y$

(2)  $x - 6 > y - 6$

(3)  $-3x > -3y$

(4)  $x + 2 > y + 2$

(5) All of them are equivalent



24. Which one of the following belongs to the solution set of the equation  $\frac{5}{x} + \frac{8}{x+3} = 1 + \frac{4}{x-1}$

- (1) 0
- (2) 1
- (3) 3
- (4) 8
- (5) -3

25. Which two of the following equations are equivalent?

I.  $2x + 5 = 3x + 2$

II.  $3x + 5 = 2x + 2$

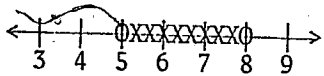
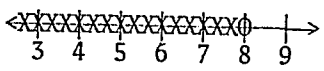
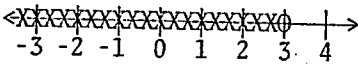
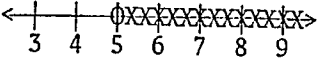
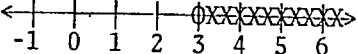
III.  $4x + 1 = 3x + 4$

- (1) I and II only
- (2) II and III only
- (3) I and III only
- (4) I, II, and III
- (5) No two are equivalent

26.  $\{2, 4, 6\} \cup \{2, 3, 4\} =$

- (1)  $\{2, 4\}$
- (2)  $\{2, 3, 4, 6\}$
- (3)  $\{4, 7, 10\}$
- (4)  $\{3, 6\}$
- (5)  $\{21\}$

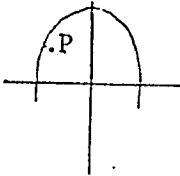
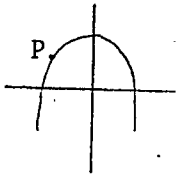
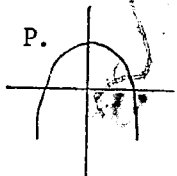
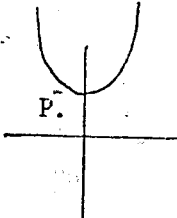
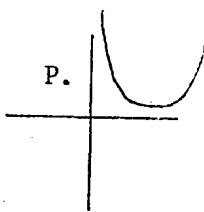
27. Which of the following is the graph of the set  $\{x: 5 + x < 8\}$

- (1) 
- (2) 
- (3) 
- (4) 
- (5) 

28. The intersection of sets  $\{2, 3, 4, 5\}$  and  $\{2, 4, 6, 8, 10\}$  is

- (1)  $\{2, 3, 4, 5, 6, 8, 10\}$
- (2)  $\{3, 5, 6, 8, 10\}$
- (3)  $\{2, 4\}$
- (4)  $\emptyset$
- (5)  $\{3, 5\}$

29. Which one of the following correctly illustrates the relationship between the point  $P(-2, 4)$  and the graph of the equation  $x^2 + 2y = 12$ ?

- (1) 
- (2) 
- (3) 
- (4) 
- (5) 

30. Given a point with coordinates  $(3, 5)$  on the graph of the equation  $x + k = 10$ , which one of the following is true?

- (1)  $k = -1$
- (2)  $(-3, -5)$  is also on the graph
- (3) The graph is a straight line
- (4) The graph is a parabola
- (5)  $(-3, 5)$  is also on the graph

31. Suppose the symbol  $\square$  represents a new binary operation on the set of real numbers. The sentence " $(a \square b) \square c = a \square (b \square c)$  for any real numbers  $a$ ,  $b$ , and  $c$ " is equivalent to which one of the following?

- (1)  $\square$  is a commutative operation.
- (2)  $\square$  is an associative operation.
- (3)  $\square$  is a symmetric operation.
- (4)  $\square$  is an ordered operation.
- (5)  $\square$  is a distributive operation.

32. Which one of the following is the converse of the statement, "If it rains, then it pours?"

- (1) If it pours then it rains.
- (2) If it doesn't rain, then it doesn't pour.
- (3) If it doesn't pour, then it doesn't rain.
- (4) If it rains, then it doesn't pour.
- (5) None of these

33. Which one of the following equations shows that 4 is a factor of 12?

(1)  $12 = 4 + 8$

(2)  $12 = 4 \cdot 3$

(3)  $4 \cdot 12 = 48$

(4)  $12 - 4 = 8$

(5)  $12 + 4 = 16$

34. Consider the numbers  $-14$ ,  $\frac{57}{18}$ ,  $\sqrt{3}$ . Which of them can be represented by points on the number line?

(1)  $-14$  only

(2)  $\frac{57}{18}$  only

(3)  $-14$  and  $\frac{57}{18}$

(4)  $-14$ ,  $\frac{57}{18}$ , and  $\sqrt{3}$

(5) None of them

35. Which one of the following ordered pairs is in the solution set of the system  $\begin{cases} 4x + 2y = 6 \\ x + y = 1 \end{cases}$  ?

