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The logistic model-generated carrying capacities, maximum sustained off-take rates and optimal stocking rates for Kenya's commercial ranches

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This paper deals with the derivation of logistic models for cattle, sheep and goats in a commercial ranching system in Machakos District, Kenya, a savannah ecosystem with average annual rainfall of 589.3 ± 159.3mm and an area of 10 117ha. It involves modelling livestock population dynamics as discrete-time logistic equations with fixed carrying capacities. The fixed carrying capacities are generated endogenously using time-series ranch data, covering a period of 15 years, from 1987 to 2001, in a commercial ranching enterprise. The model incorporates interaction parameters, generated endogenously. The estimation of the logistic models involves estimation of econometric models for each livestock species, followed by the recovery of the logistic models mathematically. Optimisation procedures are employed to determine the optimal stocking levels and the optimal off-take levels. The model-generated carrying capacities are 2 985, 791 and 201 animal units (AUs) for cattle, sheep and goats, respectively. Optimal stocking levels are 1 369, 154 and 69 AUs for cattle, sheep and goats, respectively, while the optimal off-take levels are 857, 88 and 63 for cattle, sheep and goats, respectively. This shows that the logistic models-based system analysis is applicable to the management of mixed-species commercial ranching enterprises.

Keywords: animal units, commercial ranching enterprises, livestock species mix, logistic models

Introduction

Proper ranch management requires good grazing management strategies that provide effective means of manipulating the grazing animals so as to achieve the highest level of animal production sustainably (Pratt and Gwynne 1977). The means of manipulating the grazing animals include choice of optimal species mix (optimal stocking level). The optimal stocking level is equal to or less than the 'logistic model-generated' carrying capacity (Kinyua 1998, Kinyua and Njoka 2001). The logistic model-generated carrying capacity, therefore, represents the upper-bound stocking level of the land. For a fixed area of land, the highest level of animal production is equated to the maximum sustained off-take level (Pratt and Gwynne 1977).

The methods of determining these management parameters — maximum sustained off-take, optimal species mix (optimal stocking level) and the logistic model-generated carrying capacity — have various shortcomings. The determination of species mix (common use) is based on vegetation structure, diet selection and approximated animal exchange ratios (grazing pressure equivalence) that are derived from the ratios of the metabolic body weights (Stoddart and Smith 1955, Pratt and Gwynne 1977). This method of determining species mix relies on trial and error estimates and lacks analytical advantage, making it difficult to determine the optimal species mix. Similarly, the determination of stocking level and carrying capacity involves trial and error approximations, based on the availability of forage on the one hand, and the animal forage requirements on the other (Stoddart and Smith 1955, Pratt and Gwynne 1977). As in the case of species mix, this method lacks analytical advantage, which makes it also difficult to determine the optimal stocking rates.

In order to correct these shortcomings, which underlie the determination of carrying capacity, optimal species mix (optimal stocking levels) and optimal off-take levels, systems analysis procedures are used as an alternative method of determining these parameters. The systems analysis procedures are based on the generalised Lotka-Voterra mathematical model, involving a set of logistic equations for a given ranch (Odum 1971). This approach underlies the objective of this study, which is to provide estimates of the carrying capacity, estimates of the optimal species mix (optimal stocking levels) and estimates of the optimal off-take levels in a multi-livestock species setting, based on logistically formulated discrete-time livestock population dynamic models (Equation 1).

Conceptual view

The formulation of livestock population dynamics, based on discrete-time logistic equations with fixed carrying capacity, is captured in Equation (1), which is a difference equation

(Starfield and Bleloch 1986, Anderson 1991, Caughley and Sinclair 1994).

