CONFLICT MODELLING AND RESOLUTION IN A DYNAMIC STATE

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DECLARATION AND APPROVAL

Declaration

This thesis is my original work and has not been presented in part or whole for a degree in this or any other university.

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DEDICATION

To my wife and parents

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I express my gratitude and many thanks to all those who have contributed towards the successful completion of this study. My sincere thanks to my supervisors Prof. C.B. Singh and Prof. M.M. Manene for their support and guidance. Many thanks to Prof. G.P. Pokhariyal for his encouragement and guidance. To the entire staff of the School of Mathematics, University of Nairobi I say thank you. Many thanks to Director School of Mathematics, Dr. Were for the support I received from him.

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ABSTRACT

A conflict is neither good (functional) nor bad (dysfunctional). The distinction depends on the type of conflict, one's attitude and reaction to it thereby making it constructive or destructive. The absence a clear measuring strategy or framework, against which it can be evaluated, makes it even harder to differentiate between good and bad conflict. It is however accepted that if the result of a conflict is positive, then the conflict is considered "good" and if the result is negative, then the conflict is "bad".

The formal models and quantitative analysis to explain how strategic actor's behaviour in a conflict setting are rare even-though model-based approaches are becoming more commonly used by statisticians and other scientists. These approaches to a great extent rely on fundamental or empirical models that are frequently described by systems of differential equations.

The underlying objective of this research was to develop conflict modelling and resolution models applicable to a dynamic state using ordinary differential equations (ODE) with integrated logistic model. Solutions to the ODEs were obtained by the application of Laplace transformation.

This research assumes that a conflict can be described by two main variables; control variables and state variables which reflect on the structural causes of a conflict. It is further assumed that a conflict can be described by a Bernoulli distribution with parameter y_i and that conflicts exist over a span of time with interplaying variables that can be dynamically modelled and the initial or

boundary conditions can be estimated in a dynamic state. In developing the models, the Game theory and Bayesian theorem are used as the underlying theoretical concepts. The Game theory and Bayesian theorem are used with the assumption that conflicts can be described using statistical distributions.

This research shows that modelling of a conflict requires accurate estimation of control variables (initial conditions) defined by a Bayesian probability distribution and the variables are independently and identically distributed (i.i.d). The developed model uses Baye's rule of probability distribution and the Game theory. In a dynamic state; the initial conditions are estimated as *posteriori* conditions by the model.

Using the developed model for the estimation of initial conditions, a logistic conflict prediction model that gives the trend a conflict is likely to take at time t_f has been developed. The model is derived from the solution of an exponential growth model and it integrates the initial conditions estimation model as one the parameters.

A statistical model for conflict resolution using the concept of Bargaining Game Theory has also been developed. The model assumes that in a conflict there are two parties with opposing opinions where one makes an offer with a probability of acceptance or rejection. The Ultimatum Game Theory has been used to introduce constraints on the offers made by the parties, consequently increasing the minimum threshold on the demands associated with any offers. It provides an in-built mechanism through which a conflict resolution model guides a negotiation by ensuring that any offer made is constrained with a higher likelihood of acceptance. The model compels the parties involved in a conflict to establish the demands from the other party and integrating them in any offer proposed hence boosting the chances of resolving a conflict.

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NOTATIONS AND ABBREVIATIONS

N: Total number of parties involved in a conflict

A_i: Strategy action profile

a_i: Action chosen

f_n: Real-valued tradeoff function

$P(x)$: Probability function

$E(x)$: Expectation of x

L_s: Laplace transformation

L_s^{-}: Inverse Laplace transformation

$y(t)$: Trend function

R_i: Public reservation/Information

ℓ_2: Private information

S_i: Strategy chosen by player i

L: Log likelihood

Q^*: Ultimate prize

i.i.d: Identically independent distribution

$f_{i(\cdot)}$: Density function

$F_i(\cdot)$: Cumulative distribution function

μ: Mean

σ_i: Variance

u_i: Utility function

ANC: African National Congress

CPA: Comprehensive Peace Agreement

HIIK: Heidelberg Institute for international Conflict Research

ODE: Ordinary differential equations

PBE: Perfect Bayesian equilibrium

PRIO: Peace Research Institute, Oslo

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Chapter one

INTRODUCTION

1.0.Background of conflict

Conflict is a general social form that isn't limited to just overtly violent situations, it is more than that; it doesn't necessarily tear the society apart. In fact, it might be one of the most important ways that society holds itself together and to a certain extent it forms a greater part of the long history in a society.

The close relationship that exists between society and conflict has been illustrated in a preface of the work of Karl Marx (1818-1883) by Ryanzanskaya (1971). Karl Marx (1818-1883) pointed to the fact that conflict is part of the society when he explains the concepts of class and the dialectics of economic capitalism. Karl Marx (1818-1883) further argued capitalism would produce its own grave-diggers by creating the conditions under which class consciousness and a failing economy would come into existence. At this point, the differentiated configuration of society and class-based grouping experiences will trigger the working class rebellion.

The Marxist approach to conflict emphasizes on a materialist interpretation of history which is a dialectical method of analysis and critical stance toward existing social stratifications with the objective of a political program of revolution or at least reform. Work is thus considered very central to the materialist view held by the Marxist since they presume that work is a key determinant of the social life the people live. Karl Marx believed that everything of substance in a society was a creation of human labour. According to him working men and women are angled in creating the conditions for their existence. On the other hand, Weber (1947) notes that conflict conditions do not entirely depend on the economy as thought by Karl Marx (1818-1883), but on the economy and the state. Underlying Weber's thoughts was legitimization. Weber (1947) argues that without legitimization, a conflict was bound to occur. The complexity of class as seen by Weber (1947) in contrast to Karl Marx (1818-1883) presented other inherent factors that contributed to social inequalities which manifested themselves in status and social gathering.

Sorokin (1957) views conflict as a manifestation of rapid transition between different systems of organized relationships and as such, conflict and violence appear to be "permanently working forces, inherently connected with the essence of social life itself, which do not permit either a complete elimination or the unlimited growth of disturbances".

Underlying the understanding in the various views held by various scholars about conflict is that conflict, society and people are intertwined since conflict shapes the life of people in a society.

1.1. Conflict definition

Theoretical approaches to understanding and defining conflicts are as diverse as the fields themselves. For instance, Economists approach conflict from the game-theory and decision-making, Psychologists explore interpersonal conflicts, and Sociologists take status and class conflicts as the focal point, while Political-scientists focus on intra-national and international conflicts.

According to Singer and Small (1972), conflicts are defined as violent disputes in which at least one of the combatant parties is a state, and there are at least 100 battle-deaths. In this context only combatants are covered excluding the civilian victims.

Dwan and Holmqvist (2005) indicates that a major armed conflict is the use of armed force between the military forces of two or more governments, or of one government and at least one organized armed group, resulting in the battle-related deaths of at least 1000 people in any single calendar year and in which the incompatibility concerns control of government or territory.

Heidelberg Institute for International Conflict Research (HIIK 2005) offers a broader definition of a conflict when they describe conflicts as the diametrically opposed interests on national values of some duration and magnitude between at least two parties that are determined to pursue their interests and win their cases.

1.1.Conflict Theory

Conflict theory tries to scientifically explain how conflict manifests itself in a society in terms of how it starts, progresses and the effects that accompany it. The central concerns of conflict theory are the unequal distribution of scarce resources and power. The conceptualization of what these resources are might be different for each theorist, but most conflict theorists follow Weber's three systems of classification and subdivision, that is; class, status, and power. Of the three systems, power is seen as the central nerve in a society. This is in contrast with the notion held by functionalists who see a society as a system defined by a cohesive set of norms.

1.1.1. Variation in Perception of Conflict Theory

Conflict is perceived by Coser(1956) and Dahrendorf(1959) to be part of human life and affects every facet of human life and society. On his thoughts Dahrendorf(1959), argues that conflict is a normal part of how we structure society and create social order. In this way, he argues that it is power that defines and enforces the guiding principles of the society and that power is the only primary resource in a society. In his thinking conflict can be experienced with different intensities. Coser also notes that in daily lives and relationships there is conflict which might not necessarily involve war. Further, he argues that conflict in humans is markedly different from that for other animals in the sense that human conflicts are goal driven. The existence of a goal triggers scramble from parties to achieve it through different ways. The fact that these ways are different leads to possibilities of different strategies of achieving the goals which in turn opens opportunities for negotiation, different types and levels of conflict and functional consequences. This analogy is what we aim to exploit in this study.

1.3. Basic Sources and Issues of Conflict

From the definition of conflict theory, most social conflicts are anchored on the unequal distribution of scarce resources. These resources as identified by Weber (1947) include class, status, and power. For instance, working class persons in a given sector may not share the same social interests as a working class person in a distinctively different sector. The different status associated with different positions of these two people may significantly determine the class interests. In this case the main source of conflict is the covariance of the systems of stratification. On one hand the general populace may perceive that the same group in a given class controls access to all three resources and this will skew the perception of the legitimacy of the system in promoting social mobility.

According to Karl Marx (1818-1883), a group's sense of deprivation caused by class could lead to class consciousness and produces conflict or social change. In his thought, Karl Marx (1818-1883) was concerned with the structural changes or processes that would bring the working class to this realization.

Modern conflict theory looks at deprivation as a shift from absolute to relative deprivation that is significant in producing consciousness. A condition of absolute deprivation is where life is influenced by uncertainty over basic human needs. People in such a condition have neither the resources nor the strength of will to become involved in conflict and social change. In contrast relative deprivation is a condition where one feels underprivileged relative to some other class of persons. In this condition there is a general feeling that others are doing well at our expense. Persons in relative deprivation have the emotional and material powers to get involved in demanding for a social change and conflict. The management of the transition from absolute to relative deprivation is also important to stem out the "delayed" sense of loss particularly if the change does not measure with the uncontrolled expectation where economic structural changes are involved.

Further, parties to a conflict will always have incompatible goals. These incompatible goals can be reduced into three main sociological goals wealth, power and prestige, Weber (1947). The identification of what the issues and causes are in a political conflict will facilitate the achievement of conflict resolution. Deutsch (1973) has identified five basic issues over which a conflict could arise. They include control over resources, inclinations, beliefs, values, or the type of the association. On the other hand, Singer (1996) identifies the conflict causes and issues as territory, ideology, dynastic legitimacy, religion, language, ethnicity, self-determination, resources, markets, dominance, equality, and revenge.

On their part Pfetsch and Rohloff (2000) have identified conflict issues between the states to be territory, secession, decolonization, autonomy, system of governance, national power and international power, regional predominance, resources and other salient issues. In this study, the conflict issues have been considered as state variables and control variables.

1.4. Conflict Categories

Categorization of conflict revolves around two conflict approaches: the subjectivist and the objectivist approach. Objectivist approach looks for the origin of conflict in the social-political make-up and structure of society, and considers that the goals at stake can be thoroughly compatible. On the contrary, the subjectivist point of view focuses primarily on the perceived incompatibility of goals and differences.

According to Deutsch (1991), it is incompatible differences which give rise to conflict. He further asserts that it is not the objective incompatibility that is crucial but rather the perceived incompatibility. Incompatibility of goals and interests or at least their perception as incompatible by the parties in dispute, is as well the essence of the political conflicts analysis.

The level of incompatibility is the most important variable that impacts the intensity of the dispute and the dynamics of conflicts. When conflict appears, it develops further with certain dynamics and changes its course and state as its intensity changes. Modelling conflicts in different phases has been given by Messmer's (2003). Diez, Stetter and Albert (2004), have also considered incompatibilities between parties to a conflict and have established four levels of conflict typology, and distinguish between them as:

i. Conflict episodes: which refers to isolated incompatibility articulation related to a particular issue;
ii. Issue conflicts: which refers to persistent incompatibility over a contested issue;
iii. Identity conflicts: which refers to explicit disaccord and the moves of the other side are interpreted on the basis of hostile motives;
iv. Power conflict: which implies that the communication of disaccord is no longer marked by demarcation from the 'other', but subordination, and possibly extinction of the 'other'.

On his part, Pfetsch (1994) categorizes conflict into latent conflict, manifested conflict, crisis, severe crisis, and war by using the compatibility criterion. The five stages are further classified as nonviolent conflicts (latent conflict, manifested conflict) and violent conflicts (crisis, severe crisis and war).

1.4.1. Non-Violent Conflict

This is a condition where conflicting interests can be pursued without aggression or duress. If a conflict already exists it then means only non-violent methods are employed by parties in their struggle to resolve their incompatible differences over issues. This implies that parties do not use force against each other.

Sando'e (1998) describes non-violent conflict as a situation in which at least two parties, or their representatives, pursue their perceived incompatible goals by deflating each other's capability directly or indirectly.

A conflict cannot be detected without existence of some visible signs that show positional differences or interest oppositions between two parties over a certain demand. Sometimes conditions for conflict exist, but the parties are not pursuing an open strategy to achieve their goals. In this case however, at least one party has to have positional differences articulated in some form of demands, and the other party shall be aware of such demands. This presupposition of conflict has been given by Omwenga and Mwita (2010).

Considering the expressible demands, latent conflict can be defined as a stage in the development of a conflict where one or more parties question existing values, issues or objectives that have a public relevance. Latent conflicts therefore, must carry some identifiable and observable signs in order to be recognized and noticed as such and expressed as demands. On the other hand, the manifest conflict is defined as a phase when apprehensions are present but are articulated by ways below the threshold of violence. Diez, et al (2004), have given conditions under which a latent conflict can be turned into a manifest conflict.

1.4.1.1. Examples of Non-Violent Conflicts

Gandhi was the greatest exponent of the doctrine of non-violence in modern times. Gandhi's ideas advocated for the transformation of individual ethics into a tool of social and political action. Since 1894 he consistently pleaded with the colonial regime for the removal of iniquitous curbs and disabilities from which Indian immigrants in Natal and Transvaal suffered. In 1906 a stage was reached in Gandhi's agitation when a new technique of fighting social and political injustice was introduced. This technique was called *satyagraha* (holding on to truth). Gandhi believed that this was the method without hatred and violence. By 1914, he had succeeded in reaching an agreement with the South African government on the *satyagraha* approach in addressing issues that affected the Indian immigrants. Gandhi's believe in non-violence as a means of solving conflict is best illustrated in his remarks when in 1931 he said "I will not purchase my country's freedom at the cost of non-violence". Gandhi is remembered for applying his methods of non-violent resistance not only against foreign rule, but also against social evils. In his early life in the hostile environment of South Africa, he discovered that in an imperfect and changing world, conflicts of interests within and between countries were inevitable. His technique of *satyagraha* sought reconciliation through dialogue and compromise, but if justice was denied, it provided for confrontation, but it had to be a non-violent confrontation. His technique has been described as one of 'achieving social and political reform by means of tolerance and active goodwill coupled with firmness in one's cause expressed through non-violence, passive resistance and non-cooperation.

Gandhi's methods have been applied in Asia, Africa, America, and Europe. In South Africa for instance, the African National Congress (ANC) carried on non-violent agitation and passive resistance. Chief Albert Luthuli and Nelson Mandela in their earlier struggle to liberate their people advocated for violence, but they later changed to Gandhi's methods after realising they were not achieving much. After the massacre of Sharpeville and until the release of Nelson Mandela, the major liberation movement in South Africa took to guerrilla warfare. However, the armed struggle would have been much more difficult and prolonged had not students, industrial workers, religious leaders, youth and women's organisations joined in non-violent resistance to the racist regime on such issues as rent, consumer embargoes and bus boycotts. Thus, the liberators of the blacks in South Africa were not only the guerrilla fighters, but hundreds of thousands of men, women and children, shop assistants, and workers living in shanty towns who consciously or unconsciously adopted methods that Gandhi would have approved. Martin Luther King Jr., a Baptist minister, was inspired by Gandhi's teachings and was able, in the words of an American writer, to 'meld the image of Gandhi and the image of the Negro preacher, and to use biblical symbols that bypassed cerebral centres and exploded in the well of the Negro psyche'. Martin Luther used and advocated for the non-violent method as a practical alternative not only to armed conflicts within a country but between countries. A quote from his writing in his *Stride Towards Freedom* (1958) 'is no longer between violence and non-violence. It is either non-violence or non-existence' illustrates his advocacy for non-violent means of achieving social justices and equity.

It is believed that the purpose of non-violent action is to withdraw consent from government or other authorities, rather than wrest power from them. Therefore it fosters dialogue and education which in turn gives room for maximum participation by everyone in society. Non-violence raises the bar for moral superiority of the actionists in the eyes of the general public--especially if the authorities respond to their sincere and open protest with violence.

Non-violent conflicts are based on trust that humans who share the consciousness that violence is illegitimate and are taught from childhood the many subtle and creative ways of attaining their goals without using violence, will rarely resort to it. To connect two popular sayings, if "violence begets violence" then "the only way to peace, is peace itself!" Otherwise we become willing co-creators of our violence-wrecked system. As Gandhi said, "Every citizen silently, but never the less certainly, sustains the government of the day in ways of which he has no knowledge. Every citizen, therefore, renders himself responsible for every act of his government. Violence breeds violence...Pure goals can never justify impure or violent action...They say the means are after all just means. I would say means are after all everything. As the means, so the end....If we take care of the means we are bound to reach the end sooner or later".

1.4.2. Violent Conflicts

Conflicts enter a violent phase when parties go beyond seeking to attain their goals amicably. They try to control and destroy the opposing parties' capability to pursue their demands or goals.

Davis (1973) notes that the existence of deep frustration of substantive needs is the essential condition for one non-violent conflict to escalate into violent. On their part, political conflict analysts indicate that violet conflicts are reflected by force, physical damages and human casualties.

Violent conflicts can also be associated with nature and behavior of human beings. Based on this interpretation, Waltz (1954) considers other causes of conflict to be as a consequence of the two factors. Other theorist view violent conflict from the structural point of view. They argue that conflicts are as a result of environment in which the actors to a conflict owe their existence. This represents the manner in which the society is organized and how power is shared and exercised.

Understanding the causes of the violent conflicts will underpin the interventions to be employed in solving them. For instance, if the causes are structural in nature then the interventions should be those that target the structural institutions and capacity. On the other hand if the causes are human-nature the response to the conflict should target the changing of the human nature.

As noted in section 1.4.1, a latent conflict is the likely situation that most states find themselves in. For instance, Kenya had been experiencing latent conflict just up-to the post election violence than engulfed the country in January 2008 after the 2007 general election. This kind of conflict was identifiable considering the sporadic cases of internal conflicts and violence witnessed in the country in the years 1992 and 1997, and other sporadic conflict episodes experienced in some parts of the country in the run-up to the general election.