(1)  $(1, 1)$

(2)  $(1, 0)$

(3)  $(0, 1)$

(4)  $(0, 0)$

(5)  $(2, -1)$

## ALGEBRA-TRIGONOMETRY

UNIT TEST 1

1. If the number  $b$  has two real  $n^{\text{th}}$  roots, we may conclude that

- (A)  $n$  is odd and  $b$  is positive
- (B)  $n$  is odd and  $b$  is negative
- (C)  $n$  is even and  $b$  is positive
- (D)  $n$  is even and  $b$  is negative
- (E) None of these

2. The set of real roots of  $8x^3 + 1 = 0$  is

- (A)  $\left\{\frac{1}{2}\right\}$
- (B)  $\left\{-\frac{1}{2}\right\}$
- (C)  $\left\{\frac{1}{2}, -\frac{1}{2}\right\}$
- (D)  $\emptyset$
- (E) None of these

3. The set of real roots of  $2y^4 = 18$  is

- (A)  $\{9\}$
- (B)  $\{3, -3\}$
- (C)  $\{\sqrt{3}, -\sqrt{3}\}$
- (D)  $\emptyset$
- (E) None of these

4. The set of real roots of  $x^5 = 25$  is

- (A)  $\{5\}$
- (B)  $\{5, -5\}$
- (C)  $\{\sqrt[5]{25}, \sqrt[5]{25}\}$
- (D)  $\{\sqrt[5]{25}\}$
- (E)  $\emptyset$

5. Which of the following cannot be a root of the equation

$$6x^3 - x^2 - 19x - 6 = 0 ?$$

- (A)  $-\frac{1}{3}$
- (B)  $-\frac{3}{2}$
- (C)  $\frac{4}{2}$
- (D)  $\frac{4}{3}$
- (E)  $\frac{6}{2}$

6. Which of the following is a polynomial equation in simple form with integral coefficients and having  $-\sqrt{\frac{2}{3}}$  as a root?

- (A)  $x^2 - \frac{2}{3} = 0$
- (B)  $2x^2 = 3$
- (C)  $3x^2 = 2$
- (D)  $3x^2 + 2 = 0$
- (E)  $3x^2 - 2 = 0$

7. Which one of the following numbers is irrational?

(A)  $\sqrt{1.44}$

(B)  $\sqrt{27}$

(C) 1.414

(D)  $\sqrt[3]{-8}$

(E)  $\overline{3.142857}$

8. If P and Q are irrational while R is rational, which one of the following will always be irrational?

(A)  $P + R$

(B)  $P + Q$

(C)  $PQ + R$

(D)  $PR + Q$

(E)  $PQ$

9.  $\sqrt[5]{(243)^3} =$

(A) 3

(B) 6

(C) 9

(D) 27

(E) 54

10.  $\sqrt[3]{\frac{27}{8}}$  in simple radical form is

(A)  $\frac{3}{4} \sqrt{54}$

(B)  $\frac{9}{4} \sqrt{54}$

(C)  $\frac{9}{4} \sqrt{6}$

(D)  $\frac{3}{8} \sqrt{216}$

(E) None of these

11.  $\sqrt[3]{16x^3 y^{-5}}$  ( $y \neq 0$ ) in simple radical form is

(A)  $xy \sqrt[3]{16y^{-2}}$

(B)  $\frac{x}{y} \sqrt[3]{16y^{-2}}$

(C)  $\frac{2x}{y} \sqrt[3]{\frac{2}{y^2}}$

(D)  $\frac{2x}{y^2} \sqrt[3]{2}$

(E) None of these

12.  $\sqrt{\frac{6}{35}} \cdot \sqrt{\frac{14}{15}}$  in simple form is

(A)  $\frac{1}{35} \sqrt{210} \cdot \frac{1}{5} \sqrt{210}$

(B)  $\frac{1}{50} \sqrt{210}$

(C)  $\sqrt{\frac{84}{525}}$

(D)  $\frac{4}{25}$

(E)  $\frac{2}{5}$



13.  $\frac{\sqrt[3]{64x^5y^4}}{\sqrt[2]{4xy^2}}$  ( $x \neq 0, y = 0$ ) in simple form is

(A)  $\frac{2y \sqrt{x} \sqrt[3]{xy}}{|y|}$

(B)  $\frac{2xy \sqrt[3]{xy}}{|y| \sqrt{x}}$

(C)  $\sqrt[3]{16x^4y^2}$

(D)  $2x \sqrt[3]{2xy^2}$

(E)  $4x^2y$

14.  $\sqrt{45} - \sqrt{20}$  in simple form is

(A)  $\sqrt{5}$

(B) 5

(C)  $\sqrt{\frac{45}{20}}$

(D)  $\frac{3}{2}$

(E) None of these

15.  $(3 + \sqrt{2})(\sqrt{18} - 2)$  in simple form is

(A)  $7\sqrt{2}$

(B) 6

(C) 9

(D)  $3\sqrt{18} - 2\sqrt{2}$

(E) None of these

16. Which of the following is true for any real number  $x$  ?

- (A)  $\sqrt{x^2} = x$
- (B)  $\sqrt{|x|^2} = x$
- (C)  $|\sqrt{x^2}| = x$
- (D)  $\sqrt{x^2} = |x|$
- (E) None of these

17. The solution set of  $x\sqrt{3} - 2 = \sqrt{6}$  is

- (A)  $\{ \sqrt{2} \}$
- (B)  $\{ \sqrt{2} - 2 \}$
- (C)  $\{ \sqrt{2} + \sqrt{3} \}$
- (D)  $\left\{ \frac{3\sqrt{2} + 2\sqrt{3}}{3} \right\}$
- (E) None of these

IN PROBLEMS 18 - 20 APPROXIMATE TO TENTHS, GIVEN  $\sqrt{2} \doteq 1.414$  and  $\sqrt{3} \doteq 1.732$