Equation (1):

$$L_{it+1} - L_{it} = \Delta L_{it} = \beta_i L_{it}(1 - \frac{L_{it} + \sum_{j=1}^{n-1} \sigma_{ij} L_{jt}}{K_i}) - S_{it}$$

for i, j = 1,...,n and i ≠ j

Here, S_{it} is a control variable, representing the off-take level of livestock species i during period t (time step in years) and is measured in animal units; L_{it} represents the biomass of livestock species i in period t and is measured in animal units; and K_i, also measured in animal units, represents the carrying capacity for livestock species i. The parameter β_i represents the exponential growth rate for the livestock species i. The parameter σ_{ij} represents the interaction effect of livestock species j on species i, and this interaction parameter is negative, zero or positive, depending on whether the interactive relationship is complementary, supplementary or competitive, respectively. This parameter also gives an estimate of exchange ratios or grazing pressure equivalence among species; that is, it plays the role of converting L_{jt} animal units into the equivalent L_{it} animal units. Under this formulation, both carrying capacity and exchange ratios are endogenously determined, making it possible to empirically estimate the theoretical population growth models where the logistic model-generated, rather than the theoretical, carrying capacities is determined endogenously.

Conceptually, the term K_i represents the maximum number of animal units of livestock species i that can be supported through period t, or the number of animal unit years (AUYs). Three cases can then be identified, as shown in Equations (2), (3) and (4).

Equation (2) — Case 1 (stocking level below carrying capacity):

$$K_i \succ (L_{it} + \sum_{j}^{n-1} \sigma_{ij} L_{jt})$$

for i, j = 1,...,n and i ≠ j

This implies that the AUYs of available forage exceed the AUYs of forage demand by the standing population of livestock species i. The standing livestock population increases, since there is enough forage to meet their maintenance requirements and leave a balance for growth and reproduction.

Equation (3) — Case 2 (stocking level equal to carrying capacity):

$$K_i = (L_{it} + \sum_{j}^{n-1} \sigma_{ij} L_{jt})$$

for i, j = 1,...,n and i ≠ j

This implies that the AUYs of available forage exactly match the AUYs of forage demand by the standing population of livestock species i. The standing livestock population barely meets its maintenance requirements, leaving no balance for growth and reproduction. As a result, the population change is zero, implying that L_{it} is at the maximum stocking level.

Equation (4) — Case 3 (stocking level above carrying capacity):

$$K_i \prec (L_{it} + \sum_{j}^{n-1} \sigma_{ij} L_{jt})$$

for i, j = 1,...,n and i ≠ j

In this case, the forage demand by livestock species i is greater than the AUYs of available forage, or the standing livestock population is greater than the carrying capacity. The excess number of animals will die out, resulting in the decline of the standing population of livestock species i.

The discrete-time livestock population logistic models, Equation (1), are, in fact, quadratic equations without an intercept (Beattie and Taylor 1985) and are derived from the discrete-time compounding model (Equation 5a).

Equation (5a):

$$L_{it} = L_{i0}(1 + r_i)^t - S_{i0}$$

for i = 1,...,n

where L_{it} represents the population of livestock species i in year t; L_{i0} represents the initial population (at t = 0) of livestock species i; r_i represents the rate of annual livestock population growth; S_{i0} represents the off-take level for livestock species i in time zero (t = 0); t represents discrete time-step in years. The term $L_{i0}(1+r_i)^t$ is the discrete-time compounding formula, capturing the population growth of livestock species i. When t = 1, Equation (5a) becomes Equations (5b).

Equation (5b):

$$L_{i1} = L_{i0}(1 + r_i)^1 - S_{i0}$$

for i = 1,...,n

When t = 2, Equation (5b) becomes Equation (5c).

Equation (5c):

$$L_{i2} = L_{i0}(1 + r_i)(1 + r_i) - S_{i1} = L_{i1}(1 + r) - S_{i1}$$

for i = 1,...,n

Here, β_i represents the fixed rate of population change for species i and parameter δ_i represents intraspecies competition. Parameter λ_{ij} represents interspecies interaction between the discrete-time livestock population logistic models; (Equation 1), are, in fact, quadratic equations without an intercept (Beattie and Taylor 1985) and are derived from the discrete-time compounding models (Equation 5a).

The general form of Equation (5c) is shown in Equation (5d). This equation is independent of time: that is, 't' is not a variable.

Equation (5d):

$$L_{it+1} = L_{it}(1+r_i) - S_{it}$$

for i = 1,...,n

From an ecological point of view, the rate of livestock population growth per year (r_i) is affected by intra- and interspecies competition, thus (r_i) is more realistically modelled as shown in Equation (5e).