Violent conflicts can be internal conflicts, that is, those that involve domestic parties competing or conflicting over an internal/domestic issues (for instance, conflict in Somalia and the conflict in Sudan) or external conflicts involving non-domestic parties (for example, Somalia-Ethiopia conflict). The external conflict may also be described as intra-state conflicts. In both categories there is the tendency of the conflicts taking a regional or even an international perspective depending on the interests involved as many other states join either party to that conflict.

Violent conflicts are costly in all aspects and they immensely destroy harmony and properties where they occur. The Rwanda conflict of the period 1991-1995 is perfect example of the massive destructive nature of violent conflicts. The same scenario is replicated in the Sudanese conflict that lasted over two decades until it was settled with the signing of a comprehensive peace agreement (CPA) in 2005 in Naivasha, Kenya.

The structural causes tend to be the major contributor of violent conflict in Africa and elsewhere. The Somalia conflict that has lasted for two decades and claimed many life and destroyed livelihood, is an example of a conflict caused by structural issues. The structural issues may manifest themselves in social, political and socio-economic structures that are too weak which might be perceived to be not inclusive enough and in unequal resource distribution, Seligson and Muller (1987).

In the case of resources for instance, the resource distribution might imply scarcity or abundance. If the resource is scarce there is scramble for that resource and if there is any perceived inequality in sharing it, a violent conflict is likely to occur. However, in the case of abundant resources, the human nature sets in with greedy members of society becoming more aggressive to control the resource. A violent conflict will occur when the aggressive and greedy members of the society belief that the opportunity cost in financing these conflicts are less than benefits and control they will get from the resources, Collier and Hoeffler (1998), Collier(2007). In most cases they are convinced that by fuelling despondency, the status quo will remain and will have constant supply of armed groups to their camp. The easy at which resources are looted and blockage of others from accessing them makes it difficult and prolongs the period taken to resolve most resource-based conflicts, Ross (2003, 2004).

In a political power perspective, violent conflicts can be perpetuated by actors who will manipulate the conflict issues to reflect on the external angle. It is through this that communities are galvanized against each other and self political-egos created. This case scenario is common and grave when a society is made to belief that it is through their political chiefs that a Zero-sum-game can be turned to be a non-zero-sum-game. This was noted widely to be one of the possible causes of post-election violent conflicts in Kenya.

Some violent conflicts like in Zimbabwe are derived from resource distributions. Land as a resource in Zimbabwe has been of great controversy and interest. The land redistribution programs adopted by the Government of Zimbabwe have lead to violent unrests and an almost total collapse of the economy.

1.5. Existing Conflict Resolution Mechanisms

The conflict resolution mechanisms aim at using strategies that could facilitate the exit from the destructive nature of a conflict and bringing parties in a conflict towards a peaceful settlement. Burton (1968), Fisher and Keashly(1991), Fisher (2007) and Kriesberg (1998), have given various approaches and strategies that can be used in conflict settlement. Burton (1968), fronts the idea of considering human-needs in conflict resolution in his human-needs theory. He views that an existence of a conflict is an indication of unsolved human-needs through security, justice and recognition as the fundamental values. These values cannot be suppressed if meaningful conflict resolution is to be achieved.

Methods and procedures like workshops, discussion groups, or round tables mediation, negotiations, or arbitration have been identified by Burton to convert the respective conflict into a situation acceptable for both sides. These approaches are aimed at improving communication between the conflict parties and to develop a mutual understanding so as to boost the interests of each party. In this study, a conflict resolution mechanism revolving around negotiation has been developed by considering the Ultimatum Bargaining Game Theory as discussed in chapter 5.

1.6. Perception and Interpretation of Conflicts

There has been a debate about whether conflict is a positive or negative social occurrence. According to Bailey (1997) functionalists perceive conflict as a hindrance, dysfunctional and generally bad. On his part, Kriesberg (1998) notes that conflict represent a disruption of dependable and stable conditions. These arguments calls for avoidance, repression or elimination approaches to conflict. Contrary to the perception of functionalist, Castro and Nielson (2001) argue that conflict can be constructive. This position is held by many and conflict is interpreted to promote cohesiveness if handled positively.

Foregoing the different perception of conflicts, it is now generally accepted that conflict has both positive and negative potential Kriesberg (1998). Due to this understanding, the success of conflict management is dependent on how to deal with conflict constructively so as to overcome the negative outcomes that many result from a conflict. To achieve that, establishing and understanding the trigger factors (initial conditions) that lead to a conflict is critical.

1.7. Mathematical Modelling of Conflict

In the recent past, formal models and quantitative analysis have been used in explaining how strategic actors bargain in a variety of conflict settings. In any conflict be it in the political setting or international relations bargaining play a central role in understanding and solving many conflicts; Banks (1990), Bennett (1996), Huth and Todd (2002), London (2002), Powell (1987), Powell (1996). To understanding the basics of logic of bargaining in the face of conflicting interests, game theory has played a key role in the study and modelling of conflicts. Hargreaves and Yanis (1995) describe game theory as being underpinned by three key assumptions about the parties in conflict. These assumptions are that the parties are: Instrumentally rational, Know this, and Know the rules.

In order to model a conflict, a game theorist need to:

i. Focus modelling on rules and utilities deduced from *a-priori* knowledge of the conflict,
ii. Use knowledge of game theory, or
iii. Combine modelling and knowledge of game theory.

According to Dixit and Skeath, (1999); and Hargreaves et al, (1995), Binmore (1990) game theory provides a way of modelling a conflict through predictions in the chapter authored by Johnson and Scholes pg 354. Political scientist have employed for instance, bargaining models to analyze effects of open and closed rules on the distributive politics of legislative appropriation to the study of war initiation and termination Baron and Ferejohn (1989), Mansfield, Helen and Rosenderff (2000).

Most conflicts in any setting are mostly triggered by the differences in opinions and interpretation of an idea. It is therefore imperative that these differences are understood in terms of their magnitude in a conflict and taken care of before any bargaining can commence. This will give each opinion an unbiased attention since they have been appropriately assigned attention based on their contribution in a conflict. Further, theoretical studies of the bargaining problem have pointed to the importance of asymmetric information and the "reservation values" of players in distributional politics. In general, it is important to understand the effects of substantive variables on the bargaining process. The theoretical models tell us something about the path by which these variables may influence outcomes.

To fully understand the bargaining problem, research has been conducted on the empirical relationship between substantive variables of interest, such as regime type, economic interdependence, institutional rules, legislative composition, and bargaining outcomes McCarty and Poole (1995), Werner (1999). However, lacking is an explicit model of the process that generates the empirical data, and leaving out the choice-based path by which these variables influence decisions. It is often the case that selection and omitted variable bias plague the analysis, King, Gary, Robert and Sidney (1994). In particular, Signorino (2002) demonstrates that traditional linear and categorical estimation techniques can lead to faulty inferences when the strategic data generating process is ignored during estimation, Signorino (1999, 2002).

For effective bargaining, there is need to integrate theoretical models and statistical methods, Signorino and Kristopher (2006). A statistical tool that support theoretical consistency, inferences about the relationship between substantive variables, the bargain struck, and the probability of bargaining failure is needed.

In this study a statistical conflict resolution model that uses Ultimatum Bargaining Games Theory has been developed. The developed model integrates systems in a dynamic state which address the substantive variables, the bargain struck and the probability of bargaining failure or success. The model presents the relationship between the variables that affect the parties' utilities and the outcomes of the bargaining in a strategic setting.

1.8. Conflict Modelling in a Dynamic State

Describing and modelling conflict in a dynamic state requires complex multi-players using dynamic games unlike the simple two-party arms race used during the cold war, Rapoport (1960). This is due to the interconnectivities that exist today save for the technological and socio-economic advancement.

Mathematical modelling has contributed to unfathomable understanding and the design of new approaches and tools for conflict prediction and resolution. Due to the changing nature of the society, there is a need to have dynamical systems or instruments that can respond to the changes that characterize a conflict situation. The theory of dynamic systems that determines equilibrium and stability of differential equations in a dynamic society have been developed. The optimality path in dynamic systems which is important in modelling conflict in a dynamic state can be established using control theory discussed in section 4.2.

In a conflict, various decisions by players are to be made and the rationale on the choices to be made among different options available to them is important. Game theory has been argued to be the most rational approach to this kind of situation von Neumann and Morgenstern (1944). The game theory needs also to be responsive to the dynamisms of the society and thus a new approach called dynamic games has been developed to deal with dynamic conflict situations.

Other approaches like differential games, cooperative dynamic games, coalition formation and repeated prisoners' dilemma have also been used to manage conflicts Dockner, Jorgensen, van Long and Sorger (2000) and Axelrod (1997). Conflicts have also been modelled using nonlinear dynamical systems commonly used in other science fields like physics Weidlich (2000).The applications of dynamic-game models involve a number of systems in a conflict environment, Olsder (1995).

In-spite of the existence of these models there is still a lot to be done in the optimization of the various game strategies to fully satisfy the stochastic behaviors of a conflict environment. The establishment of initial conditions using a mathematical model under the dynamic system provides a stabilization mechanism of the control variables and hence the optimization of the various strategies. In this study, we have developed a model for the estimation of the initial conditions (control variables) presented in chapter 3.

The ultimatum game theory has been used to provide a mean of establishing the equilibrium levels for a conflict in a dynamic situation. This opens another front for decision making in a conflict environment with extra constraint on the demands made by the parties to the conflict. In chapter five, we have used ultimatum game theory to develop a model for conflict resolution with a constraining factor on the demands made by the parties.

1.9. Challenges of Variables Identification in a Conflict Environment

Most conflict models like Richardson's model, makes basic structural assumption. Hess (1995) notes that Lewis F. Richardson (1881–1953), applied mathematics to understand arms race and war. His contributions to weather forecasting however failed due to the long delay time it took to give predictive values. Richardson used a set of differential equations to describe the accumulation of arms by relying on empirical data from the First World War.

These models allude that societies are structureless entities and are expressible by a single variable. Moreover, this variable might not expressly relate to a conflict. For instance, military expenditures are used as an indicator for military threat. This is erroneous in the present technological world and the expenditures information might not be available, verifiable and reliable.

The various variables in a conflict sometimes are difficult to express quantitatively. This to a large extent leads to abstraction of variables so as to capture them quantitatively. For instance, Richardson's model abstracts "politics without personalities", with state authorities as black boxes and decisions as hidden in the budget.

Further, in any conflict there are two or more parties; understanding the interplaying variables from either side is important but in most cases this is not possible since some of the variables are "private information" factors. This limits the available information on the parties to a conflict and might erroneously presuppose existence of a variable.

The dynamic nature of conflict environment and changing parties to a conflict over time can significantly influence the variables during a conflict modelling. These challenges have been addressed by this study through a model for the estimation of control variables dynamically as presented in chapter 3.

1.10. Problem Description

Our concern is to model and monitor how a conflict system evolves through time $[t_0 ; T]$. It is important to note that at any moment the system is in some state, that is, some configuration of relationships exist between the elements ϕ, θ and t of the system. The main problem is to control the system as it moves through time using ODEs that can be solved using Laplace transformations. Controlling the system can be achieved through the manipulation of the control variable θ(t), so as to make the system respond in a particular way. Control problems can be described as the determination of an optimal control input variable, θ^*(t), and the synthesis of associated elements which generate ϕ(t), such that a desired dynamic response is achieved.

To address a conflict system in a dynamic state, we have developed models for the estimation of control variables, prediction and conflict resolution using Bayesian theory and Ultimatum Game Theory. The models developed are logistic models.

1.11. Importance of Mathematical Models

Mathematical models are more expressible than verbally expressed statements since the analytical approaches associated with mathematical modelling calls for a careful and keen scrutiny of the ideas or opinions. Moreover, in oral statements implied meanings may be unnoticed but in mathematical formulae there are deductions made which make it possible to notice such implications. The brevity of mathematical models makes it easier to memorize and express an idea. The brevity also may lead to speedy conclusion of a controversial issue so that truth can be found.

Mathematical models can however prevent thoughtfulness since some are based on assumptions and strict definitions.

1.12. Basic Assumptions for the Research

The study was conducted under the following assumptions;

1. Conflicts exist over a span of time with interplaying variables that can be dynamically modelled.
2. Initial or boundary conditions can be established or estimated from a dynamic state.
3. Transformative approach to conflict and negotiation which emphasizes on deliberative processes as processes of learning and understanding will

inform the modelling process. Thus, negotiation will be seen as the process of producing fundamental change in a dispute. The change may be in the way the parties understand themselves, their conflict, their relationship, or their situation. Change can occur at the level of issues, actors, rules, structure or context.

4. Individuals to a conflict can be modelled as individual entities or as a group.

Chapter Two

LITERATURE REVIEW

2.0. Introduction to Game Theory

Game theory can be described as the study of mathematical models of conflict and cooperation between decision makers in tradeoffs. In a conflict setting the tradeoffs are seen as the demands made by the parties to a conflict. A game in this case describes any social situation that involves two or more individuals who are called players. In a conflict these players are called parties to a conflict. It is assumed that the rules governing the interaction between the parties are well defined and their actions or strategies must lead to some outcomes. It is also assumed that the parties in a game are rational.

The motivation behind game theory is based on the wise saying regarding the social effort which says that "the purpose for any game is the greatest possible good for the greatest possible number". It is however important to note that what is good for one, might turn to be bad for another and that the existence of a single optimization function for "goodness" is practically possible, but there may exist many individual and conflicting functions. Game theory therefore defines a method for individuals to obtain a desired outcome when intelligent opponents seek the same for themselves.

A game in the context of this study is a conflict involving two or more disputing factions. This game need not be necessarily a violent conflict. For instance, Berne (1964) presents another kind of game, that is, the psychological games that are played on a personal level, where the weapons are words, and the tradeoffs are good or bad feelings. In the recent past, game theorists have tried to understand conflicts by studying quantitative models. The close connection between theoretical statistics and game theory where nature takes the role of one player has been given by Ferguson (1968).

Games are generally characterized by a number of players whose interaction take certain actions under uncertain conditions, and finally receive some positive or negative benefits. They can also be described as a formal abstraction of a social interaction where:

i. there are two or more decision makers, called players,
ii. each player has a choice of two or more ways of acting, called actions or (pure) strategies or utilities,
iii. the outcome of the interaction depends on the strategic choices of all the players.

Drawing from social sciences and economics, Borel (1921), Von Neumann (1928) and von Neumann and Morgenstern (1944) developed a mathematical model for decision making called game theory. Nash (1950) introduced the concept of equilibrium as a means of controlling the utilities employed by each player in the game model. His contribution was aimed at modifying the utilities in the model. Through the concept of Nash's equilibrium, it was expected that each player's strategies will minimize his expected utility tradeoffs against the strategies of the other player. Following Nash's equilibrium concept, in 5.1.1 a model that minimizes the demands made by a party to a conflict while maximizing the chances of success to a conflict resolution is developed.

Selten (1975) demonstrated that for many games, Nash equilibrium can at times generate many equilibria, some may be implausible and extraordinary when extensively examined. Many improvements to Nash equilibrium were suggested by Harsanyi (1968) who introduced uncertainty modelling into game theory using Bayesian game models. The introduction of uncertainty modelling into Nash equilibrium extended the concept of game theory into incomplete information cases and into other economic and social sciences, Myerson (1999).

In other fields like economics, game theory has been used as a predictive tool to forecast on what is likely to happen using theoretical framework. It has enabled economists to transfer their interpretation abstracted insights on an economic concept from one context to another.

2.1. Further Developments in Game Theory

The restrictive assumption that the players in a game are aware of the tradeoff is very limiting. For instance, not all parties to a conflict are aware of why they are making certain demands and might not know the reservations from the other parties. Relaxing this assumption provides a shift in the strategy employed in the game as described in section 2.7.

Selten (1975) proposes a means of relaxing this strategies and he called it the "trembling hand perfection". The basic assumption was that with a player trembling, there will be some perturbation on the game and hence relaxing the restrictive assumption through a shift on the limit of the equilibrium when the mistake probability tends to zero.

Fudenberg and Levine (1988) provide a means of dealing with different equilibrium so as to achieve relaxation on the restrictive assumption on the tradeoffs. They propose an iterated deletion of weakly dominated strategies by considering equilibrium in games whose tradeoffs differ from those of the original game by a small amount. Their ideas were extended by Dekel and Fudenberg (1990) to other forms of games, where they found that iterated elimination of weakly dominated strategies will not be followed by rational players, where rationality is dependent on the tradeoff uncertainties in the game.

Other form of games employing the behavioral game theory that model the cognitive process and psychological aspects of the player's reasoning ability have been developed. The modelling of these aspects requires additional tradeoffs which depend on the chosen strategy. Guth (1995) has demonstrated how a behavioral theory that uses dynamic reasoning process in the ultimatum bargaining games work. Slonim and Roth (1998) have investigated how ultimatum games can be used to explain the interaction between rejection frequencies and financial stakes.

A statistical model for quantification of choice in a theoretical game setting incorporating additional tradeoffs has been given by McKelvey and Palfrey (1995). The players in this setting choose strategies based on relative expected utility and assume other players do the same. Costa-Gomez, et al (1999) argue that the success of the strategic behavior is dependent on the extent to which it reflects the players' analyses of their environment as a game, taking its structure and other players' incentives into account. In this study, extension to this concept to include conditional probabilities on the strategic utilities on the choices has been considered. This idea is the main difference between the behavioral assumptions of traditional non-cooperative and cooperative game theory, which are unlimited and evolutionary game theory or adaptive learning models which are severely limited.

Due to the emergence of a networked society, the presence of agents through social and economic networks is evident in recent interactions. The agents in a network may introduce external variables into a system if the agents are not localized. Johnson and Gilles (2000) give a discussion on social distances and their consequences in a network formation from an economical perspective. Jackson and Wolinsky (1996) have used equilibrium concept called pair-wise stability which requires agents to establish a relationship among agents themselves. It is anticipated that the cumulative effect of agents in a tradeoff will be shifted against the stability and efficiency of the network. In their arguments stability is derived from the equilibrium concept, while efficiency tries to maximize the benefits of the linkages established.