18.  $\sqrt{3}(3 - \sqrt{3}) \doteq$

- (A) 0
- (B) 2.1
- (C) 2.2
- (D) 3.0
- (E) None of these

19.  $\frac{3 + \sqrt{2}}{\sqrt{2}} \approx$

- (A) 1.5
- (B) 3.0
- (C) 3.1
- (D) 6.2
- (E) None of these

20.  $\sqrt{18} - \sqrt{8} \approx$

- (A) 0.8
- (B) 1.4
- (C) 2.8
- (D) 3.1
- (E) None of these

## ALGEBRA-TRIGONOMETRY

UNIT TEST 2

1. Simplify:  $\sqrt[3]{2^5} \cdot \sqrt[4]{2}$

(A)  $\frac{17}{2^{20}}$

(B)  $2 \cdot \sqrt[2]{2^{11}}$

(C)  $2^{\frac{6}{7}}$

(D)  $\sqrt[7]{2^6}$

(E)  $\sqrt[13]{2^6}$

2. In exponential form,  $3 \sqrt[3]{8x^{-9}w^6} =$

(A)  $6x^{-3}w^2$

(B)  $24x^{-3}w^2$

(C)  $24x^{-9}w^2$

(D)  $8x^{-9}w^6$

(E)  $8x^{-3}w^2$

3. Evaluate  $64^{\frac{2}{3}}$

(A)  $\frac{128}{3}$

(B)  $\sqrt[3]{128}$

(C) 4

(D) 16

(E) None of these

4. Evaluate  $\left(\frac{9}{25}\right)^{\frac{3}{2}}$

(A)  $-\frac{27}{50}$

(B)  $\frac{6}{25}$

(C)  $\frac{3}{5}$

(D)  $\frac{125}{27}$

(E)  $\frac{27}{125}$

5. Simplify:  $10^{2.3} \times 10^{-0.8} \div 10^{-0.45}$

(A)  $10^{2.65}$

(B)  $10^{3.55}$

(C)  $10^{0.05}$

(D)  $10^{1.95}$

(E) None of these

6. Simplify:  $\left(4^{\frac{1}{2}} \sqrt{2}\right)^{\sqrt{2}}$

(A) 4

(B) 2

(C)  $2\sqrt{2}$

(D)  $2^{\sqrt{2}}$

(E) None of these

7. Solve:  $7^n = 7^{2n} - 3$

- (A) {7}
- (B) {343}
- (C) {1}
- (D) {3}
- (E) None of these

8. State in logarithmic form:  $125 = 5^3$

- (A)  $\log_5 3 = 125$
- (B)  $\log_3 5 = 125$
- (C)  $\log_5 125 = 3$
- (D)  $\log_3 125 = 5$
- (E) None of these

9.  $\log_{64} 32 =$

- (A) 2
- (B)  $\frac{1}{2}$
- (C)  $\frac{5}{6}$
- (D)  $\frac{6}{5}$
- (E) None of these

10. Solve for  $x$ :  $\log_9 x = 3$

- (A) {3}
- (B) {9}
- (C) {27}
- (D) {729}
- (E) None of these

11. Find  $\log 0.783$

- (A) 0.8938
- (B) 9.7830 - 10
- (C) 9.8938 - 10
- (D) 6.07
- (E) None of these

12. Find antilog 2.6920

- (A) 4,920
- (B) 0.8401
- (C) 0.4301
- (D) 4.92
- (E) 492

13. If  $9^x = 27$ , then  $x =$

- (A)  $\frac{3}{2}$
- (B) 2
- (C)  $\log 4$
- (D)  $\log 8 - \log 4$
- (E) None of these

14. Find  $\log 88,280$

- (A) 4.9456
- (B) 4.9459
- (C) 3.9456
- (D) 3.9459
- (E) None of these

15. Find antilog  $7.4701 - 10$

- (A) .002952
- (B) 295.2
- (C) .006722
- (D) 8733
- (E) None of these

16. Evaluate  $N = (14.7) (0.0451)$  using logs. Give 3 significant digits in the answer.

- (A) .663
- (B) .6632
- (C) .6633
- (D) .8215
- (E) .822



17. Evaluate  $N = \frac{2370}{8948}$  using logs. Give 4 significant digits in answer.

- (A) .2120
- (B) .2714
- (C) .02714
- (D) .275
- (E) None of these

18. Write the following equation in logarithmic form.  $9^x = 4$

19. Solve for x:  $\log_{10}(x^2 + 1) = 1$

20. Solve for n:  $\log_2 n = \log_2 5 + \log_2 9 - \log_2 15$

In each of problems 21 - 25 evaluate N using common logs. Show your logarithmic equation and all of your work. Assume the data given are approximations.

21.  $N = (.721)(.894)$

22.  $N = \sqrt{5133}$

$$23. N = \frac{(10.85)(.93)}{-.672}$$

$$24. N = \sqrt[5]{.08298}$$

$$25. N = \frac{608 \sqrt{0.00859}}{9.484 \sqrt[3]{0.0368}}$$

$$26. \text{ Solve for } n: 3 \log n = 2 \log 8$$

$$27. \text{ Solve for } x: 5^x = 30 \text{ (to the nearest hundredth)}$$

## ALGEBRA-TRIGONOMETRY

UNIT TEST 3

In problems 1 - 3 the coordinates of a point P are given. Let  $\theta$  be any position angle for P.