Equation (5e):

$$r_i = \{\beta_i - (\delta_i L_{it} + \Sigma_j^{n-1} \lambda_{ij} L_{jt})\}$$

for i and j = 1,...,n and i ≠ j

Here, parameter β_i represents the fixed rate of population change for species *i* while parameter δ_i represents intraspecies competition. Parameter λ_{ij} represents interspecies interaction between species *i* and species *j*; this parameter is negative, zero or positive, depending on whether the interactive relationship is competitive, supplementary or complementary, respectively.

Substituting r_i in Equation (5d) with r_i in Equation (5e) yields Equation (5f).

Equation (5f):

$$L_{it+1} = L_{it}\{1+[\beta_i - (\delta_i L_{it} + \sum_{j=1}^{n-1} \lambda_{ij} L_{jt})]\} - S_{it}$$

for i and j = 1,...,n and i ≠j

Multiplying out Equation (5f) and rearranging terms yields Equation (6a) and, like Equation (1), it is a difference equation. Shifting S_{it} from the right to the left-hand side of Equation (6a) yields Equation (6b). Equation (6b) is the econometric (statistical) model, and although it is non-linear in variables, it is linear in parameters.

Equation (6a):

$$L_{it+1} - L_{it} = \Delta L_{it} = \beta_i L_{it} - \delta_i L^2_{it} + \sum_{j=1}^{n-1} \lambda_{ij} L_{it} L_{jt} - S_{it}$$

for i and j = 1,...,n and i ≠j

Equation (6b):

$$L_{it+1} - L_{it} + S_{it} = \Delta L_{it} + S_{it} = \beta_i L_{it} - \delta_i L^2_{it} + \sum_{j=1}^{n-1} \lambda_{ij} L_{it} L_{jt}$$

for i and j = 1,...,n and i ≠j

Materials and methods

Study area

The study area is located on Malili Ranch, 70km south-east of Nairobi on the Athi-Kapiti Plains, along the Nairobi-Mombasa road, in Makueni District, Eastern Province of the Republic of Kenya. The ranch has a land area of 10 117ha and is 1 750–1 850m above sea level. Its vegetation is typical of eco-climatic Zone 4 (Pratt and Gwynne 1977) and is mainly covered by *Acacia* species *Themeda triandra* tree grassland in association with *Chloris gayana*, *Setaria sphacelata*, *Eragrostis tenuifolia*, *Cenchrus ciliaris* and *Pennisetum mezianum* grass species, and *Acacia mellifera*, *A. drepanalobium*, *A. hockii*, *Balanites aegyptiaca*, *Combretum molle* and *Albizia gummifera* woody plant species. The ranch's operation is based on cattle, sheep and goat livestock species. Its average annual rainfall is 589.3mm for the period covering 1978–2001, with a standard deviation of 159.3; the rainfall ranges from a minimum of 177.4mm to a maximum of 924.5mm, as shown in Figure 1.

Methods

Through on-site visits to Malili Ranch in 2002, time series data for the period 1987 through 2001 on cattle, sheep and goat populations was collected from the ranch records. Animal numbers were then converted to AUs, effectively converting discrete data to continuous data, where one AU is equivalent to the metabolic body weight of 454kg. This data is shown in Tables 1, 2 and 3 for cattle, sheep and goats, respectively, while the distributions of livestock sales and the standing livestock population over time are shown in Figures 2, 3 and 4 for cattle, sheep and goats, respectively. The ranch livestock sales policy is not formalised; animal sales are dependent on the demand for slaughter stock and immature livestock by the adjacent agro-pastoral communities. Based on the data shown in Tables 1, 2 and 3, the complete econometric population growth model, Equation (6b), was estimated for cattle, sheep and goats, using the SPSS 2001 software for statistical analysis. The respective specific forms of the cattle, sheep and goat models are as shown in the three equations which follow.