Belief is another concept that has a vital role in a game. Mertens and Zamir (1985) have discussed belief as a foundation for the Bayesian formulation. Extension of this concept has been made by Aumann and Brandenburger (1995) to provide foundations associated with the Nash equilibrium. Their concept assumes that any two players' beliefs about a third player's strategy must be the same and the common beliefs viewed as mixed strategies must be the same in equilibrium. At this equilibrium on beliefs, a player's mixed strategy represents other players' beliefs about his realized pure strategy about which he himself need not be uncertain, and players' beliefs determine their optimal strategies and expected tradeoffs. In a conflict setting this mixed strategy will constitute the overall demand from groups of parties to a conflict such that if there are any fragmented small parties to a conflict they can be merged. This is quite important in reducing the conflict space.

2.2. Rational Choice and Exchange Theory in Conflicts

Rational choice and exchange theory has gained popularity in recent years. Sociologists in explaining this theory, borrow heavily on the work of economists and political scientists in their analyses of the ways that economic incentives and other material considerations affect the choices people make. Early work done on this theory was referred to as Exchange theory and has been demonstrated by Coleman (1990) in his book, *Foundations of Social Theory*. Sociologists applying this theory aim at probing the limits of rationality and on developing mathematical models of conditions needed for maintaining trust and solidarity among the various members of the society. It is through trust and solidarity that parties in a conflict will forge forward in the determination and settlement of a conflict. The choices made by the parties are derived from the gains they aim at achieving and the rationality inherent to these choices.

2.3. Game Trees

Reasoning about games of any complexity is difficult but a game tree which is a visual abstraction that relates all elements of a game makes the analysis process easy. An example of a game tree is given in the figure 1 below.

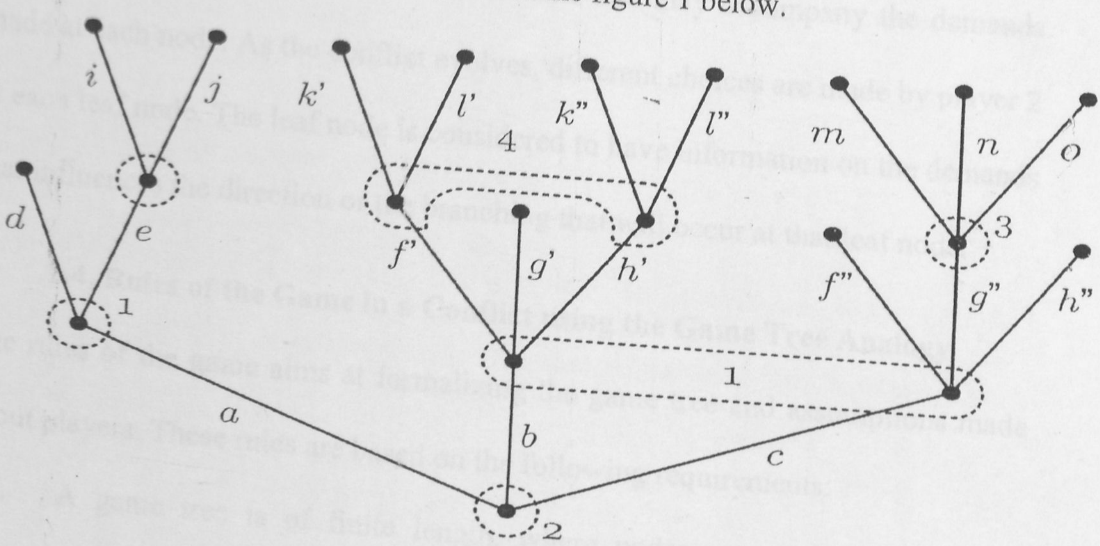

Figure 1: Game tree from Luce et al (1957)

Using the game tree analogy, it is assumed that the game progresses from the root node to a leaf node along a series of branches. Players are expected to make their selections for one branch at each node with the final outcome expected to be the value of a leaf node. The leaf nodes represent the ultimate gain from a demand made in a conflict.

The example above of the tree represents only a subset of all possible demands made. A complete tree would be prohibitively large for most games, due to combinatorial explosion. Here, the game begins with player 2 selecting one of the three demands. The probability of each demand is represented by variables a, b, and c. If the player selects the left branch, the next demand is determined by a chance operation such as drawing a random card. The player cannot distinguish between the other two branches, due to some limitation such as incomplete information. Game trees are simply formalism, enabling a precise analysis of game progression. In a conflict set-up, they will give an indication of the direction a conflict is taking and possible intervention that may accompany the demands made at each node. As the conflict evolves, different choices are made by player 2 at each leaf node. The leaf node is considered to have information on the demands that influences the direction or the branching that will occur at that leaf node.

2.4. Rules of the Game in a Conflict using the Game Tree Analogy

The rules of the game aims at formalizing the game tree and assumptions made about players. These rules are based on the following requirements:

i. A game tree is of finite length, where nodes represent demands and branches potential outcomes.
ii. A label in each node represents the player making a demand
iii. A probability associated with each demand is governed by chance
iv. Demands can be divided into subsets, called information sets, where each set contains demands that cannot be distinguished by the player due to lack of information (reservation values)
v. Each node is labeled according to the demands in the information set.
vi. The final outcome represents the terminal point (Apex) of the tree.
vii. A function called the tradeoff function defines the linear strategic function for each player.
viii. Each player has full knowledge of the rules and tradeoff functions of all players.

ix. Each player aims at optimizing their tradeoff functions.

Rule (viii) is unrealistic in practice, but gives the theory a sharp bound for modelling behavior in the strictest scenario. In this study, we have considered this scenario as the publicly observable reservation values and private information for the players in a conflict when modelling conflict resolution model.

2.5. Types of Games

2.5.1. Zero-Sum Games

The zero-sum noncooperative game is a simple system where gain for one player is balanced equally by loss to another. The sum of all gains and losses is zero. More formally:

$$\sum gain = -\sum loss$$
$$\sum gain + \sum loss = 0 \qquad (2.1)$$

Cooperation among players complicates the theory, and is outside the scope of this study.

2.5.2. Uncertainty Game

If we view the world as completely determinate we can define probability theory as Glimcher (2003) does. He describes a world as "the tool we use to describe those portions of the environment about which we have incomplete knowledge." He further states that, "by gathering more information we can reduce the uncertainty we face, and thus reduce our reliance on probabilistic models of the world, but we accomplish this at a cost in time and energy".

Consider a game where the goal is prediction of a random machine-flipped coin toss. You could collect data about the machine and the coin, such as the center of gravity, and refine these measurements over repeated trials. By doing so, you could construct a probability distribution which increasingly reduces your uncertainty. As you approach certainty, you would always predict the same outcome when the same priors are observed. This is called a pure strategy in a conflict.

2.5.3. Matching Pennies Game

The game of matching pennies illustrates a different kind of uncertainty, which is the key to understanding the dynamics of a game system. Two players place pennies heads- or tails-up on a table. If both pennies are placed with the same face up, player 1 wins. Otherwise, player 2 wins. As discussed in uncertainty in section 2.5.2 above, player 1 collects data on the behavior of player 2. If he detects a pattern, he gains an advantage in the game. The collected behaviors are the private information of the opposing player. However, unlike a simple coin flipping machine, player 2 can disrupt player 1's pattern detection by introducing randomness in her behavior. We consider this in our model to be the constraining condition that is introduced by the use of Bayes' conditional probability. From player 2's perspective, no reduction in uncertainty should be given to player 1. A formulation of the balance between strategy and randomness is a key contribution of game theory.

2.5.4. Non-Zero-Sum Games

In non-zero-sum non-cooperative games, player gains and losses are not in balance. More formally:

$$\sum gain \neq -\sum loss$$

$$\sum gain + \sum loss \neq 0 \quad (2.2)$$

This type of gain exhibits an exploitative condition and in a conflict setting it will be an unfavorable game to be played if a conflict resolution is to be attained.

2. 6. Equilibrium Points in a Game

An optimal strategy in the context of game theory allows a player to maximize gains and minimize loses. The values of an optimal mixed or pure strategy are equilibrium points, said to form an equilibrium strategy.

Following equilibrium strategy means neither party should vary their behavior from equilibrium points unless the other changes a strategy. By straying from equilibrium points a player may obtain larger gains, but does so at the risk of larger losses. This is the kind of scenario that characterizes warring factions in a violent conflict. Where the expectations are to gain more at the expense of the other, however this position is taken with some degree of risks. Violent conflicts experts advise that, if a player takes a risk, it is then better for the opponent to also adopt a non-equilibrium strategy, Glimcher (2003).

Multiple equilibrium points can co-exist, and have equal utility in zero-sum games. It is also important to note that not all games have an equilibrium strategy, Luce et al (1957).

2.7. Modelling a Game

There are a number of mathematical models of games that represent different circumstances. Some of the models are used to provide sets of actions to be taken by players and others will utilize the available information to make predictions.

To be able to present the mathematical models of game theory, we will start by defining the various terminologies used in game theory.

Suppose we let the number of players be denoted by n and identify the players with the integers 1 to n. Then we can denote the set of players by $N = \{1, 2, \ldots, n\}$, where N represents the total number of parties involved in the conflict.

In the cases when $n = 1$, the game theory is called decision theory. For instance, puzzles are examples of one-person games as are various sequential optimization problems found in operations research. When $n=0$, the game is called the "zero-person games", where once an automation gets set in motion, it keeps going without any person making decisions. In this study, we assume throughout that there are at least two parties/players, that is, $n \geq 2$. For purposes of developing and illustrating the various mathematical models, we take n to be finite large number.

2.7.1. Mathematical Models for the Study of Game Theory

There are three main mathematical models used in the study of games theory. They include;

a) The extensive form,

b) The strategic form

c) The coalitional form.

The main difference in each of the models is in the amount of detail on the play of the game built into the model. For instance, if the game has most details it is taken to be extensive form. In this type of model, key components of the game like position in the game and movement of the game from one position to the other is captured. Most of the movements are random moves associated with probabilities of the outcomes.

Some a-priori information on the movements in the game by other players is important and can influence the current game. In the case where all past information about the movements are known, the game is said to be of perfect information. If we have a two-person games of perfect information with same set of legitimate movements from each position, the game is described to be impartial and it becomes partisan when the players do not have the same set of legitimate movements. In the context of this study, as indicated earlier these movements will be considered as the demands. It can therefore be seen that the demands made in a conflict are not static but rather dynamic in nature.

Definition 2.1.: An extensive game with perfect information consists of

a) *a* set *N* (the set of players)

b) *a* set *H* of sequences (the set of terminal histories) with the property that no sequence is a proper sub-history of any other sequence

c) a player's function *P* that assigns a player to every proper subsequence of every terminal history

And it assumes that for each player$i \in N$ a preference relation $\rho_i$over the set H of terminal histories exist.

The restriction on the set *H* is necessary for its members to be interpreted as terminal histories: if *(x, y, z)* is a terminal history then *(x, y)* is not a terminal history, because *z* may be chosen after *(x, y)*. We refer to subsequences of terminal histories as *histories* Osborne (2006).

The sets of actions available to the players when making their moves, while not explicit in the definition, may be deduced from the set of terminal histories. For any history h, the set of actions available to $P(h)$, the player who moves after h, is the set of actions a for which (h, a) is a history.

Bogomolnaia and Jackson (2002) have provided the following definition of a coalitional form of game theory: A coalition formation game G is a pair $(N, (\geq_i)i \in N)$ where N is a finite player set with i being a representative element, and $\geq_i$ is a reflexive, complete, and transitive binary relation on $S_I(N) = \{S \in 2^V : i \in S\}$.

Strict preference relation and the indifference relation are Denoted by $>_i$ and $\sim_i$ respectively $S >_i T \Leftrightarrow [S \geq_i T \; and \; T \, ! \geq_i S]$ and $S \sim_i T \Leftrightarrow [S \geq_i T \; and \; T \geq_i S]$.

In developing how conceptual frame work for our study, we have considered the case of strategic form. Generally, any game can be described using a strategic form. In the strategic form, many of the details of the game such as position and movements are lost; the main concepts are those of a strategy and tradeoffs. In the strategic form, each player chooses a strategy from a set of possible strategies. Suppose we denote the action space of player i by A_i, for $i = 1, 2, \ldots, n$ to constitute the strategy set. Then, it is expected that each player considers all the other players and their possible strategies, and then chooses a specific strategy from his strategy set. To avoid biasness, all players will make such choices simultaneously, the choices are revealed and the game ends with each player receiving some tradeoffs. Each player's choice may influence the final outcome for all the other players. In this study, we have developed a model that provides some constraints on the choices to be made by the players using conditional probability concepts as given in section 3.3.1.

Similarly, by the Utility theory, tradeoffs are modelled to take numerical values and assume that each player receives a numerical tradeoff that depends on the actions chosen from the strategy set (action space) by all the other players.

Now, suppose player 1 chooses $a_1 \in A_1$, player 2 chooses $a_2 \in A_2$, etc. and player n chooses $a_n \in A_n$, where a_i represents the action chosen and A_i represents the action space. Then we denote the tradeoff to player j, for $j = 1, 2, \ldots, n$, by $f_j(a_1, a_2, \cdots, a_n)$ and call it the tradeoff function for player j.

Considering the above case the strategic form of a game is then defined by the relationships:

a) The set, $N = \{1, 2, \ldots, n\}$, of players,

b) The sequence, $A_1, \ldots, A_n$, of action space of the players,

c) The sequence, $f_1(a_1, a_2, \cdots, a_n), \ldots, f_n(a_1, a_2, \cdots, a_n)$, of real-valued tradeoff functions of the players.

If the sum of the tradeoffs to the players is zero no matter the actions chosen by the players, such a game is said to be zero-sum.

That is,

$$\sum_{i=0}^{n} f_i(a_1, a_2, \cdots, a_n) = 0, \text{ for } a_1 \in A_1, a_2 \in A_2, \ldots, a_n \in A_n. \qquad (2.3)$$

The strategy followed by a given party is depended on certain decisions that are made in terms of the choices. One of the ways of making these decisions is by using Bayesian Decision theory which is discussed in section 2.8.

2.8. Bayesian Decision Theory

Bayesian Decision Theory has been used as a fundamental statistical approach that quantifies the tradeoffs between various decisions using probabilities and costs that accompany such decisions. It first assumes that all probabilities are known and then study the cases where the probabilistic structure is not completely known. Bayesian Decision Theory is one of the methods used to solve pattern recognition problems, when those problems are posed in a particular way.

2.8.1. Decision Making Process Using Bayesian Decision Theory

Bayesian decision theory requires parties' states to be represented by a probability function, with 'subjective probabilities', and parties to be represented by a real-valued utility function. In this case the best choice is the one with highest expected utility according to the probability function. Since it is somehow difficult to represent a state by a single probability function, Jeffrey (1983) and van Fraassen (1990, 1995) have proposed the use of a set of probability functions to represent the states. Walley (1991) gives a similar proposal which allows for a complete representation of the parties by a finite number of constraints. Since the parties' state has been represented by a set of probability function, then the utility function will also constitute a set of utility function with different expected utilities for a choice. So the expected utility of an action is not a number, but a set.

To illustrate this theory, consider a case of a party which is assumed to be defined by a known probability functions represented by the indicator function;

$$P(A) = \begin{cases} 1 & \text{if } A \\ 0 & \text{otherwise} \end{cases} \qquad (2.4)$$

and wants to trade ϑ for ε.

This will be divided further into two questions: when is trade permissible, and when is it rationally required? To make a good selection consideration should be made to determine which choices are permissible from a set of available decisions. This case is more like a betting game. Consequently, for a bet ϑ, a party's probability function P will determine a range of expected values for ϑ given by $[l_\vartheta, u_\vartheta]$. This interval presents an imprecise probabilities and the decision theories which allow for imprecise belief fall into two broad categories: structured and unstructured. Unstructured decision theories say we can determine the relative merits of ϑ and ϵ by just looking at l_ϑ , u_ϑ , l_ε and u_ϑ. Structured decision theories say we need to look at more; in particular, we need to compare the values ϑ and ϵ according to particular members of P.

Bayesian decision theory therefore plays a role when there is some *a-priori* information about the things we are trying to classify. For example, consider a case of fruits processing industry and suppose that you didn't know anything about the fruits, but you knew that 80% of the fruits that the conveyer belt carried were apples, and the rest were oranges. If this is the only available information that you will use to make your decision about the type of fruits used in the industry, then you would want to classify a random fruit as an apple. The *a-priori* information in this case is the probabilities of either an apple or an orange being on the conveyer belt.

However, if a decision must be made with so little information, it makes sense to use the following rule:

$$Fruit = \begin{cases} apple & if\ P(apple) > P(orange) \\ orange & if\ P(apple) < P(orange) \end{cases} \quad (2.5)$$

where *P(apple)* is the probability of there being an apple on the belt.

In this case, *P (apple) = 0.*

2.8.2. Decision Rules for Using Bayesian Decision Theory in Cases Involving Two Variables

To illustrate on the decision rules, let us consider an apple as the object with some information we will try to classify. This can only be possible if some probability distribution of the object's characteristics or information is known. For example, we may have the probability distribution for the colour of apples, as well as that for oranges.

Now, let w_{app} represent the state of nature where the fruit is an apple, w_{org} represent the state where the fruit is an orange and x be the continuous random variable that represents the colour of a fruit. Then, the expression $p(x/w_{app})$ represents the density function for x given that the state of nature is an apple. In a typical problem, we would know the conditional densities $p(x/w_i)$ for i either an apple or an orange. We would also know the prior probabilities $p(w_{app})$ and $p(w_{org})$ which represent simply the total number of apples versus oranges that are on the conveyer belt. What we are looking for is some formula that will tell us about the probability of a fruit being an apple or an orange given that we observe a certain colour x. If we had such a probability, then for some given colour that we observed we would classify the fruit by comparing the probability that an orange had such a colour versus the probability that an apple had such a color. If it were more probable that an apple had such a colour, the fruit would be classified as an apple. Using Baye's formula, we can represent the problem as;

$$P(w_i/x) = P(x/w_i)P(w_i)/P(x) \tag{2.6}$$

What this means, is that using our *a-priori* information, we can calculate the *posteriori* probability of the state of nature being in state w_i given that the feature value x has been measured. This analogy will apply in the cases when determining the control variables in a conflict environment.