1. P:(5, 10). Find  $\sin \theta$

(A)  $\frac{1}{2}$

(B) 10

(C)  $\frac{\sqrt{5}}{5}$

(D)  $\frac{2}{\sqrt{5}}$

(E) None of these

2. P:(8, -3). Find  $\tan \theta$

(A)  $\frac{8}{-3}$

(B)  $-\frac{3}{8}$

(C)  $\frac{8}{\sqrt{73}}$

(D)  $\frac{-3}{\sqrt{73}}$

(E) None of these

3.  $P: (-1, 0)$ . Find  $\csc \theta$

- (A) 0
- (B) +1
- (C) -1
- (D) Undefined
- (E) None of these

4. For a certain angle  $\theta$  in standard position  $\sin \theta > 0$  and  $\tan \theta < 0$ .

Therefore the terminal side of  $\theta$  must be in quadrant . . .

- (A) I
- (B) II
- (C) III
- (D) IV
- (E) None of these

5. For a certain angle  $\theta$  in standard position  $\sec \theta > 0$ . Therefore

the terminal side of  $\theta$  must be in quadrants . . .

- (A) I or II
- (B) II or III
- (C) III or IV
- (D) IV or I
- (E) None of these

6.  $\cos 60^\circ =$

- (A)  $\frac{1}{2}$
- (B)  $\frac{\sqrt{2}}{2}$
- (C)  $\frac{\sqrt{3}}{3}$
- (D)  $\frac{\sqrt{3}}{2}$
- (E) None of these

7.  $\cot 45^\circ =$

- (A)  $\frac{1}{2}$
- (B)  $\frac{\sqrt{2}}{2}$
- (C)  $\frac{\sqrt{3}}{3}$
- (D)  $\frac{\sqrt{3}}{2}$
- (E) None of these

8.  $\sin 180^\circ =$

- (A) 0
- (B) +1
- (C) -1
- (D) Undefined
- (E) None of these

9.  $\sec 30^\circ =$

- (A)  $\frac{1}{2}$
- (B)  $\frac{\sqrt{2}}{2}$
- (C)  $\frac{\sqrt{3}}{3}$
- (D)  $\frac{\sqrt{3}}{2}$
- (E) None of these

10.  $\cos 45^\circ =$

- (A)  $\frac{1}{2}$
- (B)  $\frac{\sqrt{2}}{2}$
- (C)  $\frac{\sqrt{3}}{3}$
- (D)  $\frac{\sqrt{3}}{2}$
- (E) None of these

11.  $\csc 270^\circ =$

- (A) 0
- (B) +1
- (C) -1
- (D) Undefined
- (E) None of these

In problems 12, 13, 14, 15, 17, and 18 use Table 6 or Table 7 (whichever is appropriate) to look up the values.

12.  $\cot 25^{\circ} 37' =$

- (A) .4781
- (B) .4795
- (C) 2.086
- (D) .4745
- (E) None of these

13.  $\tan \theta = 2.115$ . Find  $\theta$ .

- (A)  $64^{\circ}42'$
- (B)  $64^{\circ}52'$
- (C)  $64^{\circ}48'$
- (D)  $25^{\circ}18'$
- (E) None of these

14.  $\log \sin 64^{\circ}23' =$

- (A) .9017
- (B) 9.9551 - 10
- (C) 9.9547 - 10
- (D) 9.6358 - 10
- (D) None of these

15.  $\log \cos \theta = 9.9720 - 10$ . Find  $\theta$ .

- (A)  $20^{\circ}22'$
- (B)  $20^{\circ}28'$
- (C)  $70^{\circ}38'$
- (D)  $69^{\circ}38'$
- (E) None of these

16. Express  $\sec 500^{\circ}$  as a function of an acute angle.

- (A)  $+\sec 40^{\circ}$
- (B)  $+\sec 50^{\circ}$
- (C)  $-\sec 40^{\circ}$
- (D)  $-\sec 50^{\circ}$
- (E) None of these

17.  $\cot 260^{\circ} =$

- (A) .1763
- (B) - .1763
- (C) 5.671
- (D) - 5.671
- (E) None of these

18.  $\sin (-430^{\circ})$

- (A) - .3420
- (B) - .9397
- (C) + .3420
- (D) + .9397
- (E) None of these



19. Give exact values.  $\sin 240^\circ =$

(A)  $+\frac{1}{2}$

(B)  $-\frac{1}{2}$

(C)  $+\frac{\sqrt{3}}{2}$

(D)  $-\frac{\sqrt{3}}{2}$

(E) None of these

20. Give exact values.  $\tan 150^\circ =$

(A)  $+\frac{1}{2}$

(B)  $-\frac{1}{2}$

(C)  $+\frac{\sqrt{3}}{3}$

(D)  $-\frac{\sqrt{3}}{3}$

(E) None of these

21. Give exact values.  $\cos 315^\circ =$

(A)  $+1$

(B)  $-1$

(C)  $+\frac{\sqrt{2}}{2}$

(D)  $-\frac{\sqrt{2}}{2}$

(E) None of these

22. Another way of indicating the direction  $N40^{\circ}W$  is

- (A)  $040^{\circ}$
- (B)  $140^{\circ}$
- (C)  $220^{\circ}$
- (D)  $320^{\circ}$
- (E) None of these