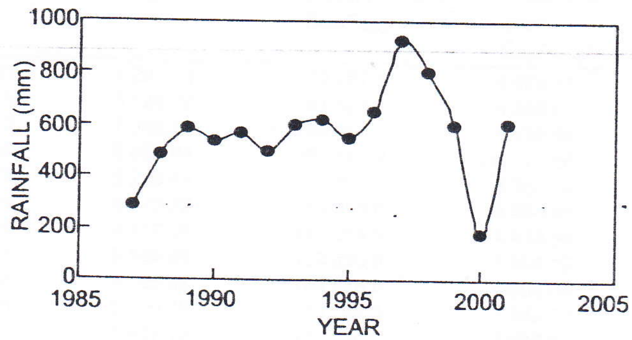

Figure 1: Malili Ranch annual rainfall distribution from 1987 to 2001

$$L_{Ca,t+1} - L_{Ca,t} + S_{Ca,t} = \Delta L_{Ca,t} + S_{Ca,t} = \beta_{Ca} L_{Ca,t} - \delta_{Ca} L^2{}_{Ca,t} + \lambda_{CaSh} L_{Ca,t} L_{Sh,t} + \lambda_{CaGo} L_{Ca} L_{Go}$$

$$L_{Sh,t+1} - L_{Sh,t} + S_{Sh,t} = \Delta L_{Sh,t} + S_{Sh,t} = \beta_{Sh} L_{Sh,t} - \delta_{Sh} L^2{}_{Sh,t} + \lambda_{ShCa} L_{Sh,t} L_{Ca,t} + \lambda_{ShGo} L_{Sh} L_{Go}$$

$$L_{Go,t+1} - L_{Go,t} + S_{Go,t} = \Delta L_{Go,t} + S_{Go,t} = \beta_{Go} L_{Go,t} - \delta_{Go} L^2{}_{Go,t} + \lambda_{GoCa} L_{Go,t} L_{Ca,t} + \lambda_{GoSh} L_{Go} L_{Sh}$$

It is notable that all the variables in the aforementioned econometric models are contemporaneous, although they are derived from time series data. These models can also be estimated using cross-sectional data.

After the estimation of the econometric models (Equation 6b), the logistic models (Equation 1) were recovered from the former (see box containing seven equations, which follows at the top of the next page).

Table 1: Malili Ranch data on cattle off-take rates and on variables included in Equation (6b), in AUs

Time in years (t)	Cattle sales ($S_{Ca,t}$)	Change in standing population ($\Delta L_{Ca,t}$)	Total cattle population change ($\Delta L_{Ca,t} + S_{Ca,t}$)	Standing population of cattle ($L_{Ca,t}$)	Intraspecies competition variable ($L^2{}_{Ca,t}$)	Interspecies interaction variable: cattle and sheep ($L_{Ca,t}{}^*L_{Sh,t}$)	Interspecies interaction variable: cattle and goats ($L_{Ca,t}{}^*L_{Go,t}$)
1987	504.24	154.99	659.23	2 017.51	4 070 334.50	132 207.4	151 777.3
1988	390.75	132.63	523.38	2 172.50	4 719 773.63	161 525.4	141 038.7
1989	524.42	91.45	615.87	2 305.13	5 313 638.15	196 212.7	175 996.7
1990	536.73	−17.30	519.43	2 396.58	5 743 595.70	198 892.2	140 655.3
1991	743.79	−142.43	601.36	2 379.28	5 660 982.84	171 903.0	144 232.0
1992	617.96	−183.64	434.32	2 236.85	5 003 493.45	160 919.0	173 154.6
1993	654.66	98.88	753.54	2 053.21	4 215 663.09	141 014.5	132 103.5
1994	565.57	−63.90	501.67	2 152.09	4 631 487.06	160 050.9	140 875.8
1995	666.44	−72.79	593.65	2 088.19	4 360 516.59	144 628.0	144 816.0
1996	523.72	39.58	563.30	2 215.51	4 908 502.28	173 164.3	225 140.1
1997	423.18	−1.35	421.83	2 255.09	5 085 448.95	170 056.3	227 200.3
1998	424.92	208.78	633.70	2 253.74	5 079 343.99	57 109.77	167 159.9
1999	322.29	−39.46	282.83	2 462.52	6 063 989.98	99 879.81	183 753.2
2000	453.81	−208.85	244.96	2 423.06	5 871 200.38	92 924.35	177 343.8
2001	589.45	131.68	721.13	2 214.20	4 902 690.50	81 216.86	175 121.1

Table 2: Malili Ranch sheep data, in AUs, for the variables included in Equation (6)