So, if you observe a certain x for a random fruit on the conveyer belt, then by calculating$P(w_{app}/x)$ and $P(w_{org}/x)$ we would be inclined to decide that the fruit was an apple if the first value were greater than the second one. Similarly, if $P(w_{org}/x)$ is greater, we would decide that the fruit was most likely an orange. Therefore, *Baye's Decision Rule* can be stated as:

$$Decision = \begin{cases} w_{app} & if\ P(w_{app}/x) > P(w_{org}/x) \\ w_{org} & if\ P(w_{org}/x) > P(w_{app}/x) \end{cases}, \tag{2.7}$$

Since $P(x)$ occurs on both sides of the comparison, the rule is equivalent to saying:

$$Decision = \begin{cases} w_{app} & if\ P(x/w_{app})P(w_{app}) > P(x/w_{org})P(w_{org}) \\ w_{org} & if\ P(x/w_{org})P(w_{org}) > P(x/w_{app})P(w_{app}) \end{cases}, \tag{2.8}$$

The probability that an error can be made in the decision is given by;

$$P(error|x) = min[P(w_{app}|x), P(w_{org}|x)]. \tag{2.9}$$

This formula represents the probability of making an error for a specific measurement x. But it is often useful to know the *average* probability of error over all possible measurements. This can be calculated using Baye's Law of total Probabilities, which implies that

$$P(error) = \int_{\forall x} \left[p(x) min\left[P\left(w_{app}|x\right), P\left(w_{org}|x\right)\right]\right] \quad (2.10)$$

2.8.2.1. Generalization of the Bayesian Decision Theory to Include More Variables:

In a more general case, there are several different features that we measure, so instead of x we have a feature vector $\boldsymbol{x}$ in $\mathbb{R}^d$ for d different features. We also allow for more than 2 possible states of nature, where $w_1 \dots w_c$ represent the c states of nature. Baye's formula can be computed in the same way as:

$$P(w_i|x) = \frac{P(x|w_i)P(w_i)}{P(x)},$$

$for\ i = 1 \dots c.$ (2. 11)

But now $P(x)$ can be calculated using the Law of Total Probabilities so that

$$P(x) = \sum_{i=1}^{c} P(x|w_i)\, P(w_i) \quad (2.12)$$

As before, if we measure feature vector **x**, we want to classify an object into class i if $P(w_i|\mathbf{x})$ is the maximum of all the probability densities $for\ i = 1 \dots c$. This is the same as the Baye's decision rule for the 2 category case in section 2.8.2.

2.9. Utility Theory

Sometimes decisions can be made by individuals or groups, under certainty, risk, or uncertainty. A decision made in certainty always has a known outcome to the decision maker. A risky decision is one where known outcomes have known probabilities with some degree of intrinsic losses such as betting on tossing a dice.

An uncertain decision involves potentially unknown outcomes and definitely unknown probabilities. The distinction between individual and group is somewhat artificial, as a group can behave as a collective individual. As this is not always the case, such a distinction gives the model flexibility.

When making decisions, some quantities are usually maximized or minimized. Under certainty, this reduces to a system of constraints, solvable by linear programming. More interesting is what happens under uncertainty or risk. With risk, we know the probabilities $p_1, \cdots, p_n$ and can first approach the optimization problem using the expected value given by:

$$E(x) = \sum_{i=1}^{n} p_i x_i \tag{2. 13}$$

This essentially gives a weighted sum of values produced by each outcome $x_1, \cdots, x_n$. However, there is a problem with using the unmodified expected value. For instance, considering a case of a wager on a coin flip, where the winner is paid a million dollars by the loser. This may be a reasonable bet for a multi-millionaire, but could bankrupt anyone else and as such will be a risky undertaking.

It is commonly accepted that there is a non-linearity in an individual's preference for a wager's outcome. The transition from one million dollars to zero is more dramatic than one hundred million to ninety-nine million. Alternatively, a million dollars means much more to a penniless individual than to a multi-millionaire. We therefore need a function which transforms the raw value of an outcome, such as monetary gain, into something representing an individual's preference and ability.

This preference is called the utility, and a function transforming an outcome to a preference is a utility function. This function has certain formal consistency requirements proposed by Von Neumann and Morgenstern (1944):

i. Any two outcomes must be comparable in terms of preference
ii. Preferences are transitive that is if A > B and B > C, then A > C
iii. A wager whose outcome is another wager (said to be compound) can be probabilistically separated into individual wagers.
iv. If a person is indifferent towards two wagers, they are interchangeable in a compound wager.
v. If two wagers share identical preferred outcomes, the wager with a higher probability of occurring is preferred.
vi. If there exist outcomes A, B, C where A > B and B > C, a wager exists involving A and C where a player is indifferent to B.

This formalism simply says that to determine preference, the value of an outcome needs to be scaled by its' net worth to an individual. Any pair of outcomes can be numerically compared in terms of utility. In fact, there exists a linear transformation $au+b$ between outcomes, where u is the first outcome's utility and a, b are constants.

It is important to keep in mind that an outcome has a larger utility because it is preferred, and not vice versa. Also, because preferences are relative to an individual, they cannot be meaningfully compared between people.

2. 10. Mathematical Contributions to Conflict Modelling

Bowen and McNaught (1996), gives a review of Lanchester's equations which were among the earlier mathematical modelling of conflict by Taylor (1983a). Alberts and Hayes (2003), indicates that the modelling equations should take into considerations the emergent behaviour of self-organising future forces in conflict. To illustrate the mathematical contribution to conflict, we consider a two-sided conflict between Red and Blue parties in which each side uses "aimed fire" to attack the other side Taylor (1983b). Principally, we assume that;

i. each unit on each side is within weapon range of all units on the other side;
ii. units on each side are identical but the units on one side may have a different kill rate to the units on the opposing side;
iii. each firing unit is sufficiently well-aware of the location and condition of all enemy units so that when a target is killed, fire may be immediately shifted to a new target;
iv. new target is randomly chosen from the surviving targets.

Then at time t, and considering the small increment of time between t and $t + \delta t$, the number of Blue casualties $\delta b(t)$ is given by the number of remaining Red units times the number of targets they kill, that is, $r(t)kr\delta t$, where kr is the effectiveness of a single Red unit engagement, and $r(t)\delta t$ is a measure of the number of such engagements. From this we have the relationship $db/dt = kr\, r(t)$. By symmetry we also have a similar relationship for the attrition of Red units, namely, $dr/dt = k_b\, b(t)$.

Hence, dividing one equation by the other we have $db/dr = k_r r / k_b b$. Integrating both sides of this equation, and using the initial values r_0 and b_0 for the number of Red and Blue units at the start of the conflict, we have the following relationship:

$$k_b(b_0^2 - b^2) = k_r(r_0^2 - r^2) \tag{2.14}$$

Now suppose we define the non-dimensional variables $x = 1 - r/r_0$ and $y = b/b_0$. We also define the non-dimensional "Lanchester number:" $L = k_r r_0^2 / k_b b_0^2$ and it is referred to as a "similarity parameter," by which we mean that battles with the same value of L will evolve in a similar way.

From the expression (2.14) above we have

$$\frac{k_b}{k_r} b_0^2 \left\{1 - \left(\frac{b}{b_0}\right)^2\right\} = r_0^2 \left\{1 - \left(\frac{r}{r_0}\right)^2\right\}, \tag{2.15}$$

Thus,

$$\frac{k_b b_0^2}{k_r r_0^2}\{1 - y^2\} = \left\{\left(1 - \frac{r}{r_0}\right)^2 + 2\left(\frac{r}{r_0}\right)\left(1 - \frac{r}{r_0}\right)\right\}, \tag{2.16}$$

or

$$L^{-1}(1 - y^2) = x^2 + 2(1 - x)x = x(2 - x),$$

$$(1 - y^2) = Lx(2 - x), \tag{2.17}$$

from which it follows that

$$y = \left\{1 - 2Lx\left(1 - \frac{x}{2}\right)\right\}^{1/2}, \tag{2.18}$$

A first-order expansion of the expression (2.18) is of the form

$$y \cong 1 - lx\left(1 - \frac{x}{2}\right) \cong 1 - lx \tag{2.19}$$

The relationship $y = 1-Lx$ corresponds to Lanchester's linear law Taylor (1983b). Thus we can see that when x, the proportion of red units destroyed, is small, the linear and square law relationships evolve in the same way, and then diverge as the nonlinear terms grow larger. This example demonstrates how a violent conflict like war can be modelled or described using mathematical expressions.

2.10. 1. Laplace Transforms

Theorem 2.1. (Differentiation Theorem): The differentiation theorem for Laplace transforms states that:

$$\dot{x}(t) \leftrightarrow sX(s) - x(0), \tag{2. 20}$$

where $\dot{x}(t) \triangleq \frac{d}{dt}x(t)$, and $x(t)$ is any differentiable function that approaches zero as t goes to infinity. In operator notation,

$$L_s\{\dot{x}\} = sX(s) - x(0). \tag{2. 21}$$

Proof

This follows immediately from integration by parts:

$$\begin{aligned} L_s\{\dot{x}\} &\triangleq \int_0^\infty \dot{x}(t)e^{-st}dt, \\ &= x(t)e^{-st}|_0^\infty - \int_0^\infty x(t)(-s)e^{-st}dt, \\ &= sX(s) - x(0). \end{aligned} \tag{2. 22}$$

since $x(\infty) = 0$ by assumption.

The differentiation theorem can be used to convert differential equations into *algebraic* equations, which are easier to solve.

Corollary: *Integration Theorem* (Smith (2007))

$$L_s\left\{\int_0^t x(\tau)d\tau\right\} = \frac{X(s)}{s} \tag{2. 23}$$

Thus, successive time derivatives correspond to successively higher powers of s, and successive integrals with respect to time correspond to successively higher powers of $1/_s$.

Theorem 2.2. (Initial value theorem Franklin et al 1993, pg 105.): The continuous time form of the initial value of the function $f(t)$ given by: $\lim_{t\to 0} f(t) = \lim_{s\to\infty} sF(s)$ can be determined from its Laplace transform as;

$$\lim_{s\to\infty} sF(s) = f(0^+)\,. \tag{2.24}$$

Proof

By the definition of the Laplace transform

$$L\left\{\frac{df}{dt}\right\} = sF(s) - f(0^-) = \int_{0^-}^{\infty} e^{-st}\frac{df}{dt}dt \tag{2.25}$$

Consider when $s \to \infty$ and rewrite as;

$$\int_{0^-}^{\infty} e^{-st}\frac{df}{dt}dt = \int_{0^+}^{\infty} e^{-st}\frac{df(t)}{dt}dt + \int_{0^-}^{0^+} e^{-st}\frac{df(t)}{dt}dt \tag{2.26}$$

Taking the limit of equation (1) after substituting equation (2) as $s \to \infty$, we get

$$\lim_{s\to\infty}[sF(s) - f(0^-)] = \lim_{s\to\infty}\left[\int_{0^-}^{0^+} e^{-st}\frac{df(t)}{dt}dt + \int_{0^+}^{\infty} e^{-st}\frac{df(t)}{dt}dt\right] \tag{2.27}$$

The 2[nd] term on the right hand side approaches 0 (zero) because $e^{-st} \to 0$ when $s \to \infty$. Thus we get;

$$\lim_{s\to\infty} [sF(s) - f(0^-)] = \lim_{s\to\infty} [f(0^+) - f(0^-)] \tag{2.29}$$

$$= f(0^+) - f(0^-)$$

Or

$$\lim_{s\to\infty} sF(s) = f(0^+) \blacksquare. \tag{2.30}$$

2.10.1.1. Application of Differentiation theorem

Singh et al (2009) have used the method of Laplace transform by converting boundary value problem into initial value problem. They note that through Laplace transform the difficulties encountered in solving a differential equation by applying boundary conditions for a particular problem are overcome. They considered equation of motion given by;

$$\frac{d^2q_x}{dy^2} - M^2q_x = -P \tag{2.31}$$

Taking Laplace transform of the equation (2.31), we have;

$$L\left[\frac{d^2q_x}{dy^2} - M^2q_x\right] = L[-P]$$

Or

$$L\left[\frac{d^2q_x}{dy^2}\right] - M^2L[q_x] = L[-P]$$

Or

$$s^2\bar{q}_x - sq_x(0) - \acute{q}_x(0) - M^2\bar{q}_x = -\frac{P}{s}$$

Or

$$s^2\bar{q}_x - M^2\bar{q}_x = -\frac{P}{s} + sq_x(0) + \acute{q}_x(0) \tag{2.32}$$

Let

$q_x(0) = l_1$ and $\acute{q}_x(0) = l_2$

Then equation (2.32) becomes

$$s^2\bar{q}_x - M^2\bar{q}_x = \frac{-P+l_1s^2+l_2s}{s}$$

Or

$$\bar{q}_x = \frac{l_1s^2+l_2s-P}{s(s^2-M^2)} \tag{2.33}$$

Using partial fractions we get

$$\frac{l_1 s^2+l_2 s-P}{s(s^2-M^2)}=\frac{E}{s}+\frac{F}{s+M}+\frac{G}{s-G} \tag{2.34}$$

which gives

$$l_1 s^2+l_2 s-P=(E+F+G)s^2+(G-F)Ms-EM^2$$

Giving

$$E=\frac{P}{M^2}, F=\frac{l_1}{2}-\frac{l_2}{2M}-\frac{P}{2M^2} \;\; and\; G=\frac{l_1}{2}+\frac{l_2}{2M}-\frac{P}{2M^2}$$

Putting values of E, F, and G from above in equation (2.34) we get;

$$\bar{q}_x=\frac{P}{sM^2}+\frac{l_1 s}{s^2-M^2}+\frac{l_2 M}{M(s^2-M^2)}-\frac{Ps}{M^2(s^2-M^2)} \tag{2.35}$$

Now, taking inverse Laplace transform of (2.35) we get;

$$q_x=\frac{P}{M^2}+l_1\cosh My+\frac{l_2}{M}\sinh My-\frac{P}{M^2}\cosh My \tag{2.36}$$

Using boundary condition $q_x=0$ when $y=\pm 1$ we get from (2.36)

When y=1

$$0=\frac{P}{M^2}+l_1\cosh M+\frac{l_2}{M}\sinh M-\frac{P}{M^2}\cosh M \tag{2.37}$$

And when y=-1 we have

$$0=\frac{P}{M^2}+l_1\cosh(-M)+\frac{l_2}{M}\sinh(-M)-\frac{P}{M^2}\cosh(-M)$$

Or

$$0 = \frac{P}{M^2} + l_1 \cosh M - \frac{l_2}{M}\sinh M - \frac{P}{M^2}\cosh M \tag{2. 38}$$

Solving (2. 37) and (2. 38) we get

$$l_1 = \frac{P}{M^2} - \frac{P}{M^2 \cosh M} \quad and \quad l_2 = 0$$

Putting values of $l_1 and\ l_2$ from above in equation (2.38) we get

$$q_x = \frac{P}{M^2}\left\{1 - \frac{\cosh My}{\cosh M}\right\} \tag{2. 39}$$

which is the solution of differential equation.

In our study we will use the differentiation theorem for Laplace transformations to be able to solve conflict situations that are expressible using differential equations with initial conditions. This is because differentiation theorem can be used to convert differential equations into algebraic equations, which are easier to solve.

Summary of the chapter

The Game Theory that studies the dynamics of a group of players who repeatedly play a game and adjust their behavior (strategies) over time as a result of their experience (through e.g. reinforcement, imitation, or belief updating) has been used as the theoretical framework of this study. The mathematical representation of conflict has been given by using the Bayesian decision theory. A review of differentiation theorem for Laplace transformation which is has been used in solving the ODEs has been given.

Chapter Three

INITIAL CONDITIONS ESTIMATION IN A CONFLICT

3.0. Introduction

Initial conditions are all the factors that will instantiate any conflict. In a conflict setting, initial conditions constitute the critical component in the determination of the nature and characteristics of a conflict. These factors vary from one conflict environment to another and from time to time. The estimation and/or establishment of these initial conditions should therefore take into consideration the state and the time.

In this chapter, the dynamic time varying model for estimating initial conditions (control variables) which play a significant role in modelling and the success of conflict resolution estimated using a logistic probability model is presented. The model uses the *a-priori* conditions to estimate the *posterior* conditions in a dynamic state. The concept of conditional Bayesian rule has been used in the development of the model in order to obtain the *posterior* conditions from the *a-priori* conditions.

The development of the initial condition estimation model has been driven by strong desire for the application of formal models and quantitative analysis techniques in explaining how strategic actors bargain in a variety of conflict settings. The use of model-based estimators is increasingly becoming popular among statisticians and other scientists in dealing with conflict situations. These models to a great extent rely on fundamental or empirical models that are frequently described by systems of differential equations.

For instance, in the political setting or international relations, bargaining theory plays a central role in solving many conflicts and thus the masterly of the concept of bargaining is very important, Banks (1990); Huth and Todd (2002), London (2002), Powell (1987, 1996), Omwenga (2010).

The basics of logic of bargaining in the face of conflicting interests are best explained and presented by Game theory. Political scientist for instance, have employed bargaining models based on the Game theory to analyze effects of open and closed rules on the distributive politics of legislative appropriation to the study of war initiation and termination, Baron and Ferejohn, (1989); Mansfield et al. (2000).

In developing the model for the estimation of initial conditions, we considered that most conflicts are generally triggered by the differences in opinions and interpretation of an idea Omwenga (2010). We further considered that these differences in opinions have interplaying factors that can be categorized into two broad distinct variables, that is, control variables and state variables. Control variables are the most critical factors to any conflict. According to Omwenga (2010), the control variables are represented as private information values. These variables are considered substantive since, they directly influence individual/group demands made in a conflict environment. Moreover, understanding the interplaying factors in a conflict is very important in solving the conflict and modelling any conflict. In the likelihood that the factors are not known, a reliable model that uses the *a-priori* conditions or belief can be used to predict them.

A conflict situation can therefore be modelled by integrating the control and state variables into statistical and numerical models of the system dynamics. The fundamental or empirical models that are frequently described by systems of ODEs have been used to describe a conflict situation, Signorino (1999, 2002). These systems of ODEs can be used to predict the future dynamics of the conflict, provided that the initial conditions (states) of the conflict are known. An account on the modelling of a conflict from the perspective of social welfare theory and social choice theory has been given by Gordon (2007).

The model we present for the estimation of initial conditions in this study is based on the state dynamics that can be represented using ordinary differential equations. The initial conditions estimated are integrated into the exponential dynamic models to predict on the future trends of a conflict. A method for the estimation of the initial conditions (initial control conditions) in a dynamic state system is given in section 3.3.

What follows is a brief description of the linear and exponential dynamic system models where the estimated initial conditions can be integrated to form a predictive model for modelling a conflict in a dynamic state.