23. Another way of indicating the direction  $150^{\circ}$  is

- (A)  $S30^{\circ}E$
- (B)  $S30^{\circ}W$
- (C)  $S50^{\circ}E$
- (D)  $S50^{\circ}W$
- (E) None of these

24. Point T has position angle  $120^{\circ}$  and is 14 units from the origin.

The horizontal component vector of vector  $\vec{OT}$  is

- (A) 7 right
- (B) 7 left
- (C)  $7\sqrt{3}$  up
- (D)  $7\sqrt{3}$  down
- (E) None of these

25. Simplify  $i^{47}$

- (A)  $i$
- (B)  $-i$
- (C)  $+1$
- (D)  $-1$
- (E) None of these

26. Simplify  $3\sqrt{-18} - \frac{1}{2}\sqrt{-32}$

- (A)  $38i$
- (B)  $-38i$
- (C)  $7\sqrt{2}i$
- (D)  $-7\sqrt{2}i$
- (E) None of these

27. Simplify  $-2\sqrt{-2} \cdot 3\sqrt{-32}$

- (A)  $-24\sqrt{2}i$
- (B)  $10\sqrt{2}i$
- (C)  $-48$
- (D)  $+48$
- (E) None of these

28. The conjugate of the complex number  $2i - 3$  is

- (A)  $-3 - 2i$
- (B)  $3 - 2i$
- (C)  $2i + 3$
- (D)  $-2$
- (E) None of these

29.  $\frac{7 + 2i}{2i}$  in standard form is

- (A)  $\frac{7}{2}i + 1$
- (B)  $1 + \frac{7}{2i}$
- (C)  $1 + \frac{7}{2}i$
- (D)  $1 - \frac{7}{2}i$
- (E) None of these

30.  $\frac{3 - 8i}{3 - 2i}$  in standard form is

- (A)  $\frac{25}{13} - \frac{18}{13}i$
- (B)  $5 - \frac{18}{5}i$
- (C)  $\frac{-7}{13} - \frac{18}{13}i$
- (D)  $\frac{9}{5} - \frac{34}{5}i$
- (E) None of these

31. The roots of  $3x^2 - 4x + 3 = 0$  are

(A)  $-\frac{2}{3} \pm \frac{\sqrt{5}}{3}i$

(B)  $\frac{2}{3} \pm \frac{\sqrt{5}}{3}i$

(C)  $\frac{2}{3} \pm \frac{20}{3}i$

(D)  $-\frac{2}{3} \pm \frac{20}{3}i$

(E) None of these

## ALGEBRA-TRIGONOMETRY

UNIT TEST 4

1. Simplify:  $\frac{1}{\sec \phi}$
  
2. Simplify:  $1 - \cos^2 A$
  
3. Simplify:  $\sin B \cot B$
  
4. Simplify:  $\frac{\tan \phi \cos \phi}{\sin \phi}$
  
5. Simplify:  $\frac{1 - \sin^2 X}{\sin^2 X}$
  
6. Simplify:  $\sec \theta \tan \theta \cos \theta$

7. Simplify:  $\frac{\sin^2 A}{\cot^2 A + 1}$

8. Simplify:  $\sin \theta \tan \theta - \csc \theta \tan \theta$

State whether the following identities are TRUE or FALSE identities.

9.  $\sin \theta + \cos \theta = 1$

True

False

10.  $\sin(\alpha + \beta) = \cos \alpha \sin \beta + \sin \alpha \cos \beta$

True

False

11.  $\sin(-\beta) = \sin \beta$

True

False

12.  $\cos(-\beta) = \cos \beta$

True

False

$$13. \cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

True

False

$$14. \cos \frac{\theta}{2} = \pm \sqrt{\frac{1 + \cos \theta}{2}}$$

True

False

$$15. \cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha$$

True

False

$$16. \cos(90^\circ - \theta) = \sec \theta$$

True

False

$$17. 2 \sin \rho \cos \rho = 2 \sin 2\rho$$

True

False

$$18. \cos \theta = \frac{1}{\csc \theta}$$

True

False



19.  $\sec^2 \alpha - \tan^2 \alpha = 1$

True

False

20. Prove the identities:  $\cos^3 \theta + \sin^2 \theta \cos \theta = \cos \theta$

21. Prove the identities:  $\frac{\csc B}{\sec B} = \cot B$

22. Prove the identities:  $\frac{\tan \phi}{1 + \tan^2 \phi} = \sin \phi \cos \phi$

23. Find the exact value of the following:

$$\cos 200^\circ \cos 80^\circ + \sin 200^\circ \sin 80^\circ$$

24. Find the exact value of the following:  $\sin 45^\circ \cos 75^\circ$

25. Find the exact value of the following:  $\sin^2 38^\circ + \cos^2 38^\circ$

19.  $\sec^2 \alpha - \tan^2 \alpha = 1$

True

False

20. Prove the identities:  $\cos^3 \theta + \sin^2 \theta \cos \theta = \cos \theta$

21. Prove the identities:  $\frac{\csc B}{\sec B} = \cot B$

22. Prove the identities:  $\frac{\tan \phi}{1 + \tan^2 \phi} = \sin \phi \cos \phi$

23. Find the exact value of the following:

$$\cos 200^\circ \cos 80^\circ + \sin 200^\circ \sin 80^\circ$$

24. Find the exact value of the following:  $\sin 45^\circ \cos 75^\circ$ 25. Find the exact value of the following:  $\sin^2 38^\circ + \cos^2 38^\circ$

26. Find the exact value of the following:  $\frac{2 \tan 22\frac{1}{2}^\circ}{1 - \tan^2 22\frac{1}{2}^\circ}$

(27-29) Solve completely the following triangles using this procedure:

- Draw and label a sketch of the triangle(s).
- State which case is involved (e.g. SAS, SSA) and give the number of possible triangles.
- Find one unknown part of each possible triangle. (Lengths should be given in three significant figures, and angles should be given correct to the nearest ten minutes.)
- Show all your work.