Time in years (t)	Sheep sales ($S_{Sh,t}$)	Change in standing population ($\Delta L_{Sh,t}$)	Total sheep population change ($\Delta L_{Sh,t} + S_{Sh,t}$)	Sheep standing population ($L_{Sh,t}$)	Intraspecies competition variable ($L^2{}_{Sh,t}$)	Interspecies interaction variable: sheep and cattle ($L_{Sh,t}{}^*L_{Ca,t}$)	Interspecies interaction variable: sheep and goats ($L_{Sh,t}{}^*L_{Go,t}$)
1987	29.47	8.83	38.30	65.53	4 293.53	132 207.4	4 929.71
1988	26.08	10.77	36.85	74.35	5 528.22	161 525.4	4 826.63
1989	25.64	−2.12	23.52	85.12	7 245.24	196 212.7	6 498.84
1990	25.22	−10.73	14.49	82.99	6 887.84	198 892.2	4 870.86
1991	25.51	−0.31	25.20	72.25	5 220.64	171 903.0	4 380.04
1992	19.21	−3.26	15.95	71.94	5 175.22	160 919.0	5 568.80
1993	21.34	5.68	27.02	68.68	4 717.08	141 014.5	4 418.94
1994	11.98	−5.11	6.87	74.37	5 530.60	160 050.9	4 868.13
1995	9.52	8.91	18.43	69.26	4 796.67	144 628.0	4 803.04
1996	13.50	−2.75	10.75	78.16	6 109.45	173 164.3	7 942.92
1997	7.66	−50.08	−42.42	75.41	5 687.12	170 056.3	7 597.86
1998	10.57	15.21	25.78	25.34	642.01	57 109.77	1 879.32
1999	9.21	−2.21	7.00	40.56	1 644.79	99 879.81	3 026.29
2000	4.85	−1.67	3.18	38.35	1 470.65	92 924.35	2 806.76
2001	8.20	0.00	8.20	36.68	1 345.64	81 216.86	2 901.26

$$L_{it+1} - L_{it} = \Delta L_{it} = \beta_i L_{it} - \delta_i L^2_{it} + \sum_{j=1}^{n-1} \lambda_{ij} L_{it} L_{jt} - S_{it}$$

$$= \beta_i L_{it}(1 - \frac{\delta_i}{\beta_i} L_{it} + \sum_{j=1}^{n-1} \frac{\lambda_{ij}}{\beta_i} L_{jt}) - S_{it}$$

$$= \beta_i L_{it}(1 - \alpha_i L_{it} + \sum_{j=1}^{n-1} \psi_{ij} L_{jt}) - S_{it}$$

$$= \beta_i L_{it}(1 - \alpha_i \{ L_{it} + \sum_{j=1}^{n-1} \frac{\psi_{ij}}{\alpha_i} L_{jt} \}) - S_{it}$$

$$= \beta_i L_{it}(1 - \alpha_i \{ L_{it} + \sum_{j=1}^{n-1} \sigma_{ij} L_{jt} \}) - S_{it}$$

$$= \beta_i L_{it}(1 - \frac{L_{it} + \sum_{j=1}^{n-1} \sigma_{ij} L_{jt}}{\frac{1}{\alpha_i}}) - S_{it}$$

$$= \beta_i L_{it}(1 - \frac{L_{it} + \sum_{j=1}^{n-1} \sigma_{ij} L_{jt}}{K_i}) - S_{it}$$

Under the steady state conditions, when S_{it} equals ΔL_{it} we for i and j = 1,...,n and j and i ≠ j[1]

The respective specific form of the logistic models for cattle, sheep and goats is as follows: (see equations at the base of this page).

Under the steady state condition, when S_{it} equals ΔL_{it} we have constant herds and flocks, and Equation (1) becomes Equation (7). Under this condition, the system is in equilibrium. However, when S_{it} is less than ΔL_{it}, the system is in disequilibrium, tending to a higher stocking level or higher standing livestock population. Similarly, when S_{it} is greater than ΔL_{it}, the system is again in disequilibrium, tending to lower stocking level or standing population.

Equation (7):

$$L_{it+1} - L_{it} = 0 = \beta_i L_{it}(1 - \frac{L_{it} + \sum_{j=1}^{n-1} \sigma_{ij} L_{jt}}{K_i}) - S_{it}$$

for i, j = 1,...,n, and i ≠j

Solving Equation (7) for S_{it} yields (Equation 8).