3.1. Ordinary Differential Equation Models (ODE)

a). A Linear Model:

As discussed in section 1.8, a conflict situation/or environment is best described by a dynamic system due to the dynamism of the society in which it occurs. Moreover, the current world which is characterized with great dynamism; a conflict is best described using a dynamical system. In the dynamic system which uses continuous time formulation, it is assumed that the absolute change with respect to time of the series is equal to a constant, that is, the average change is constant over a period of time. Hence, the system dynamics can be represented using a linear model given by;

$$\frac{dy}{dt} = \emptyset, \qquad y(0) = \theta, \tag{3. 1}$$

where θ is the initial condition of the series.

Using Laplace transform the general solution to equation (3.1) is given by;

$$L[y'] = L[\emptyset], \tag{3. 2}$$

where L represents Laplace's transformation.

From (3. 2) we get

$$[s\bar{y} - y(0)] - \emptyset L[1],$$

Or

$$s\bar{y} - \theta = \frac{\emptyset}{s},$$

$$s\bar{y} = \theta + \frac{\emptyset}{s}, \tag{3. 3}$$

Or

$$\bar{y} = \frac{\theta}{s} + \frac{\emptyset}{s^2}, \tag{3. 4}$$

Then, applying inverse Laplace transform, we get,

$$L^{-}[\bar{y}] = L^{-}\left[\frac{\theta}{s} + \frac{\emptyset}{s^2}\right],$$

$$y = L^{-}\left[\frac{\theta}{s}\right] + L^{-}\left[\frac{\emptyset}{s^2}\right], \tag{3. 5}$$

By applying the initial conditions on equation (3.5) we get

$$y = \theta + \emptyset t, \tag{3. 6}$$

which is the linear model in time. Thus, we can view the estimation of the parameters in (3.1) as fitting the solution (3. 6) to a discrete data set.

b). Exponential Model:

Similarly a conflict situation can be modelled using an exponential model in a dynamic system represented by;

$$\frac{dy}{dt} = \emptyset y, \qquad y(0) = \theta, \tag{3. 7}$$

where ϕ is representative for exponential growth rate of a conflict and y is a reducing factor depending on environmental factors.

Due to the in-deterministic nature of the environmental factors, y might remain missing. To overcome this, we modify the model to include logistic model that takes care of the environmental dynamics. The new exponential model now becomes;

$$\frac{dy}{dt} = \emptyset y\left(1 - \frac{y}{y_t}\right), \tag{3. 8}$$

where y_t is the threshold values for conflict occurrence.

To solve (3.8), we rewrite the equation into the form

$$\frac{dy}{dt} - \emptyset y = -\emptyset y\left(\frac{y}{y_t}\right), \tag{3.9}$$

which is a Bernoulli equation.

Dividing (3.9) by y^2 gives;

$$-y^{-2}\frac{dy}{dt} + \emptyset y^{-1} = \left(\frac{\emptyset}{y_t}\right), \tag{3.10}$$

Since letting $u(y) = y^{1-a}$ converts the non-linear Bernoulli into 1st order linear system, we let; $u = y^{1-a} = y^{-1}$ (since a=2 for the case above).

From the above relationship, we have,

$$u(t) = \frac{1}{y} ,$$

whose differentiation yields

$$\frac{du}{dt} = -y^{-2}\frac{dy}{dt} \tag{3.11}$$

Substituting (3.11) in (3. 10), we get

$$\frac{du}{dt} + \emptyset u = \frac{\emptyset}{y_t}, \tag{3.12}$$

Equation (3.12) is the first order linear ODE. Applying Laplace transformation to solve equation (3.12), we get

$$L\left[\frac{du}{dt} + \emptyset u\right] = L\left[\frac{\emptyset}{y_t}\right] \tag{3.13}$$

From (3.13), we get

$$[s\bar{u} - u(0)] + \emptyset\bar{u} = \frac{\emptyset}{y_t s} ,$$

Or

$$(s+\emptyset)\bar{u} - u(0) = \frac{\emptyset}{y_t s}, \tag{3.14}$$

But $u(0) = \frac{1}{\theta}$, hence we rewrite (3.14) as;

$$(s+\emptyset)\bar{u} - \frac{1}{\theta} = \frac{\emptyset}{y_t s}, \tag{3.15}$$

Or

$$(s+\emptyset)\bar{u} = \frac{\emptyset}{y_t s} + \frac{1}{\theta}, \tag{3.16}$$

Then,

$$\bar{u} = \frac{\emptyset}{y_t s(s+\emptyset)} + \frac{1}{\theta(s+\emptyset)}, \tag{3.17}$$

Or

$$\bar{u} = \frac{\emptyset}{y_t}\left[\frac{1}{s(s+\emptyset)}\right] + \frac{1}{\theta}\left[\frac{1}{(s+\emptyset)}\right], . \tag{3.18}$$

Or

$$\bar{u} = \frac{\emptyset}{y_t}\left[\frac{1}{\emptyset s} - \frac{1}{\emptyset(s+\emptyset)}\right] + \frac{1}{\theta}\left[\frac{1}{(s+\emptyset)}\right] \tag{3.19}$$

Further simplification gives,

$$\bar{u} = \frac{1}{y_t}\left[\frac{1}{s} - \frac{1}{(s+\emptyset)}\right] + \frac{1}{\theta}\left[\frac{1}{(s+\emptyset)}\right], \tag{3.20}$$

Taking inverse Laplace's transformation on equation (3.20), gives,

$$u = \frac{1}{y_t}\left[L^{-}\left\{\frac{1}{s}\right\} - L^{-}\left\{\frac{1}{(s+\emptyset)}\right\}\right] + \frac{1}{\theta}\left[L^{-}\left\{\frac{1}{(s+\emptyset)}\right\}\right], \tag{3.21}$$

Or

$$u = \frac{1}{y_t}\left[(1) - e^{-\emptyset t}\right] + \frac{1}{\theta}\left[e^{-\emptyset t}\right], \tag{3.22}$$

Or

$$u = \frac{1}{y_t} - e^{-\emptyset t}\left[\frac{1}{y_t} - \frac{1}{\theta}\right],$$

(3. 23)

$$u = \frac{1}{y_t} - e^{-\emptyset t}\left[\frac{\theta - y_t}{y_t\theta}\right],$$

$$u = \frac{\theta - (\theta - y_t)e^{-\emptyset t}}{y_t\theta},$$

(3. 24)

Or

$$u = \frac{\theta + (y_t - \theta)e^{-\emptyset t}}{y_t\theta},$$

(3. 25)

Since, $u(t) = \frac{1}{y}$, $\Rightarrow y(t) = \frac{1}{u}$, we can write the general solution equation (3.25) as;

$$y(t) = \frac{y_t\theta}{\theta + (y_t - \theta)e^{-\emptyset t}},$$

(3. 26)

when $t=0$, $y(0)$ is the initial conditions and y_t is the threshold condition for the occurrence of a conflict at time t.

3.2. Dynamic Representation of the System Models

As discussed in section 3.1, above, purely linear and exponential functions of time can be used for the estimation of the trend of the conflict as a solution to their corresponding time dynamics equations, i.e., equations used to describe how systems change or evolve over time. This is important because understanding the relationships can be very useful to researchers in a conflict situation. It is often the case that reality necessitates the relaxation of the linearity assumptions in a numbers of situations like conflict and economic environments giving rise to nonlinear dynamic systems. Analytical solutions of these systems are in general unattainable for some relatively more complicated dynamics and the only method of estimation may be the dynamic approach.

In static system, the initial data point is used as the initial condition of the differential equation, while in the dynamic option; the initial condition(s) is estimated as an additional parameter. The nice thing about this procedure is that the dynamics are written as they occur in the model equations. It is very important to understand the difference between the static and dynamic options when fitting dynamic models to data.

The model we have developed can be used for the estimation of the initial conditions for both static and dynamic systems.

3.3. Model for the Estimation of Initial Conditions

As indicated in section 3.1, control variables are crucial in modelling any conflict. These control variables as indicated earlier can be any private information that is relevant to the party's decision making in a conflict environment. In most cases the control variables are hidden or reserved from the other parties to the conflict. Suppose we denote the private information by ℓ_i *(where i= 1...N, are parties to the conflict)*, these reserved information usually will lead to a conflict in opinion since they are based on personal considerations and belief. As a consequence of conflict of opinion, a conflict state described by these divergent opinions is developed. This conflict state is characterized by initial conditions (control variables) which must be understood and quantified to successfully model and solve any conflict.

The estimation of the initial conditions in a conflict situation can be compared to the estimation of types in a Bayesian game theory. In this study, the control variables are modelled to constitute the following components:

1. Demand to the other parties.
2. Demands from the other parties.

The two components are considered as the conditioning variables to a probability of one another. Assuming that a conflict is most likely to arise if the demands from one party are not met by the other party and these demands are private information, these demands can be defined by Baye's probability distribution described in section 2.8.

Now, suppose the initial conditions are the state set, θ, (current demands), they can be represented by:

$$\theta = X_{i \in N} \theta_i \tag{3.27}$$

where N is the total number of parties to a conflict, X is the state vector, θ_i is the state of the system at *i*.

Initially, the state is assigned *a-priori* belief $P(\theta)$ which reflects existing knowledge about the conflict state. As the system evolves, some new private information and data say D will become available. To estimate these new outcomes, the available beliefs can be updated using the Baye's rule which states that;

$$Posterior = P(\theta/_D) = \frac{P(D/_\theta)P(\theta)}{\int P(D/\theta)P(\theta)d\theta} = \frac{likelihood \times prior}{nomalizer} \tag{3.28}$$

From equation (3. 28) a new set of initial conditions can be obtained for a conflict state. Since, these set of initial conditions are dependent on the *a-priori* conditions, extra conditioning is required so as to limit and provide constraints on the demands at the same time provide dynamism to the model. This can be achieved by using the concept of ultimatum game theory discussed in section 3.3.1 below.

3.3.1. Constraining the Initial Condition Estimation Model

Suppose in a conflict the first party has made an offer y based on the state set θ given by (3.27), then the second party will chose between the offer and her reservation value given by $R_2 + \ell_2$ where R_2 is public information for the second party and ℓ_2 is the private or reserved information for the second party. Then the equilibrium and hence settlement of a conflict can be achieved if the second party will play the cut-point strategy given by:

$$s_2(y, \ell_2) = \begin{cases} accept & if\ \ y \geq R_2 + \ell_2 \\ reject & if\ \ y < R_2 + \ell_2 \end{cases} \tag{3. 29}$$

From a negotiation stand point, the first party does not observe ℓ_2 but must assess the probability that the second party will accept or reject his offer, where;

$$\begin{aligned} \Pr(\ accept \ \ / \ y) &= \Pr(\ y \geq R_2 + \ell_2) \\ &= \Pr(\ \ell_2 \leq y - R_2) \\ &= F_{\ell_2}(y - R_2) \end{aligned} \tag{3. 30}$$

Considering the optimization problem for the first party, given the second party's strategy (3.30), then the expected utility for the first party is:

$$E_{U_1}(y/Q^*) = F_{\ell_2}(y - R_2) \cdot (Q^* - y) + \left(1 - F_{\ell_2}(y - R_2)\right) \cdot (R_1 + \ell_1) \tag{3. 31}$$

By the first order condition (F.O.C) and the log-concavity of f_{ℓ_2}, the first party's optimal offer is the unique y* that implicitly solves

$$y^* = Q^* - R_1 - \ell_1 - \frac{F_{\ell_2}(y^* - R_2)}{f_{\ell_2}(y^* - R_2)} \qquad (3.32)$$

However, $0 \le y^* \le Q^*$ and sometimes y^* will be outside the feasible set. We can then show that an end-point (0 or Q^*) is optimal and in any perfect Bayesian equilibrium (PBE), the first party will have the strategy:

$$s_1(\ell_1 / R_1, R_2, Q^*, F_{\ell_2}(.)) = \begin{cases} Q^*, & \ell_1 \le -R_1 - \frac{F_{\ell_2}(Q^* - R_2)}{f_{\ell_2}(Q^* - R_2)} \\ y^*, & -R_1 - \frac{F_{\ell_2}(Q^* - R_2)}{f_{\ell_2}(Q^* - R_2)} < \ell_1 < Q^* - R_1 - \frac{F_{\ell_2}(-R_2)}{f_{\ell_2}(-R_2)} \\ 0, & \ell_1 \ge Q^* - R_1 - \frac{F_{\ell_2}(-R_2)}{f_{\ell_2}(-R_2)} \end{cases} \qquad (3.33)$$

Taking variables $\delta_k, k \in \{0, y, 1\}$ such that $\delta_0 = 1$ if y = 0, $\delta_y = 1$ if $0 < y < Q^*$ and $\delta_1 = 1$ if $y = Q^*$ that is, a censored model with a "latent" best offer in the constraint set. Otherwise there is the best feasible offer, at, a boundary point.

Taking the second party's acceptance as $\delta_{accept} = 1$ if she accepted the offer and $\delta_{accept} = 0$ if she rejected the offer and assuming we have data on both parties actions (i.e., y and δ_{accept}) from the state set, θ, then the likelihood would be

$$L = \prod_{i=1}^{n} \Pr(y^* < 0)^{\delta_0} . \Pr(y^* = y)^{\delta_y} . (1 - \Pr(y^* < Q^*))^{\delta_1} X \Pr(accept)^{\delta_{accept}} . \Pr(reject)^{1-\delta_{accept}}$$

(3.34)

Equation (3.34) is begged on the existing control variables in θ. It gives the log-likelihood function for our data in terms of distributions already derived, which are functions of regressors.

Using equation (3.34), the Likelihood, $P(D/\theta)$, which is a measure of the probability of seeing particular realization of the state θ, can therefore be estimated, where y =ultimatum offer from the first party to the conflict, Q^* =upper bound of the contested prize, δ_i = actions, $\sigma_i : \ell_i \rightarrow A^i$, $i = \{1, 2\}$, where A^i defines the action set for the i^{th} party.

Since, party 1 is making the ultimatum offer, $A^1 = \{y: y \in [0; Q^*]\}$, the second party is then left to accept or reject the offer, so A^2 = {accept; reject}.

$$\Pr(accept / y) = \wedge(y - Z\gamma) \tag{3.35}$$

Suppose the public portion of the parties' reservation values are $R_1 = \beta X$ and $R_2 = \gamma Z$, where X and Z are sets of substantive regressors.

Then, for party 1, logistic distribution of y^* implies that

$$_1y^* = Q^* - \beta X - \ell_1 - \frac{\wedge(y^* - \gamma Z)}{\lambda(y^* - \gamma Z)} \tag{3.36}$$

which is the optimal offer, where $\wedge()$ is the logit cumulative distribution function (c.d.f) and $\lambda()$ is the logit probability density function (p.d.f). Solving for y^* gives

$$y^* = Q^* - \beta X - \ell_1 - 1 - \omega\left(e^{(Q^* - \beta X - \gamma Z - \ell_1 - 1)}\right) \tag{3.37}$$

where ω is Lambert's ω, which solves transcendental functions of the form $z = \omega e^{w}$ for ω. Lambert's ω is useful here because it is known to have nice properties. First, Lambert's ω is single valued on R+. Second, ω's first and second derivatives exist and are well behaved, making it easy to show that y^* is a monotonic function of ℓ_1 and allowing for the derivation of the probability density function for equilibrium offers.

From (3.28), the new initial conditions estimates of θ in a dynamic system estimated as posteriors can then be given by:

$$\theta^* = \frac{LP(\theta)}{\int P(D/\theta)P(\theta)d\theta} \tag{3. 38}$$

where $\int p(D/\theta)p(\theta)d\theta$ is used to ensure that the values of $P(D/\theta)$ sum up to one and thus define a proper probability distribution, L is the constrained log-likelihood given by equation (3.34) and θ is the *a-priori* conditions.

3.4. Application of the Model to an Armed Conflict

We examine the application of the model in the estimation of initial conditions in an armed conflict situation. Modelling the initial conditions in this situation can be compared to the modelling of the risk related to the previous conflict Clementine, Dirk and Francois (2008).

It is believed that countries that have experienced an armed conflict are more prone to another conflict in the future and thus their risk levels of an armed conflict are high. We have developed a model that estimates the initial conditions which can act as the pointer to the current risk levels using the past and current state control variables. The estimates of the initial conditions can be used to make predictions for the future trends of a conflict in a dynamic state system.

Assuming that all countries in the world are a universal set Ψ and the countries that are likely to be in a conflict are its subset denoted by Q^*. Our concern is on the subset which can be described as the "prize". A country becomes an element (member) of Q^* if it has experienced an armed conflict at any time in the period of interest. The set Q^* is described as a semi-open space since it allows individuals to become members but does not allow them to get out.

We can therefore define an indicator variable X_{tc}, such that

$$X_{tc} = \begin{cases} 0 & if\ c\ is\ not\ in\ conflict\ in\ year\ t \\ 1 & if\ c\ is\ is\ in\ conflict\ in\ year\ t \end{cases} \qquad (3.39)$$

Thus,

The total number of countries in a conflict in year *t* is:

$$s_t = \sum_{c=1}^{n} X_{tc} \qquad (3.40)$$

The number of countries that are at conflict in year *t* and have experienced at least one armed conflict in the past is:

$$m_t = \sum_{c=1}^{n} X_{tc} \quad if\ X_{tc} = 1\ and \ni y < t / X_{yc} = 1 \qquad (3.41)$$

The number of countries that have experienced an armed conflict before year *t*, they are not at conflict in year *t*, but are reported to have experienced another conflict later is:

$$z_t = \sum_{c=1}^{n} X_{tc} + 1 \quad if\ X_{tc} = 0\ and\ \ni y < t,\ j > t / X_{yc} = 1,\ X_{jc} = 1 \qquad (3.42)$$

The number of countries at conflict in year t that are reported to be still at conflict at any later period is:

$$r_t = \sum_{c=1}^{n} X_{tc} \quad if\ X_{tc} = 1\ and\ \ni y > t / X_{yc} = 1 \qquad (3.43)$$

The tetal of armed conflicts in a country which is subset of Q^* is:

$$a_c = \sum_{c=1}^{n} X_c \qquad (3.44)$$

The probability, $P(D/\theta)$, given by equation(3.34) that an armed conflict is likely to occur given that a country is a member of Q^* in t can be estimated by:

$$P(D/\theta) \equiv L = \frac{m_t r_t}{m_t r_t + s_t z_t} \qquad (3.45)$$

And the prior belief $P(\theta)$ can be obtained as:

$$P(\theta) = \frac{a_c}{s_t} \qquad (3.46)$$

Using the data set in appendix A extracted from PRIO/Uppsala Conflict Data Project, obtained from *http://www.prio.no/cwp/ArmedConflict* and estimated values by equation (3.45) and (3.46), our estimated initial condition $\hat{\theta}$, for the various conflict situations in the various countries in the year 2000, 2003 and 2004 can be estimated using equation (3.38). These estimates are shown in table 1 below:

Table1: Estimated initial conditions as posterior

Country	2000		Country	2003		Country	2004	
	$\hat{\theta}$	No. of conflicts		$\hat{\theta}$	No. of conflicts		$\hat{\theta}$	No. of conflicts
India	0.68	8	India	0.69	7	India	0.74	6
Nepal	0.60	1	Nepal	0.60	1	Nepal	0.67	1
DRC	0.50	1	DRC	0.34	-	DRC	0.56	-
Colombia	0.68	1	Colombia	0.68	1	Colombia	0.74	1
Peru	0.14	0	Peru	0.14	0	Peru	0.18	0
Pakistan	0.49	1	Pakistan	0.49	1	Pakistan	0.56	2
Ethiopia	0.68	3	Ethiopia	0.69	2	Ethiopia	0.74	2
Turkey	0.68	1	Turkey	0.69	1	Turkey	0.74	1
Indonesia	0.55	1	Indonesia	0.55	1	Indonesia	0.52	1
Mali	0.25	0	Mali	0.25	0	Mali	0.31	0
Nigeria	0.14	0	Nigeria	0.14	0	Nigeria	0.18	1
Niger	0.37	0	Niger	0.25	0	Niger	0.31	0
Thailand	0.55	0	Thailand	0.55	1	Thailand	0.62	1

From the table $\hat{\theta}$ represents the estimated initial conditions for the various countries based on the past armed conflicts and the current state conditions. The estimates reflect the risk level of an occurrence of an armed conflict and can given a pointer to the future trends of the existing conflict. The result gives a positive prediction of the occurrence of a conflict in a given year considering the previous conflicts experienced by the country.