27. In  $\Delta ABC$ ,  $a = 63.0$ ,  $b = 77.0$ ,  $c = 58.0$

28. In  $\Delta PQR$ ,  $R = 97^\circ 40'$ ,  $r = 27.8$ ,  $p = 30.3$

29. In  $\Delta XYZ$ ,  $X = 14^\circ 30'$ ,  $y = 35.0$ ,  $x = 15.0$

(30-32) Find the area of  $\triangle ABC$  correct to three places.

30.  $a = 369$ ,  $b = 246$ ,  $c = 491$

31.  $b = 107$ ,  $c = 118$ ,  $A = 56^{\circ}10'$

32.  $A = 39^{\circ}00'$ ,  $B = 58^{\circ}00'$ ,  $c = 15.2$

## ALGEBRA-TRIGONOMETRY

UNIT TEST 5

1. Find the
- exact value
- :

$$\cos \frac{\pi}{4}$$

2. Find the
- exact value
- :

$$\tan \frac{7\pi}{12}$$

3. Find the
- exact value
- :

$$\sec \left( -\frac{\pi}{3} \right)$$

4. Find the value to the nearest hundredth:

$$\tan (2.4) \quad (\text{Use } 1^{\text{R}} \doteq 57^{\circ}18' \quad .1^{\text{R}} \doteq 5^{\circ}44')$$

5. Give the exact radian measure:
- $270^{\circ}$

6. Give the exact radian measure:  $-135^\circ$

Give the amplitude and period of each of the following functions

f whose values are given by:

7.  $f(x) = \frac{1}{2} \cos 2x$

8.  $f(x) = 3 \sin x$

9.  $f(x) = \sin 4x$

10.  $f(x) = \cos \pi x$

Sketch the graphs of each of the following functions through an interval of one period.

11.  $\{(x, y): y = 2 \sin \frac{1}{2}x\}$

12.  $\{(x, y): y = \cos 3x\}$

TEST  
OF  
LOGIC AND REASONING

DIRECTIONS:

1. On the separate answer sheet, print your name, date, and other requested information in the proper spaces.
2. Wait for further instructions. Do not turn the page until you are told to do so.



## LOGIC AND REASONING

PART 1

Directions: Look at the sample problems below. Each problem contains a statement and a conclusion. The statements and conclusions are given a code as follows:

- = means "is equal to."
- > means "is larger than."
- < means "is smaller than."
- $\neq$  means "is not equal, and so is larger or smaller."
- $\not>$  means "is not larger, and so is equal or smaller."
- $\not<$  means "is not smaller, and so is equal or larger."

Your task is to decide whether each conclusion is always true, always false, or impossible to definitely determine on the basis of the statement.

In the first example below, the statement says that X is equal to Y, and Y is equal to Z. The conclusion says that X is equal to Z. Since X, Y, and Z are all equal to each other, the conclusion is always true. Therefore, the answer space under "T" has been marked.

- |   |                                     |                          |                          |
|---|-------------------------------------|--------------------------|--------------------------|
|   | T                                   | F                        | ?                        |
| 1. $X = Y$ and $Y = Z$ , therefore, $X = Z$ | <input checked="" type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |

In the next problem, the statement says that X is larger than Y, and Y is larger than Z. The conclusion says that X is equal to Z. Since Y is larger than Z, X must be even larger, and the conclusion is always false. Therefore, the answer space under "F" has been marked.

- |  |                          |                                     |                          |
|--|--------------------------|-------------------------------------|--------------------------|
|  | T                        | F                                   | ?                        |
| 2. $X > Y$ and $Y > Z$ , therefore $X = Z$ | <input type="checkbox"/> | <input checked="" type="checkbox"/> | <input type="checkbox"/> |

In the third sample problem, the statement says that X is not equal to Y, and Y is not equal to Z. The conclusion says that X is not larger than Z. The statement indicates that neither X nor Z is equal to Y, but there is no way to tell whether X is larger, smaller, or equal to Z. Therefore, the conclusion is uncertain, and the answer space under "?" has been marked.

- |   |                          |                          |                                     |
|---|--------------------------|--------------------------|-------------------------------------|
|   | T                        | F                        | ?                                   |
| 3. $X \neq Y$ and $Y \neq Z$ , therefore, $X > Z$ | <input type="checkbox"/> | <input type="checkbox"/> | <input checked="" type="checkbox"/> |

To make sure you understand, mark the next three problems yourself.

- |   | T                        | F                        | ?                        |
|---|--------------------------|--------------------------|--------------------------|
| 4. $X < Y$ and $Y < Z$ , therefore, $X < Z$       | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| 5. $X = Y$ and $Y > Z$ , therefore, $X = Z$       | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| 6. $X \neq Y$ and $Y \neq Z$ , therefore, $X > Z$ | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |

You should have marked "T" for problem 4, "F" for problem 5, and "?" for problem 6. Are there any questions?