Equation (8):

$$S_{it} = \beta_i L_{it}(1 - \frac{L_{it} + \sum_{j=1}^{n-1} \sigma_{ij} L_{jt}}{K_i})$$

for i, j = 1,...,n, and i ≠j

In order to solve for maximum sustained off-take levels, Equation (8) was optimised for cattle, sheep and goats, and the three equations comprising the first-order conditions were solved simultaneously to yield the optimal livestock species mix and the optimal stocking levels. Substituting the 'optimal species mix' values into Equation (8) yielded the optimal off-take values.

Results and discussion

The results of cattle, sheep and goat model parameter estimates for the econometric models, Equation (6b), are presented in Table 4.

Based on the F-test, the whole of the cattle model is significant ($P < 0.05$). The cattle exponential growth rate parameter (β_{Ca}) is positive, as expected, and is also significant ($P < 0.05$). The cattle intraspecies parameter (δ_{Ca}) is negative, as expected, and is significant ($P < 0.05$). The cattle and sheep interspecies competition parameter (λ_{CaSh}) is positive, implying complementarity, but not significantly so ($P > 0.05$); from an ecological point of view, this implies that sheep do not have a major impact on the cattle population. The cattle and goat interspecies competition parameter (λ_{CaGo}) is negative, implying competition, but not significantly so ($P > 0.05$); from an ecological point of view, this implies that goats do not have a major impact on the cattle population.

Based on the F-test, the whole of the sheep model is significant ($P < 0.05$). The results of the sheep model show that the exponential growth rate parameter (β_{Sh}) is positive, as expected, and is also significant ($P < 0.05$). The intraspecies competition parameter (δ_{Sh}) is negative, as expected, but is not significantly so ($P > 0.05$); this implies that sheep intraspecies competition does not exert a strong influence on the sheep population. The sheep and cattle interspecies competition parameter (λ_{ShCa}) is negative, implying competition, but not significantly so ($P > 0.05$); from an ecological point of view, this implies that cattle do

$$L_{Ca,t+1} - L_{Ca,t} = \Delta L_{Ca,t} = \beta_{Ca} L_{Ca,t}(1 - \frac{L_{Ca,t} + \sigma_{CaSh} L_{Sh} + \sigma_{CaGo} L_{Go}}{K_{Ca}}) - S_{Ca,t}$$

$$L_{Sh,t+1} - L_{Sh,t} = \Delta L_{Sh,t} = \beta_{Sh} L_{Sh,t}(1 - \frac{L_{Sh,t} + \sigma_{ShCa} L_{Ca,t} + \sigma_{ShGo} L_{Go,t}}{K_{Sh}}) - S_{Sh,t}$$

and

$$L_{Go,t+1} - L_{Go,t} = \Delta L_{Go,t} = \beta_{Go} L_{Go,t}(1 - \frac{L_{Go,t} + \sigma_{GoCa} L_{Ca,t} + \sigma_{GoSh} L_{Sh,t}}{K_{Go}}) - S_{Go,t}$$

Table 3: Malili Ranch goat data, in AUs, for the variables shown in Equation (6b)

Time in years (t)	Goat sales ($S_{Go,t}$)	Change in standing population ($\Delta L_{Go,t}$)	Total goat population change ($\Delta L_{Go,t} + S_{Go,t}$)	Goat standing population n($L_{Go,t}$)	Intraspecies competition variable ($L^2_{Go,t}$)	Interspecies interaction variable: goats and cattle ($L_{Go,t}*L_{Ca,t}$)	Interspecies interaction variable: goats and sheep ($L_{Go,t}*L_{Sh,t}$)
1987	44.09	−10.31	33.77	75.23	5 660.15	151 785.12	4 929.71
1988	31.59	11.43	43.02	64.92	4 214.09	141 030.27	4 826.63
1989	18.60	−17.66	0.94	76.35	5 829.32	175 996.90	6 498.84
1990	30.97	1.93	32.90	58.69	3 444.52	140 655.28	4 870.86
1991	13.51	16.79	30.30	60.62	3 674.78	144 232.07	4 380.04
1992	11.64	−13.07	−1.43	77.41	5 992.31	173 154.48	5 568.80
1993	24.74	1.12	25.86	64.34	4 139.64	132 103.40	4 418.94
1994	16.23	3.89	20.12	65.46	4 285.01	140 875.75	4 868.13
1995	15.79	32.27	48.06	69.35	4 809.42	144 815.63	4 803.04
1996	10.47	−0.87	9.60	101.62	10 326.62	225 140.53	7 942.92
1997	14.48	−26.58	−12.10	100.75	10 150.56	227 200.72	7 597.86
1998	15.12	0.45	15.57	74.17	5 501.19	167 159.90	1 879.32
1999	15.31	−1.43	13.88	74.62	5 568.14	183 753.02	3 026.29
2000	10.85	5.90	16.75	73.19	5 356.78	177 343.47	2 806.76
2001	13.63	3.87	17.50	79.09	6 255.23	175 121.24	2 901.26