3.5. Conclusion

In this chapter, the dynamic- time-varying model for estimating control variables (initial conditions) which play a significant role in the success of conflict resolution is estimated using a logistic probability model has been developed. A real conflict data set, from International Peace Institute, Oslo (PRIO), was used to test on the workability of the model.

The model gives initial conditions based on the previous and available conditions for the country in conflict. The estimated initial conditions gives the probability of the occurrence of a conflict and can thus forms the basis for further investigation and prediction of the trend that a conflict is likely to take as other new interplaying factors come into play. The model is dynamic in the sense it can be adjusted over the time under investigation.

Chapter Four

CONFLICT PREDICTION IN A DYNAMIC STATE

4.0. Introduction

As pointed out in chapter 2, conflict situations can be described by statistical and numerical models of the system dynamics. Most of these models are described by systems of ordinary differential equations (ODEs), Signorino (1999, 2002). The models can be used to predict the future dynamics of the conflict, provided that the initial states of the conflict are known or can be estimated using equation 3.38. In order to understand and precisely model a given conflict, it is fundamental to identify potential or existing conflict causes, as well as possible factors contributing to peace. The causes of conflicts can be defined as those factors which contribute to people's grievances and can be categorised as:

i. Structural causes – pervasive factors that have become built into the policies, structures and fabric of a society and may create the pre-conditions for violent conflict.

ii. Proximate causes – factors contributing to a climate conducive enough for violent conflict or its further escalation, sometimes apparently symptomatic of a deeper problem.

iii. Triggers – single key acts, events, or their anticipation that will set off or escalate violent conflict.

All these factors are classified as observable variables in conflict modelling. Protracted conflicts also tend to generate new causes (e.g. weapons circulation, war economy, culture of violence), which help to prolong them further.

As the main factors contributing to conflict and to peace are identified, it is also important to acknowledge that conflicts are multi-dimensional and multi-causal phenomena – that there is no single cause of conflict. It is also essential to establish linkages and synergies between causes and factors, in order to identify potential areas for intervention and further prioritise them for consideration in modelling the conflict.

Complete data defining all of the states of a conflict system at a specific time are, however, rarely available. This challenge can be handled using missing data analysis techniques, Rubin (1996), Harzog and Rubin (1993) and Omwenga (2004). In a conflict, for instance, there are some underlying issues that can be described to be private and as such may not be available. Furthermore, both the models and the available initial data contain inaccuracies and random noise that can lead to significant differences between the predicted states of the system and the actual states of the system. In such a case, observations of the system over time can be incorporated into the model equations to derive improved estimates of the states and also to provide information about the uncertainty in the estimates.

Due to the popularity of model-based algorithms in a number of systems like conflict control and process optimization there has been increased interest in developing fundamental models with precise parameter estimates, Biegler and Grossman (2004), El-Farra and Christofides, (2003). In this study, we have developed a conflict prediction model based on the state dynamics as represented by the ordinary differential equations (ODEs) from exponential dynamic models given in section 3.1. The model also uses initial conditions that are dynamically estimated as discussed in chapter 3. This model is expected to adjust as per the existing conflict conditions and time.

4.1. Dynamic Model Conceptualization

As discussed in section 1.8, a conflict is generally dynamic and therefore, to effectively describe and modelling a conflict in such a situation will require a dynamic model.

Considering the linear and exponential dynamic models discussed in section 3.1, and also considering the observable variable using the criteria discussed in section 4.2, we have developed a conflict prediction model for a dynamic state given by equation (4.9).

Now, suppose we define an indicator variable Y_i that fully describes a conflict situation as;

$$Y_i = \begin{cases} 1 & if\ there\ exist\ a\ conflict\ for\ year\ i \\ 0 & if\ there\ exist\ a\ peace\ for\ year\ i \end{cases} \tag{4.1}$$

Then a Bernoulli distribution with parameter π_i(probability of occurrence of a conflict) fully describes this variable if conflict and peace are exhaustively described by the above indicator variable.

Let a vector of a constant term and n explanatory variables be denoted by; $X_i = (1, X_{1i}, X_{2i}, \cdots, X_{ni})$.

Then a linear function that describes the conflict situation is given by;

$$Y_i \sim Bernoulli(\pi_i) \tag{4.2}$$

with $\pi_i = X_i\beta = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \cdots + \beta_n X_{ni}$, where $X_i\beta$ is a matrix showing a linear relationship between π_i and X_i and the $(k+1) \times 1$vector β indicates a constant term and k coefficients on each of the k explanatory variables.

Therefore;

$$\pi_i = linear(X_i). \tag{4.3}$$

The linear probability function has a challenge of generality values of π_i greater than 1 or less than 0 which are outside the boundaries defined by the indicator variable Y_i. Due to these challenges a linear model we replaced π_i with logit models given by;

$$\pi_i = logit(X_i\beta) = \frac{1}{1+e^{-X_i\beta}} \tag{4.4}$$

which maps the linear function form $X_i\beta$ by taking the interval $[0, 1]$required for π_i by applying logit function.

From (4.4), the underlying probability of a conflict π_i is therefore given as a logit function of a linear function of X_i.

That is;

$$\pi_i = logit(\ linear(X_i)) \tag{4.5}$$

Equation (4.5) is therefore the generalization of the linear probability model (4.2), obtained by adding an extra level of hierarchy.

Equation (4. 5) further estimates the impossible values which are given by model (4. 2) and it assumes a more acceptable relationship between the explanatory variable and the probability outcome. The effect of each explanatory variable varies across observations and depends on other explanatory variables.

To examine the changes in the effects of explanatory variables, derivative of probability π_i with respect to one of the explanatory variables, say X_{1i} is considered. For a linear model, this derivative is β (which is a constant) whereas for a logit model the derivative is;

$$\pi_i(1-\pi_i)\beta_i \quad (4.6)$$

The logit derivative given by (4.6) is better compared to the constant value given by a linear model. But since π_i is within a small range above zero for most of the observations; this is a highly restrictive and nearly constant specification. To improve on this weakness, we specify a random effect model which is the initial condition estimation model discussed in section 3.3. Instead of leaving β fixed at one set of values as in equation (4.4), we let it vary randomly over observations in some form given by equation (3.28).

Inclusion of a random estimation model into (4.5) gives an additional variable in $X_i$and considers more state variables which influence the conflict. This strategy works well by considering the *a-priori* conditions in the development of the initial condition estimation model as discussed in section 3.3

4.2. Controllability and Observability in a Conflict Environment

In general if the desired state is specified for all time, the requirements for the existence of a control variable $\theta(t_0)$ (initial condition) that will generate the desired state $\phi(f)$, the new state condition are very stringent. A less ambitious but more realistic goal is to require only a partial specification of the state variables. One such partial specification is forcing the state of a given system to attain a specified value at some finite time in the future. That is, given an initial time, t_0 an initial state $\phi(t_0) = x_0$, and a final state $\phi(f)$, and a control variable $\theta(t), t_0 \le t \le t + T$, for some finite T, such that $\phi(t_0 + T) = \phi_f$, there may or may not be a control variable, $\theta(t)$ which can force the system to attain the state $\phi(f)$. Thereafter it may be desirable to maintain the state $\emptyset(f)$. However, by a suitable choice of error coordinates such as state variables, the problem of reaching and maintaining a specified state is the problem of matching a desired dynamic response.

Now, considering the first order differentiable dynamic system (3.7) that describes a state given by;

$$\frac{dy}{dt} = \emptyset(y), \qquad y(t_0) = \theta \tag{4.7}$$

And since a conflict environment has many interplaying factors, then equation (4.7) can be modified by considering;

$$\emptyset(y) = A(t)y(t) + B(t)\theta(t),$$

where $A(t)$ is an $(n \times n)$ square matrix and $A: T^m \to B \in \mathbb{R}^n$ is regressive and rd-continuous, since it is continuous in right dense points and $\lim f(s)$ as $s \to t$ exists for all right dense points at $t \in T$.

Such that equation (4.7) becomes;

$$\frac{dy}{dt} = A(t)y(t) + B(t)\theta(t), \qquad y(t_0) = y_0 \tag{4.8}$$

The concepts of controllability and observability for the differentiable dynamic system equation (4.8) is considered in this section, while noting that controllability is influenced by a specific finite time interval $[t_0, t_f]$.

Definition 4.1: *A dynamic system given by (2) is said to be completely state controllable if, for any initial state* $y(t_0) = \theta$, *initial time* t_0, *it is possible to generate an unconstrained control variable vector* $\emptyset(y)$, *that will take any given original conflict state* $\phi(t_0)$ *to any final state* $\phi(t_f)$ *in a finite time interval* $t_0 \leq t \leq t_f$.

And if equation (4.8) is controllable for all y_0 at $t = t_0$ and for all y_f at $t = t_f$, then the system (4.8) is said to be completely controllable. Then, suppose that $T^m = (a, b) \cap T$ and the associated standardized system is

$$\frac{dy}{dt} = A(t)y(t), \qquad y(t_0) = \theta = y_0 \tag{4.9}$$

If we let $\Phi_A(t, t_0)$ be a basic matrix solution of (4.9), then any solution $y(t)$ for (4.8) has the form;

$$y(t) = \Phi_A(t, t_0)y_0 + \int_{t_0}^{t} \Phi_A(t, \sigma(s))B(s)\theta(s)ds \tag{4.10}$$

from equation (4.10), we get;

$$\bar{y}(t) = \int_{t_0}^{t} \Phi_A(t, \sigma(s))\, B(s)\theta(s)ds, \tag{4.11}$$

which is a particular solution of the dynamic system (4.8), Lakshmikantham, Sivasundaram and Kaymakelan (1996).

Theorem 4.1: The system defined by equation (4.8) is completely controllable on the closed interval $F = [t_0, t_f]$ if and only if the $(n \times n)$ symmetric matrix

$$W(t_0, t_f) = \int_{t_0}^{t_f} \Phi(t, \sigma(s)) B(s)B^*(s)\Phi^*(t, \sigma(s))ds \qquad (4.12)$$

is non-singular.

Proof

Assuming that $W(t_0, t_f)$ is non-singular, then the dynamic system (4.8) is completely controllable. For given $(n \times 1)$ vector y_0, we choose;

$$\theta(t) = -B^*(t)\Phi^*(t, \sigma(t))W^{-1}(t_0, t_f)y_0 \qquad (4.13)$$

From above, the control variable θ is continuous on F and the corresponding solution of (4.8) with the initial condition $y(t_0) = y_0 = \theta$ is given by;

$$y(t_f) = \Phi(t_f, t_0)y_0 + \int_{t_0}^{t_f} \Phi(t_0, \sigma(s))B(s)\theta(s)ds \qquad (4.14)$$

Substituting for $\theta(t)$ and using $W(t_0, t_f)$ as given in the theorem, we get

$$y(t_f) = \Phi(t, t_0)x_0 - \int_{t_0}^{t} \Phi(t, \sigma(s))B(s)B^*(s)\Phi^*(t, \sigma(s))W^{-1}(t_0, t_f)y_0 ds$$

$$= \Phi(t, t_0)y_0 - \Phi_A(t, t_0) \int_{t_0}^{t} \Phi(t, \sigma(s))B(s)B^*(s)\Phi^*(t, \sigma(s))W^{-1}(t_0, t_f)y_0 ds$$

$$= 0$$

Thus the dynamic system is controllable for all $t_0 \leq t \leq t_f$, and it follows that the system given by equation (4.8) is completely controllable.

Next assuming that the dynamic system (4.8) is completely controllable on F and suppose that $W(t_0, t_f)$ is singular. Then since $W(t_0, t_f)$ is non-invertible and there exists a non-zero $(n \times 1)$ vector y, such that;

$$y^* W(t_0, t_f)C = \int_{t_0}^{t_f} y^* \Phi(t_0, \sigma(s)) B(s) B^*(s) \Phi^*(t_0, \sigma(s)) y ds \tag{4.15}$$

Because of the fact that the integrand in this expression is non-negative continuous function, we have;

$$\|y^* \Phi(t_0, \sigma(s)) B(s)\| = 0, \tag{4.16}$$

It therefore follows that;

$$y^* \Phi(t_0, \sigma(s)) B(s) = 0,\ s \in F. \tag{4.17}$$

Since the state equation is completely controllable on F, choosing $y_0 = y$, there exists a continuous control variable $\theta(t)$ such that,

$$0 = \Phi(t_f,\ t_0) y + \int_{t_0}^{t_f} \Phi\left(t_f, \sigma(s)\right) B(s) \theta(s) ds$$

Or

$$y = -\int_{t_0}^{t_f} \Phi^{-1}(t_f,\ t_0) \Phi\left(t_f, \sigma(s)\right) B(s) \theta(s) ds$$

$$= -\int_{t_0}^{t_f} \Phi(t_0, \sigma(s)) B(s) \theta(s) ds$$

Thus,

$$y^*y = -\int_{t_0}^{t_f} y^*\Phi(t_0,\sigma(s))B(s)\theta(s)ds \tag{4.18}$$

And since (4.17) holds, it follows that $y^*y = 0$, and thus it contradicts the fact that $y \neq 0$. Thus $W(t_0, t_f)$ is non-singular■.

Definition 4.2: *A system is completely observable on* $[t_0, t_f]$*, if, for some arbitrary initial state* $y(t_0) = y_0$*, there is a finite output uniquely determined such that from measurements of the output, (y (0), y (I),... ,y(m)), the initial state,* $y(t_0) = y_0 = \theta$*, can be computed.*

Theorem 4.2: The dynamic system (2) is completely observable on $[t_0, t_f]$ if and only if the $(n \times n)$ symmetric observability matrix

$$M[t_0, t_f] = \int_{t_0}^{t_f} \Phi^*(s, t_0)C^*(s)C(s)\Phi(s, t_0)ds \tag{4.19}$$

is non-singular.

Proof

Suppose that $M[t_0, t_f]$ is non-singular, then the solution expression with $\theta(t) = 0$ is given by $y(t) = C(t)\Phi(t, t_0)y_0$,

Or

$$\Phi^*(t, t_0)C^*(t)y(t) = \Phi^*(t, t_0)C^*(t)C(t)\Phi(t, t_0)y_0 \tag{4.20}$$

Hence

$$\int_{t_0}^{t_f} \Phi^*(s, t_0)C^*(s)y(s)ds = \int_{t_0}^{t_f} \Phi^*(s, t)C^*(s)C(s)\Phi(s, t_0)y_0 ds$$

$$= M(t_0, t_f)y_0$$

Since M is non-singular, y_0 can be determined uniquely. Thus the dynamical system (4.8) is completely observable.

In a conflict the concept of observability is employed so as to analyze the state $\emptyset(f)$. Observability implies the determinability of a state $\emptyset(f)$ from an observation of the output over a finite time interval, starting from the instant at which the state is desired.

In complex systems the observability of the system can be determined by examination of the coordinates of a transformation of the state vector $\phi(t)$. In some control problems, it is necessary to determine the state of the system in order to generate the appropriate control input. Observation of the output of a completely observable system, over a finite time interval, yields sufficient information to determine the state of the system at the beginning of the time interval.

By selecting different trajectories for the control variables θ over time a set of future 'histories' or behaviour can be built for the system. The problem is, however, to choose between the essentially infinite possible future histories, by no less than rigorous means. The most 'appropriate' history can be selected by choosing certain values of $\theta(t)$ through time. By 'appropriate' it is meant that the choice of the values of the control variables should be governed by some objective or goal that the parties are attempting to achieve.

2.3.1. Objective Function

The main reason of having a controllable and observable dynamic system is to be able to build on the objective function that represents the desired yield from the dynamic system at each moment in time and is expressed as a function of the state variable, control variable and time given by;

$$F(\emptyset(t), \theta(t), t) \tag{4.21}$$

where F is the objective function.

Thus for a finite time horizon, the general statement of the optimization problem is to find a particular $\theta(t)$, call this $\theta^*(t)$, which will optimize :

$$F = \int_{t_0}^{T} \lambda(\phi(t), \theta(t), t)\, dt \tag{4.22}$$

subject to the initial condition $\theta(t)$ and possibly other conditions.

The use of the integral in (4.21) indicates that the parties to a conflict desires to optimize over the entire time period from t_0 to T. Equation (4.22) is commonly referred to as objective function and it describes the utility function of the parties to a conflict who are to make a decision that will optimize their tradeoffs.

We can therefore infer that a conflict problem has three parts:

a) an objective function of the form given in equation (4.21)

b) a state equation $\emptyset(t)$

c) a set of initial conditions $\theta(t)$ and possibly additional constraints on the values of variables and parameters through time or at the initial or terminal time points.

Now, suppose the state variables ϕ(t), the control variables θ(t) and time (t) comprise a complete and closed system, then an objective function of a dynamic conflict system can be described by a set of differential equations of the form in (1), i.e.,

$$\frac{dy}{dt} = F(\phi(t), \theta(t), t) \quad y(0) = \theta \tag{4.23}$$

where F is generally some linear objective function and $\frac{dy}{dt}$ is the rate of change in y.