You have 10 minutes to answer 20 items.

## LOGIC AND REASONING

PART I

1.  $A > B$  and  $B = C$ , therefore  $A = C$ .
2.  $A > B$  and  $B > C$ , therefore  $A > C$ .
3.  $A < B$  and  $B < C$ , therefore  $A > C$ .
4.  $A \neq B$  and  $B = C$ , therefore  $A \neq C$ .
5.  $A > B$  and  $B < C$ , therefore  $A < C$ .
6.  $A \neq B$  and  $B = C$ , therefore  $A = C$ .
7.  $A < B$  and  $B = C$ , therefore  $A = C$ .
8.  $A < B$  and  $B > C$ , therefore  $A > C$ .
9.  $A \neq B$  and  $B \neq C$ , therefore  $A \neq C$ .
10.  $A \neq B$  and  $B \neq C$ , therefore  $A \neq C$ .
11.  $A \neq B$  and  $B > C$ , therefore  $A > C$ .
12.  $A > B$  and  $B \neq C$ , therefore  $A \neq C$ .
13.  $A = B$  and  $B < C$ , therefore  $A \neq C$ .
14.  $A > B$  and  $B \neq C$ , therefore  $A < C$ .
15.  $A < B$  and  $B \neq C$ , therefore  $A < C$ .
16.  $A \neq B$  and  $B \neq C$ , therefore  $A \neq C$ .
17.  $A > B$  and  $B \neq C$ , therefore  $A > C$ .
18.  $A \neq B$  and  $B < C$ , therefore  $A \neq C$ .
19.  $A > B$  and  $B \neq C$ , therefore  $A \neq C$ .
20.  $A < B$  and  $B \neq C$ , therefore  $A \neq C$ .

S T O P. DO NOT TURN TO PART II UNTIL YOU ARE TOLD TO DO SO.

## LOGIC AND REASONING

PART II

DIRECTIONS: Problems 21 through 32 on the following two pages refer to the story below. Read the story and then answer the multiple choice items, which follow by shading the one appropriate space with your pencil on the answer sheet. You have 20 minutes to answer items 21 through 32.

One afternoon at their regular club meeting at school, all of the members of the Mystery Trip Club of East High School were given a copy of the following street map and list of directions for getting to a party at the home of Jane Howard, the club president.

You are to read and follow all directions in numerical order unless the directions specify otherwise.

1. Go outside the school building to the street corner.
2. If you are not a sophomore, ignore instruction 3 and go directly to instruction 4.
3. From where you are standing, walk two blocks west.
4. If you are a senior, ignore instructions 5 and 6 and go directly to instruction 7.
5. From where you are standing now, walk two blocks south.
6. If you are a sophomore, ignore instructions 7, 8, and 9, and go directly to instruction 10.
7. Walk one block west.
8. If you are a senior, ignore instructions 9, 10, 11, and 12, and go directly to instruction 13.
9. Ignore instruction 10 and go directly to instruction 11.
10. Walk one block east.
11. Walk one block north.
12. Ignore instruction 13 and go directly to instruction 14.
13. Walk one block south.
14. If you have correctly followed the preceding instructions, you are now standing in front of Jane's house. Ring the doorbell and introduce yourself to Mrs. Howard. Refreshments will be served inside.

A	B	C	D	E
F	G	H	school □	J
K	L	M	N	O
P	Q	R	S	T



## LOGIC AND REASONING

PART II

21. Instruction 3 applies to
- A. Sophomores only
  - B. Juniors only
  - C. Seniors only
  - D. Juniors and Seniors only
  - E. Sophomores, Juniors, and Seniors
22. Instruction 4 will be read by
- A. Sophomores only
  - B. Juniors only
  - C. Seniors only
  - D. Juniors and Seniors only
  - E. Sophomores, Juniors, and Seniors
23. Jane's house is at location
- A. L
  - B. M
  - C. Q
  - D. R
  - E. S
24. How many blocks did Seniors walk?
- A. 2
  - B. 3
  - C. 4
  - D. 5
  - E. 6
25. How many blocks did Juniors walk?
- A. 2
  - B. 3
  - C. 4
  - D. 5
  - E. 6

26. Which one of the following points did the Sophomores not pass?

- A. N
- B. M
- C. L
- D. G
- E. H

27. How many blocks did Sophomores walk?

- A. 2
- B. 3
- C. 4
- D. 5
- E. 6

28. Instruction 9 will be read by

- A. Sophomores only.
- B. Juniors only
- C. Seniors only
- D. Sophomores and Juniors only
- E. Sophomores, Juniors, and Seniors

29. Instruction 10 will be read by

- A. Sophomores only
- B. Juniors only
- C. Seniors only.
- D. Sophomores and Juniors only
- E. Sophomores, Juniors, and Seniors.

30. Instruction 11 will be read by

- A. Sophomores only
- B. Juniors only
- C. Seniors only
- D. Sophomores and Juniors only
- E. Sophomores, Juniors, and Seniors

31. Which one of the following points did none of the students pass?

- A. G
- B. H
- C. K
- D. L
- H. M

32. Instruction 13 applies to

- A. Sophomores only
- B. Juniors only
- C. Seniors only
- D. Sophomores and Seniors only
- E. Sophomores and Juniors only

S T O P . DO NOT TURN TO PART III UNTIL YOU ARE TOLD TO DO SO.