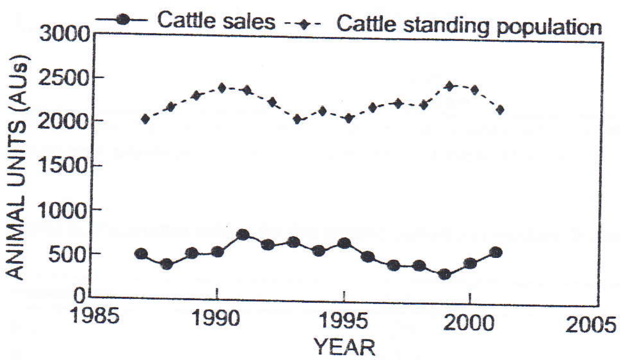

Figure 2: Cattle sales and standing populations over time

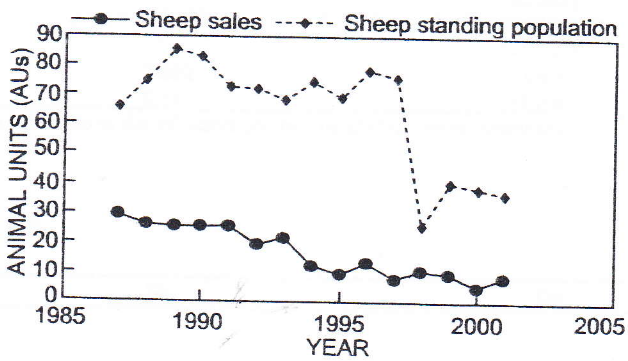

Figure 3: Sheep sales and standing populations over time

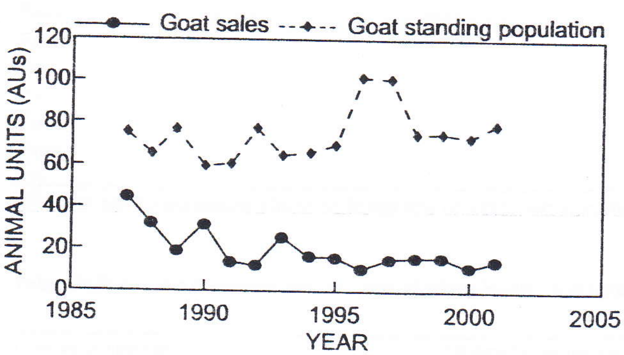

Figure 4: Goat sales and standing populations over time

not have a major impact on the sheep population. The sheep and goat interspecies competition parameter (λ_{ShGo}) is negative, implying competition, and is significant ($P < 0.05$); from an ecological point of view, this implies that goats are competitive to sheep.

Based on the F-test, the whole of the goat model is significant ($P < 0.05$). The goat exponential growth rate parameter (β_{Go}) is positive, as expected, and is also significant ($P < 0.05$). The goat intraspecies parameter (δ_{Go}) is negative, as expected, and is significant ($P < 0.05$). The goat and sheep interspecies competition parameter (λ_{GoSh}) is positive, implying complementarity, but not significantly so ($P > 0.05$); from an ecological point of view, this implies that sheep do not have a major impact on the goat population. The goat and cattle interspecies competition parameter (λ_{GoCa}) is negative, implying competition, and is significantly so ($P < 0.1$); from an ecological point of view, this implies that cattle have a major impact on the goat population.