Based on the work of Bellman, the Soviet mathematician Pontryagin showed the general problem defined by (4.23) and (4.22) could be solved by defining a function known as the Hamiltonian. In effect, the Hamiltonian function combines the objective function (4.22) and the state equations (4.23) through the use of special auxiliary variables, also functions of time. The auxiliary variables are known as the adjoint vector and are denoted here as *p(t)*.

The Hamiltonian function can be written in its general form as:

$$H(\phi(t), \theta(t), p(t), t) \tag{4.24}$$

Specifically, the Hamiltonian is:

$$H(\phi, \theta, p, t) = \lambda(\phi, \theta, t) + F.p'(\phi, \theta, t) \tag{4.25}$$

and p satisfies:

$$-\frac{dp}{dt} = dH \Big/ d\phi \tag{4.26}$$

It is important to note that the adjoint vector *p(t)* which allows for the combination of the objective function and state equations must satisfy the partial derivative in equation (4.26). The optimal control variable $\theta^{*}(t)$ can be obtained by observing that:

$$\frac{dH}{d\theta} = 0 . \tag{4.27}$$

In this approach, the procedure followed is to form the Hamiltonian function (4.24) and with equation (4.26) and (4.27) together with the initial conditions, solve for $\theta^*(t)$, the optimal control variable or optimal strategy. Having obtained the optimal strategy, it is then possible to substitute the expression for $\theta^*(t)$ into the state equation to obtain the optimal trajectory $\phi^*(t)$.

The approach of identifying the appropriate control variables dependent on time will influence the attainment of optimization and achievement of the desired objective in a conflict; conflict modelling and resolution. In section 4.3, a model for conflict prediction in a dynamic state through careful analysis of the control variable in a conflict is given.

4.3. The Conflict Prediction Model Components

In developing a model for conflict prediction, objective function, state condition (observable variable) and a set of initial conditions (control variables) including possible additional constraints on values of variables and parameters through time or at initial/terminal time points must be considered. Generally, analytical solutions or prediction can be found to conflict problems if the objective function is quadratic in form and the state equation is a linear system. Conflict problems which do not satisfy these conditions can on occasion be solved but the solutions are more complex and frequently require the use of numerical estimation techniques.

In this study, a model that uses the three components objective function, state condition (control variable) and a set of initial conditions to predict conflicts is developed.

4.3.1. Objective Function Model

The general objective function for the model is;

$$F(\emptyset(t), \theta(t), t) \tag{4.28}$$

with a target to optimize the function through $\theta^*(t)$ which is the estimation of $\theta(t)$ given by equation(3.38).

4.3.2. State Equation

According to Omwenga and Mwita (2010), a conflict with control variable ℓ_i,can be defined by a Bayes' probability distribution which is drawn independently and identically distributed (i.i.d) from a logistic distribution function $F_i(.)$ with a corresponding everywhere positive density $f_i(.)$, mean $\mu_i = 0$ and variance $\sigma_i^2 < \infty$ assuming that f_i's are continuously differentiable. Therefore, a conflict environment is defined by the logistic relationship;

$$\frac{dy}{dt} = \phi y\left(1 - \frac{y}{y_t}\right) \tag{4. 29}$$

whose solution gives the state equation

$$y(t) = \frac{y_t\theta}{\theta+(y_t-\theta)e^{-\emptyset t}} \tag{4.30}$$

4.3.3. Initial Conditions (Control Variable)

Initial conditions are vital in determining the trajectory of the equation (3.26); solution of the state equation. Further, control variables in a conflict setting are to large extent private information and they significantly influence the decisions made by the parties to the conflict and hence the direction (trajectory) a conflict takes. And therefore by using equation (3.38), the initial conditions in a dynamic system can be estimated as;

$$\theta^* = \frac{Lp(\theta)}{\int_N p(D/\theta)p(\theta)d\theta} \tag{4.31}$$

4.4. Conflict Prediction Model in a Dynamic State

In developing the conflict prediction model, we have considered purely linear and exponential functions of time. This is because they can be used for trend estimation as a solution to their corresponding time dynamics equations, i.e., equations used to describe how systems change or evolve over time.

From the state equation defined by equation (4.30), that is,

$$y(t) = \frac{y_t\theta}{\theta + (y_t - \theta)e^{-\emptyset t}}$$

with the estimates of θ given by (4.31), the conflict trend model can be given by:

$$y(t) = \frac{y_t\theta^*}{\theta^* + (y_t - \theta^*)e^{-\emptyset t}} \tag{4.32}$$

4.5. Application of the Model on Environmental Conflicts

According to Libiszewski (1992), an environmental conflict is caused by the environmental scarcity of the resource that means: caused by a human-made disturbance of ecosystem's normal regeneration rate. Environmental conflicts are therefore the result of anthropogenic activities that strain and damage the environment. If the activities exceed environmental thresholds, y, there is an increased probability of armed conflicts. Sprinz (1998) describes environmental thresholds as the states in which the functioning of natural systems changes fundamentally. They can be estimated as rations based on the current state and future capacities of the environment.

In applying the model it is assumed that the threshold, y_i and the state conditions $\phi(t)$ are known and generally have a marginal change on the overall model. They are therefore, assumed to be constants over time. Further, the application of the model to environmental conflict is depended on the conditions; that the conflict follows a Bernoulli distribution with parameter y_i defined by the indicator values given by:

$$Y_i = \begin{cases} 1, & \textit{if there is environmental conflict} \\ 0, & \textit{if there is peace.} \end{cases} \tag{4. 33}$$

The model will work effectively when the environmental control variables are identically independent distribution *(i.i.d)* defined by Bayes' probability distribution drawn from a logistic distribution $F_i(.)$ with a corresponding everywhere positive density $f_i(.)$, mean $\mu_i = 0$, variance $\sigma_i^2 < \infty$ and f_i's are continuously differentiable.

4.5. Conclusion

In this chapter, the dynamic time varying model for predicting the environmental conflict is developed, using Ultimatum game theory and Bayesian theory. The initial (state) conditions which play a significant role in the success of conflict modelling as given in chapter 3 are used in the conflict prediction model. An analogy on the application of the model in the modelling of environmentally-induced conflict is given.

In the model we have the ultimatum game theory which is in-built in the initial condition estimation model, we have provided a mechanism for the restrictions on the choices for the parties to the conflict and this has enabled us to express the outcomes as probability (weighted) of individual ideas. The restriction has the advantage of encapsulating most of the inherent optimality conditions in the Game theory. Through this approach we are able to estimate the likelihood of an occurrence of a conflict and make a prediction in a dynamic state.

The model developed is a prediction model for the trend and can be used to project on the anticipated outcomes considering the initial conditions and the state variables. In the context of determining environmentally-induced conflicts, environmental threshold values play a decisive role, since exceeding them is the sufficient condition for *environmentally-induced* armed conflicts.

Chapter Five

CONFLICT RESOLUTION

5.0. Introduction

To fully understand the bargaining problem and its contribution to conflict analysis, research has been conducted on the empirical relationship between substantive variables of interest, such as regime type, economic interdependence, institutional rules, legislative composition, and bargaining outcomes, McCarty and Poole (1995). However, lacking is an explicit model of the process that generates the empirical data, and leaving out the choice-based path by which these variables influence decisions. It is often the case that selection and omitted variable bias plague the analysis King et al (1994). In particular, Signorino (1999, 2002) demonstrates that traditional linear and categorical estimation techniques can lead to faulty inferences when the strategic data generating process is ignored during estimation.

Therefore, for effective bargaining there is need to integrate theoretical models and statistical methods Signorino et al (2006). A statistical tool that support theoretical consistent inferences about the relationship between substantive variables, the bargain struck, and the probability of bargaining failure is need. An Ultimatum bargaining games model which is a statistical model has been developed which address the substantive variables, the bargain struck and the probability of bargaining failure Signorino et al (2006).The model presents the relationship between the variables that affect the players' utilities and the outcomes of the bargaining in a strategic setting.

Using the Ultimatum bargaining model, we present a case of conflict in a social context where the contested opinions are seen as the regressor variable(s) and the outcome are seen as the dependent variable(s).

5.1. The Ultimatum Game in a Conflict Resolution

Suppose that a conflict is characterized by a contested prize that may be due to divergent opinion on how to share the prize. Assuming a scenario of two players in a bargaining arena as shown in Figure 2, where the two players must divide a contested prize, which is represented as Q. Let the prize $Q \subset \mathbb{R}_+$ be compact and convex, with lower and upper bounds $\underline{Q} < \bar{Q}$. Without loss of generality, rescale the bounds of the prize $\left[\underline{Q}, \bar{Q}\right] = \left[0, Q^*\right]$.

The game then proceed as follows: Player 1 first offers some division of the prize (Q*- y; y), where player 1's allocation is Q*-y and player 2's is y. Player 2 then decides whether to accept or reject player 1's offer. If player 2 accepts, they divide the prize according to player 1's offer. If player 2 rejects the offer, they receive some reservation amount, which may differ between the players.

Assuming each player's utility for bargaining failure has two components: one that is public knowledge and one that is private, then, we denote player 1's reservation value as $R_1 + \ell_1$ and player 2's as $R_2 + \ell_2$, where R_i is player i's publicly observable reservation value and ℓ_i is private information. We assume that nature draws the type ℓ_i of each player i from a well defined probability distribution.

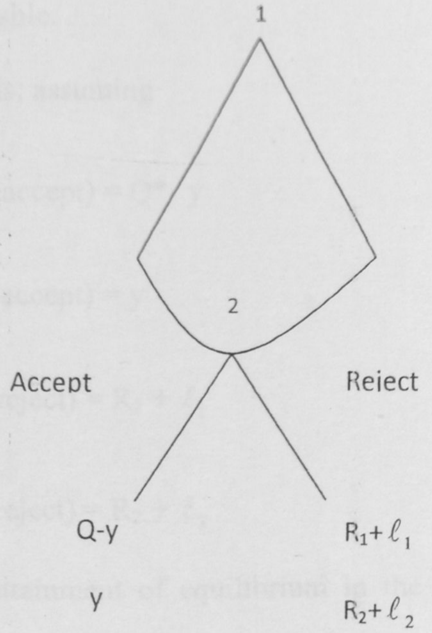

Figure 2: The Ultimatum Game Tree

Assuming that each player has a well defined *a-priori* beliefs about the distribution of these types and that each type is drawn identically and independently distributed (i.i.d.) from the cumulative distribution function $F_i(.)$, with a corresponding everywhere positive density $f_i(.)$, mean $\mu_i = 0$ and variance $\sigma_i^2 < \infty$. We also assume the f_i's are continuously differentiable.

Then, each player's strategy can be characterized by a mapping from types into actions: $\sigma_i : \ell_i \rightarrow A^i$, $i = \{1, 2\}$, where A^i defines the action set for player i. Since player 1 is making the ultimatum offer, $A^1 = \{y: y \in [0, Q^*]\}$. Player 2 is then left to accept or reject the offer, so $A^2 = \{accept; reject\}$.

If it is further assumed that both players' utilities are strictly increasing and continuous in their amount of the disputed good, and by the random utility structure, the public and private components of the players' utilities are additively separable.

That is; assuming

u_1 (y; accept) = Q*- y

u_2 (y; accept) = y

u_1(y; reject) = $R_1 + \ell_1$

u_2(y; reject) = $R_2 + \ell_2$

The attainment of equilibrium in the statistical Ultimatum game has player 1 making an offer from his strategic action profile that balances and maximizes the marginal utility thereby increasing the probability that an offer is accepted and the marginal utility of a larger amount of y is achieved. Player 2, knowing her own type, chooses the alternative that maximizes her utility. These conditions are summarized in the following theorem.

Theorem 5.1: Every conflict with a finite number of players N and finite number of action profiles A_i has Nash equilibrium.

Proof

Given a strategy profile $s \in S$ for all $i \in N$ and $a_i \in A_i$, where S is the set of strategy action profiles; then we define

$$\varphi_i a_i(s) = max\{0,\ u_i(a_i, s_{-i}) - u_i(s)\} \qquad (5.1)$$

where u_i is the expected utility function given by;

$$u_i(s) = \sum_{a \in A} u_i(a) \prod_{j=1}^{n} s_i(a_j) \tag{5.2}$$

We then define the function

$$f: s \to S \ by \ f(s) = s', \tag{5.3}$$

where

$$s_i'(a_i) = \frac{s_i(a_i)+\varphi_i a_i(s)}{\sum_{b_i \in A_I} s_i(b_i)+\varphi_i b_i(s)} \tag{5.4}$$

$$= \frac{s_i(a_i)+\varphi_i a_i(s)}{1+\sum_{b_i \in A_I} +\varphi_i b_i(s)}$$

By intuitive, the function maps a strategy profile s to a new strategy profile s' in which each agent's actions that are better responses to s receive increased probability mass.

The function f is continuous since each $\varphi_i a_i$ is continuous. Since S is convex and compact and $f: s \to S$ by Brouwer's fixed point theorem, f must have at least one fixed point.

We m ıst now show that the fixed points of f are the Nash equilibrium. First, if s is Nash equilibrium, then all $\varphi's$ are 0 making s a fixed point of f. Conversely, consider an arbitrary fixed point of s of f. By the linearity of expectation there must exist at least one action in the support of s, say a_i' for which

$$u_i a_i'(s) \leq u_i(s) \tag{5.5}$$

From the definition of φ in equation (5.1), we have $\varphi_i a_i'(s) = 0$.

Since s is a fixed point of f, $s_i'(a_i') = s_i(a_i')$.

Consider equation (5.4), the expression defining $s_i'(a_i')$, the numerator simplifies to $s_i'(a_i')$ and is positive since a_i' is in i's support. Hence, the denominator must be 1. Thus for any i and $b_i \in A_i$, $\varphi_i b_i(s)$ must be equal to 0. From the definition of φ, this can only occur when no player can improve his expected payoff by moving to a pure strategy. Therefore s is Nash equilibrium ■

5.1.1. Uniqueness of Equilibrium and its Existence in Ultimatum Game

In a game players will set strategies that map a random variable to their action space so as to win a game, such as a traditional Bayesian game or random utility model, the player's actions are however probabilistic rather than deterministic. Noting that a Nash equilibrium of a statistical Ultimatum bargaining game, where each player knows the other has random utilities, is equivalent to a perfect Bayesian Nash equilibrium of an underlying Bayesian game, where the types of the players are private information, we can use well-known game theoretic tools to begin to specify both our theoretical predictions and our empirical estimator. If the perfect Bayesian Nash equilibrium (PBE) of this underlying game can be shown to be unique, then we can solve for the equilibrium strategies and characterize an equilibrium probability distribution over observable outcomes. It is this characteristic of the Ultimatum model that will allow for its structural estimation.

Proposition 5.1. If F_{ℓ_2} is log-concave, then there exists a unique perfect Bayesian-Nash equilibrium to the statistical Ultimatum game.

Proof

Assuming player 1 has made an offer y, player 2 chooses between that offer and her reservation value $R_2 + \ell_2$. Player 2 will therefore reject the offer if and only if $y < R_2 + \ell_2$ .Generally in any equilibrium, player 2 plays the cut-point strategy:

$$S_2(y, \ell_2) = \begin{cases} accept\ if\ y \geq R2 + \ell_2 \\ reject\ if\ y < R2 + \ell_2 \end{cases} \tag{5.6}$$

Considering that

$$Pr(accept/y) = Pr(y > R_2 + \ell_2) = Pr(\ell_2 < y - R_2) \tag{5.7}$$

and

$$Pr(\ell_2 < y - R_2) = F_{\ell_2}(y - R_2) \tag{5.8}$$

Now assuming F_{ℓ_2} is log-concave and considering the optimization problem for player 1, given player 2's strategy, then his expected utility for an offer y is:

$$E_{U_1}(y, Q^*) = F_{\ell_2}(y - R_2) \cdot (Q^* - y) + \left(1 - F_{\ell_2}(y - R_2)\right) \cdot (R_1 + \ell_1) \tag{5.9}$$

Differentiation show that $E_{U_1}(y,)'$ is positive when,

$$0 < f_{\ell_2}(y - R_2) \cdot (Q^* - y) - F_{\ell_2}(y - R_2) - f_{\ell_2}(y - R_2)(R_1 + \ell_1) \tag{5.10}$$

which implies

$$\frac{f_{\ell_2}(y-R_2)}{F_{\ell_2}(y-R_2)} > \frac{1}{Q^*-y-R_1-\ell_1} \tag{5.11}$$

By inspection, the right hand side of the above equation is strictly increasing in y. Suppose equation (5.11) holds when $y = Q^*$, then equation (5.11) becomes;

$$\frac{f_{\ell_2}(Q^*-R_2)}{F_{\ell_2}(Q^*-R_2)} > \frac{1}{-R_1-\ell_1} \tag{5.12}$$

If we replace for $y < Q^*$, instead of $y = Q^*$, the left hand side becomes non-decreasing and the right hand side becomes strictly decreasing. Therefore, the derivative of $E_{U_1}(y)$is positive over the entire interval $[0, Q^*]$ and $y = Q^*$ is optimal offer when

$$\ell_1 < -R_1 - \frac{F_{\ell_2}(Q^*-R_2)}{f_{\ell_2}(Q^*-R_2)} \tag{5.13}$$

Similarly, differentiation shows that $E_{U_1}(y,)'$ is negative when,

$$0 > f_{\ell_2}(y - R_2) \cdot (Q^* - y) - F_{\ell_2}(y - R_2) - f_{\ell_2}(y - R_2)(R_1 + \ell_1) \tag{5.14}$$

Implying

$$\frac{f_{\ell_2}(y-R_2)}{F_{\ell_2}(y-R_2)} < \frac{1}{Q^*-y-R_1-\ell_1} \tag{5.15}$$

If we evaluate equation (5.15) at of $y = 0$, we get

$$\frac{f_{\ell_2}(-R_2)}{F_{\ell_2}(-R_2)} < \frac{1}{Q^*-R_1-\ell_1} \tag{5.16}$$

So, when we move from $y = 0$ to $y > 0$, the left hand side is non-increasing and the right hand side is strictly increasing. Therefore, the derivative of $E_{U_1}(y)$is negative over the entire interval $[0, Q^*]$ and $y = 0$ is optimal offer when;

$$\ell_1 > Q^* - R_1 - \frac{F_{\ell_2}(-R_2)}{f_{\ell_2}(-R_2)} \tag{5.17}$$

By examination of equation (5.13) and equation (5.17), it is clear that sometimes neither equation is satisfied. To demonstrate this, consider that;

$$-R_1 - \frac{F_{\ell_2}(Q^*-R_2)}{f_{\ell_2}(Q^*-R_2)} \leq Q^* - R_1 - \frac{F_{\ell_2}(-R_2)}{f_{\ell_2}(-R_2)} \tag{5.18}$$

Multiplying through by -1 and taking the inverse of each side, we get,

$$\frac{f_{\ell_2}(Q^*-R_2)}{F_{\ell_2}(Q^*-R_2)} \leq \frac{f_{\ell_2}(-R_2)}{F_{\ell_2}(-R_2)-Q^* f_{\ell_2}(-R_2)} \tag{5.19}$$

At $Q^* = 0$ the left hand side and right hand side are equal. However as Q^*increases the left hand side is non-increasing and the right hand is strictly increasing.