## LOGIC AND REASONING

PART III

**DIRECTIONS:** In order to answer questions 33 through 37 you must first follow the instructions below. Follow the instructions in numerical order unless directed otherwise. Whenever you are directed to write a number in a box, you are first to erase whatever number is already in the box. You have 10 minutes to answer items 33 through 37.

---

1. Choose any one of the numbers of the set (7, 11, 19) and write it in the box labelled I.
2. Multiply the number in box I by two, and write the answer in the box labelled C.
3. Subtract 4 from the number in box C, and write the answer in box C.
4. If the number in box C is greater than 8, your next instruction is instruction 3.
5. Subtract 1 from the number in box C and write the answer in box C.
6. If the number in box C is positive, your next instruction is instruction 3.
7. Stop

I

C

---

**TRUE or FALSE?** With your pencil, shade the appropriate space on the answer sheet which best corresponds with your answer.

33. If a person chooses 7 in instruction 1, the number in box C when he reaches instruction 7 will be 0.
34. If a person chooses 11 in instruction 1, the number in box C when he reaches instruction 7 will be 9.
35. If a person chooses 19 in instruction 1, the number in box C when he reaches instruction 7 will be 25.
36. Regardless of the number chosen in instruction 1, the instruction 3 will be executed exactly three times.
37. Regardless of the number chosen in instruction 1, the instruction 5 will be executed exactly twice.

WHEN YOU HAVE COMPLETED PART III, CLOSE YOUR TEST BOOKLET;  
YOU MAY NOT RETURN TO PART I OR PART II.



## LOGIC AND REASONING

ANSWER SHEETPART I

- |     | T                        | F                        | ?                        |
|-----|--------------------------|--------------------------|--------------------------|
| 1.  | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| 2.  | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| 3.  | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| 4.  | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| 5.  | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| 6.  | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| 7.  | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| 8.  | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| 9.  | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| 10. | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| 11. | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| 12. | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| 13. | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| 14. | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| 15. | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| 16. | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| 17. | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| 18. | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| 19. | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| 20. | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |

PART II

- |     | A                        | B                        | C                        | D                        | E                        |
|-----|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|
| 21. | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| 22. | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| 23. | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| 24. | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| 25. | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| 26. | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| 27. | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| 28. | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| 29. | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| 30. | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| 31. | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| 32. | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |

PART III

- |     | T                        | F                        |
|-----|--------------------------|--------------------------|
| 33. | <input type="checkbox"/> | <input type="checkbox"/> |
| 34. | <input type="checkbox"/> | <input type="checkbox"/> |
| 35. | <input type="checkbox"/> | <input type="checkbox"/> |
| 36. | <input type="checkbox"/> | <input type="checkbox"/> |
| 37. | <input type="checkbox"/> | <input type="checkbox"/> |
| 38. | <input type="checkbox"/> | <input type="checkbox"/> |

APPENDIX D  
Opinion Instruments

## COMPUTER-ASSISTED PROBLEM-SOLVING

## MATHEMATICS

STUDENT OPINION INVENTORY

## INSTRUMENT A

Name of Student: \_\_\_\_\_

Date: \_\_\_\_\_

Boy: \_\_\_\_\_ Girl: \_\_\_\_\_ (check one)

Grade Level: \_\_\_\_\_

INSTRUCTIONS: Please circle the response which best describes your feelings.

1. I like mathematics.

- a. Agree
- b. Don't know
- c. Disagree

2. I like using a computer to learn mathematics.

- a. Agree
- b. Don't know
- c. Disagree

3. Using a computer helps me to understand mathematics.

- a. Agree
- b. Don't know
- c. Disagree

4. Using a computer has increased my interest in mathematics.

- a. Agree
- b. Don't know
- c. Disagree

5. In addition to mathematics, I would like the opportunity to use a computer to learn subject-matter in other courses (for example: social studies or science).
- a. Agree
  - b. Don't know
  - c. Disagree
6. I have a better understanding of the computer because of the opportunity I have had to use a computer to learn mathematics.
- a. Agree
  - b. Don't know
  - c. Disagree

## COMPUTER-ASSISTED PROBLEM-SOLVING

## MATHEMATICS

STUDENT OPINION INVENTORY

## INSTRUMENT B

Name of Student: \_\_\_\_\_ Date: \_\_\_\_\_

Boy: \_\_\_\_\_ Girl: \_\_\_\_\_ (check one)

Grade Level: \_\_\_\_\_

INSTRUCTIONS: Please circle the response which best describes your feelings.

1. I like mathematics.
  - a. Agree
  - b. Don't know
  - c. Disagree
  
2. I would have liked to have had the opportunity to use a computer to learn mathematics this semester.
  - a. Agree
  - b. Don't know
  - c. Disagree
  
3. Using a computer would have helped me to better understand mathematics.
  - a. Agree
  - b. Don't know
  - c. Disagree
  
4. Students who used a computer to learn mathematics this semester had an advantage over me.
  - a. Agree
  - b. Don't know
  - c. Disagree

5. If I take an additional mathematics course I would like to use the computer in the course to learn mathematics.
  - a. Agree
  - b. Don't know
  - c. Disagree
  
6. I would like the opportunity to use a computer to learn subject-matter in other than mathematics courses (for example: social studies or science).
  - a. Agree
  - b. Don't know
  - c. Disagree

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