The parameter values of Equation (1), after recovery from Equation (6b), are shown in Table 5. These results show the following: that sheep are complementary to cattle, while goats are competitive; that both cattle and goats are competitive to sheep; and that cattle are competitive to goats, while sheep are complementary. The results also show that the logistic model-generated carrying capacities are 2 985.07, 791.14 and 200.92 AUs for cattle, sheep and

Table 4: Parameter estimates for the econometric models, Equations (6b) (n = 15)[a]

Parameters	Ca	Sh	Go
β_{Ca}	1.283 (4.540)**		
δ_{Ca}	–0.000430 (–4.247)**		
λ_{CaSh}	0.00003377 (0.045)		
λ_{CaGo}	–0.00106 (–0.941)		
β_{Sh}		2.942 (2.325)*	
δ_{Sh}		–0.00372 (–0.687)	
λ_{ShCa}		–0.000699 (–1.343)	
λ_{ShGo}		–0.0121 (–2.664)*	
β_{Go}			2.632 (2.871)*
δ_{Go}			–0.00131 (–4.053)**
λ_{GoSh}			0.0002091 (0.083)
λ_{GoCa}			–0.000618 (–1.647)
R^2	0.967	0.650	0.815
F	79.513**	5.111*	12.090**

[a] Results are for equations identified in top row of table, with parameters indicated in the left-hand column; t-statistics are in parenthesis: * indicates significant at the 0.05 level and ** at the 0.01 level

Table 5: Parameter values for the logistic population models, Equations (1)[a]

Parameters	Ca	Sh	Go
β_{Ca}	1.283		
σ_{CaSh}	-0.0785		
σ_{CaGo}	2.464		
K_{Ca}	2 985.07		
β_{Sh}		2.942	
σ_{ShCa}		0.18857	
σ_{ShGo}		3.254	
K_{Sh}		791.14	
β_{Go}			2.632
σ_{GoCa}			0.04718
σ_{GoSh}			-0.01596
K_{Go}			200.92

[a] Results are for equations identified in top row of table, with parameters indicated in the left-hand column

Table 6: The optimal species mix (optimal stocking levels) and optimal off-take levels, in AUs

Livestock species	Optimal species mix (optimal stocking levels)	Optimal off-take levels
Cattle	1 368.79	856.94
Sheep	153.96	88.18
Goats	69.40	63.09

goats, respectively.

The results of the optimal species mix (optimal stocking levels) and optimal off-take levels are presented in Table 6. These results show that the whole ranch stocking level is 1 592.15 AUs, which is equivalent to 6.4ha per livestock unit. The results from this study shed some light on the grazing management. The objective of grazing management is to manipulate the grazing animals so as to realise highest animal production while maintaining or improving the range condition. This objective involves the determination of optimal stocking levels and livestock species mix. The current practice involves the use of 4ha per AU as the stocking levels that are arbitrarily determined (Pratt and Gwynne 1977). The results from the present study recommend a livestock stocking level of 6.4ha per AU, which is not only optimal but also empirically determined, based on the fixed carrying capacity logistic model. This logistic model has also enabled us to determine the optimal species mix: 1 369, 154 and 69 AUs of cattle, sheep and goats, respectively; based on the 15-year mean stocking level, the observed species mix in this ranch is 2 107, 64 and 74 of cattle, sheep and goats, respectively.

In conclusion, the results confirm that it is possible to derive the fixed logistic model-generated livestock carrying capacity, optimal species mix (optimal stocking levels) and sustained optimal off-take levels. This has important application in the management of both wildlife and livestock species. Further research in this area should investigate the economic aspects of livestock and wildlife management, based on systems analysis.

Notes

[1] The following is a logistic model, with rainfall-dependent carrying capacity, and its statistical estimation is also based on time series data:

for i, j = 1,...,n, and i ≠ j, where ϕ_i is the rainfall parameter for animal species *i*, and R_t is the rainfall in year *t*. Note that this model is non-linear in parameters.

$$L_{t+1} - L_t = \Delta L_t = \beta_i L_{it}\left(1 - \frac{L_{it} + \sum_{j=1}^{n-1} \sigma_{ij} L_{jt}}{\phi_i R_t}\right) - S_{it}$$

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