When reither (5.13) or (5.17) hold, then for some $y \in [0, Q^*]$, the derivative of $E_{U_1}(y)$ is zero, implying

$$\frac{f_{\ell_2}(Q^*-R_2)}{F_{\ell_2}(Q^*-R_2)} = \frac{1}{Q^*-y-R_1-\ell_1} \tag{5.20}$$

Since the left hand side is non-increasing on $[0, Q^*]$ and the right hand side is strictly increasing on the same interval, equation (5.20) can have at most one solution, say y^*, and note that it implicitly solves as;

$$y^* = Q^* - R_1 - \ell_1 - \frac{F_{\ell_2}(y^*-R_2)}{f_{\ell_2}(y^*-R_2)} \tag{5.21}$$

Since we have not assumed that $E_{U_1}(y)$ is concave, we must demonstrate that y^* maximizes player 1's expected utility. Obviously, since $E_{U_1}(y)$ is continuous for every ℓ_1 on the interval, the utility maximizing offer exists, and must be a critical point like y^* or a boundary point. If neither (5.13) nor (5.17) hold, then there are two cases. First, if one of the end points is the unique solution to equation (5.20) we are done. Second, if y^* is interior the derivative of $E_{U_1}(y)$ at $y = 0$ is positive and $E_{U_1}(y,)'$ at $y = Q^*$ is negative by (5.13) and (5.17). Thus, the interior critical point is a local and global maximum ■.

Now consider the optimization problem for player 1, given player 2's strategy and his expected utility function given by equation (5.9) whose solution is given by equation (5.21). Then by this optimal offer is the unique y* that implicitly solves to equation (3.37). The first party will therefore have a strategy defined by equation (3.33). Using these results we present in section 5.2 an empirical model using the ultimatum game theory to provide constraints on the strategies employed by each party in the resolution of a conflict.

5.2. Empirical Model of the Ultimatum Game

The application of the Ultimatum game in empirical analysis requires that a distribution for the ℓ_i, and the appropriate likelihood are specified given the dependent variable(s).

Assuming we have data on both player 1's and player 2's actions, that is, assume we can measure and code y and Q^* for each observation, as well as whether player 2 accepted or rejected the offer. Let the public portion of the players' reservation values be $R_1 = \beta X$ and $R_2 = \gamma Z$, where X and Z are sets of substantive regressors for player 1 and player 2 respectively. Our interest is in estimating the effects of X and Z on y and player 2's decision.

Since the outcome of the bargaining model consists of two dependent variables i.e. 1's offer and 2's decision, then the probability model is a joint density over these random variables. The estimator can be obtained by assuming that the types of players 1 and 2 are drawn i.i.d. from a logistic cumulative distribution function which is log-concave.

Proposition 5.2. If F_{ℓ_2} is logistic, then it is log-concave.

Proof

Suppose F_{ℓ_2} is a logistic distribution. Then it is everywhere positive and continuously differentiable on the open interval$(-\infty, +\infty)$. By calculus, we get,

$$\left(\ln F_{\ell_2}(x)\right)'' = \frac{-(\lambda_2)^2 e^{-\lambda_2 x}}{\left(1+e^{-\lambda_2 x}\right)^2} \leq 0 \tag{5.22}$$

and since a continuously differentiable function $f: \mathbb{R} \to \mathbb{R}^+$, is log-concave on an interval (a, b) if and only if $\left(\ln f_{\ell_2}(x)\right)'' \leq 0$, then the logistic cumulative distribution function is log-concave■.

Now considering player 2's decision with a logistic error term in the random utility equation, then the probability that player 2 accepts the offer y is just the logit probability given by;

$$P(accept/y) = \Lambda(y - Z\gamma). \tag{5. 23}$$

For player 1, logistic distribution of y^* is given by equation (3.36).

Whose probability density $f_{y^*}\left(y^* / \beta X, \gamma Z, Q^*\right)$ is given by;

$$f_{y^*}\left(y^*\right) = \frac{e^{\left(Q^*-1-\beta X-e^{\left(y^*-\gamma Z\right)}-y^*\right)}.\left(1+e^{y^*-\gamma Z}\right)}{\left(1+e^{\left(Q^*-1-\beta X-e^{\left(y^*-\gamma Z\right)}-y^*\right)}\right)^2} \tag{5. 24}$$

with the cumulative density function given by;

$$F_{y^*}\left(y^*\right) = \frac{1}{\left(1+e^{\left(Q^*-1-\left(e^{\left(y^*-\gamma Z\right)}+\beta X+y^*\right)\right)}\right)} \tag{5. 25}$$

The constraint on the action space of player 1, however, implies that the observed y^* is censored both from above and below.

Remark Suppose player 1 plays the strategy s_1 above, then the distribution of y^* is the truncated distribution of the unconstrained y^*, where the truncation points are from below at 0 and above at Q^*.

Take variables $\delta_k\ k \in \{0, y, 1\}$ such that $\delta_0 = 1$ if $y = 0$, $\delta_y = 1$ if $0 < y < Q^*$ and $\delta_1 = 1$ if $y = Q^*$. That is, a censored model with a "latent" best offer in the constraint set. Otherwise there is the best feasible offer, at, a boundary point.

Taking player 2's decision to be defined by;

$$decision = \begin{cases} \delta_{accept} = 1, \ if\ offer\ is\ accepted \\ \delta_{accept} = 0, \ if\ offer\ is\ rejected \end{cases}, \tag{5.26}$$

and assuming we have data on both player 1's and player 2's actions (i.e., y and δ_{accept}), then the likelihood would be;

$$L = \prod_{i=1}^{n} \Pr(y^* < 0)^{\delta_0} . \Pr(y^* = y)^{\delta_y} . (1 - \Pr(y^* < Q^*))^{\delta_1}\ X \Pr(accept)^{\delta_{accept}} . \Pr(reject)^{1-\delta_{accept}} \tag{5.27}$$

From equation (5.27), we can derive the log-likelihood function for our data in terms of distributions already derived, which are functions of our regressors, and which explicitly models the Ultimatum game.

5.3. Application of Bargaining Behaviour to Industrial Conflict Resolutions

The theory of bargaining is important due to its nature of cutting across the various disciplines. The concept has been employed in areas like multinational cooperation and states over terms of foreign investment, to the resolution of territorial disputes, to social issues in relationships.

At this point, however, we examine the application of the bargaining model to conflict resolution in a society with keen emphasis to its statistical interpretation. As discussed elsewhere in this chapter, there are basically two players to a game. This could be seen as parties to any dispute, i.e., the proponent of a given view and the oposer of the given view. Thus considering player 1 as the proponent of a given view and player 2 as the opposser to the ideas or views as presented or adduced by player 1, then we can apply the bargaining model to a conflict resolution set up.

For instance, in an industrial strike, player 1 may involve the employer and player 2 may involve the employees. If there is a conflict between the employer and the employees, one could expect that there exists grounds for some miss-understandings. If the issues are well defined, it is possible to quantify them or even model them. Suppose the conflict between the employer and the employee is on salaries. The employer may make an offer after taking into consideration a number of factors, e,g, economic factors, motivational factors e.t.c, let us take all these factors to be the variables. It is possible that among these factors there are those which are public and those which are private. Elsewhere in this thesis these factors have been identified as player's utility for bargaining failure and have been denoted by $R_i + \ell_i$ where R_i are the publicly known variables and ℓ_i are the private variables. Similarly, the employees will be making demands with both public and private variables. To avoid any conflict, if a demand is adduced by the employees to the employer, we propose a model that will give an employer an opportunity to make an offer that will be acceptable to the employees.

For illustration, let us take the variables that the employer will be taking into consideration in order to make any offer to be denoted by $\sum_i X$ and the variables possibly considered by employees in making a given demand to be denoted by $\sum_i Z$. Of critical concern is for the estimation of the effects of these variables on the outcome i.e. on y (employer's allocation or demand) and the decision on the employees (accept or reject an offer). We assume that the concerns of both parties are drawn i.i.d from a logistic distribution.

Let $R_1 = \beta \sum_i X$ be public reservation values for employer and $R_2 = \gamma \sum_i Z$ be the public reservation values for employees.

Then the probability that the employees accept the offer y is a logit probability

$$P(accept/y) = \Lambda(y - \gamma Z). \tag{5. 28}$$

But the optimal offer by the employer y^* will be given by

$$y^* = Q^* - \beta X - \ell_1 - \frac{\wedge\left(y^* - \gamma Z\right)}{\lambda\left(y^* - \gamma Z\right)} \tag{5. 29}$$

Solving for y^* will give the best offer so that the employees will accept the offer and a conflict will be settled which will present an equilibrium strategy for the employer.

5.4. Conclusion

In this chapter a statistical model for conflict resolution using the concept of bargaining game theory is developed. We make assumptions that in a conflict there are generally two parties with opposing opinions where one makes an offer with a probability of a higher value of acceptance otherwise it will be rejected and a conflict will escalate. The model gives estimators of the offers that are to be made in a conflict and it minimizes the chances of failure since any offer made is tied to the likelihood of it being accepted as it takes into considerations the demands from the other party to the conflict.

The logitic bargaining model developed can be used to address and mitigate failures in a conflict by enabling the parties to a conflict make reasonably acceptable offers and demands. The bargaining games can be applied to a number of situations to assist in solving a conflict.

Chapter Six

CONCLUSIONS AND RECOMMENDATIONS

The research dwelt on the study of mathematical modelling of conflicts in a dynamic state. The Game theory and Bayesian theorem are used as the underlying theoretical framework with assumptions that conflicts can structurally be described. The research outcome indicate that a conflict can be modelled in three levels; the instantiation level, the prediction level and the at the resolution level. The instantiation level establishes the initial conditions which can be considered as the trigger factors. The prediction level establishes the trend a given conflict will follow taking into account the initial conditions (control variables) and other interplaying variables like state variables. The Resolution stage establishes the appropriate conflict resolution mechanism through the Ultimatum Game Theory. The research has shown that the probability of occurrence of a conflict in a dynamic state can be estimated by the use of initial conditions (control variables). These conditions are estimated as *posteriori* conditions in the dynamic environment. The application of the model to analyze the probability of occurrence of an armed conflict given the *a-priori* conditions that existed in various countries prior to the years 2000, 2003 and 2004, gave accurate predictions compared to the actual occurrence of the conflicts in those countries as shown in *table 1*. These probabilities can therefore be used as the indicators of risk levels of a conflict occurrence given the prevailing conditions and prior conditions. A common probability threshold cannot however be generalised to apply to all cases.

Logistic model developed from the solution of an exponential model which integrates the initial condition estimation model is used as the conflict prediction model. Since the model integrates the initial conditions that are estimated as *posteriori* conditions, it fits well into a dynamic state and can be used to give a trend of a conflict. The application of this model in predicting the conflict trend assumes that the state variables Ø and threshold factor y_t will remain constant over time period t. We further assume that a conflict follows a Bernoulli distribution with parameter y_i. The logistic model developed can be used to predict student unrests in universities where the student issues are modelled as control variables (initial conditions), the state conditions being the institutional policies and procedures and the threshold being the level upon which both parties cannot engage in negotiations. By applying the model it is anticipated that the patterns exhibited by the predictor model will give an indication of the direction a given stand-off is likely to take.

The conflict resolution model developed has a constraining factor on the choices to be made by the parties to the conflict by use of the Ultimatum Game Theory. From the model, the inclusion of the constraint factors through Ultimatum Theory raises the minimum threshold for any demand to be made. This is a self assessment mechanism that limits the parties involved in a conflict to underlying issues to the conflict whereby the demands made are based on probabilities of acceptance rather than rejection. The model assumes that the contested prize is well defined and the variables in questions are public reservations and private information. We further assume that the parties to a conflict are drawn from an i.i.d logistic distribution. The model can be applied in solving industrial related problems whereby the demands or offer made by the union leaders or employers are optimally assessed using the model with an aim of establishing higher probability of acceptance than rejection before the demand is put forward. A lower probability value implies a higher degree of rejection and hence a high likelihood of failure to solve a conflict. The model can work well in solving a political problem considering the various warring parties as players having different demands. Through the model their demands are constrained to the overall aim of acceptability and hence solving of the problem.

Future extensions on the model for conflict prediction to remove the structural or distributional assumptions to take care of the ever changing conflict environment may be explored. Testing on the reliability of the estimates obtained by the initial conditions estimation model needs to be investigated.

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Appendix A

Data set Extract on conflicts from Conflict Data Project

Source: PRIO/Uppsala Conflict Data Project 2009,

http://www.prio.no/cwpArmedConflict

ID	Location	SideA	SideB	Terr	YEAR
92	Colombia	Colombia	FARC		1964
92	Colombia	Colombia	FARC		1965
92	Colombia	Colombia	FARC		1966
92	Colombia	Colombia	FARC		1967
92	Colombia	Colombia	FARC		1968
92	Colombia	Colombia	ELN		1969
92	Colombia	Colombia	ELN		1970
92	Colombia	Colombia	FARC		1971
92	Colombia	Colombia	FARC		1972
92	Colombia	Colombia	ELN		1973
92	Colombia	Colombia	FARC		1974
92	Colombia	Colombia	FARC		1975
92	Colombia	Colombia	FARC		1976
92	Colombia	Colombia	FARC		1977
92	Colombia	Colombia	FARC, M-19		1978
92	Colombia	Colombia	FARC		1979
92	Colombia	Colombia	FARC		1980
92	Colombia	Colombia	FARC, M-19		1981
92	Colombia	Colombia	FARC, M-19		1982
92	Colombia	Colombia	FARC, M-19		1983
92	Colombia	Colombia	ELN, FARC, M-19		1984
92	Colombia	Colombia	FARC, M-19		1985
92	Colombia	Colombia	ELN, FARC, M-19		1986
92	Colombia	Colombia	ELN, EPL, FARC		1987
92	Colombia	Colombia	ELN, EPL, FARC, M-19		1988
92	Colombia	Colombia	ELN, EPL, FARC		1989
92	Colombia	Colombia	ELN, FARC		1990
92	Colombia	Colombia	ELN, FARC		1991
92	Colombia	Colombia	ELN, FARC		1992
92	Colombia	Colombia	ELN, FARC		1993
92	Colombia	Colombia	ELN, FARC		1994
92	Colombia	Colombia	ELN, FARC		1995
92	Colombia	Colombia	ELN, FARC		1996
92	Colombia	Colombia	ELN, FARC		1997
92	Colombia	Colombia	ELN, FARC		1998
92	Colombia	Colombia	ELN, FARC		1999
92	Colombia	Colombia	ELN, FARC		2000
92	Colombia	Colombia	ELN, FARC		2001
92	Colombia	Colombia	ELN, FARC		2002

92	Colombia	Colombia	ELN, FARC		2003
92	Colombia	Colombia	ELN, EPL, FARC		2004
92	Colombia	Colombia	ELN, FARC		2005
92	Colombia	Colombia	ELN, FARC		2006
92	Colombia	Colombia	FARC		2007
92	Colombia	Colombia	ELN, FARC		2008
214	Congo	Congo	Cobras, Ninjas		1993
214	Congo	Congo	Ninjas		1994
214	Congo	Congo	Cobras, Cocoyes		1997
214	Congo	Congo	Cocoyes, Ninjas, Ntsiloulous		1998
214	Congo	Congo	Cocoyes, Ninjas, Ntsiloulous		1999
214	Congo	Congo	Ntsiloulous		2002
225	Cote D'Ivoire	Cote D'Ivoire	MJP, MPCI, MPIGO		2002
225	Cote D'Ivoire	Cote D'Ivoire	MJP, MPIGO		2003
225	Cote D'Ivoire	Cote D'Ivoire	FN		2004
260	Djibouti, Eritrea	Djibouti	Eritrea	Common border	2008
68	DRC	DRC	State of Katanga	Katanga	1960
68	DRC	DRC	State of Katanga	Katanga	1961
68	DRC	DRC	State of Katanga	Katanga	1962
69	DRC	DRC	South Kasaii	South Kasai	1960
69	DRC	DRC	South Kasaii	South Kasai	1961
69	DRC	DRC	South Kasaii	South Kasai	1962
86	DRC	DRC	CNL		1964
86	DRC	DRC	CNL		1965
86	DRC	DRC	Opposition militias		1967
86	DRC	DRC	FLNC		1977
86	DRC	DRC	FLNC		1978
86	DRC	DRC	AFDL		1996
86	DRC	DRC	AFDL		1997
86	DRC	DRC	MLC, RCD		1998
86	DRC	DRC	MLC, RCD, RCD–ML		1999
86	DRC	DRC	MLC, RCD, RCD–ML		2000
86	DRC	DRC	MLC, RCD, RCD–ML		2001
86	DRC	DRC	CNDP		2006
86	DRC	DRC	CNDP		2007
86	DRC	DRC	CNDP		2008
254	DRC	DRC	BDK	Kongo Kingdom	2007
254	DRC	DRC	BDK	Kongo Kingdom	2008
215	Eritrea, Ethiopia	Eritrea	Ethiopia	Badme	1998
215	Eritrea, Ethiopia	Eritrea	Ethiopia	Badme	1999
215	Eritrea, Ethiopia	Eritrea	Ethiopia	Badme	2000
70	Ethiopia	Ethiopia	Military faction		1960
70	Ethiopia	Ethiopia	EPRP, TPLF		1976
70	Ethiopia	Ethiopia	EDU, EPRP		1977
70	Ethiopia	Ethiopia	EDU, TPLF		1978
70	Ethiopia	Ethiopia	TPLF		1979
70	Ethiopia	Ethiopia	TPLF		1980

70	Ethiopia	Ethiopia	TPLF		1981
70	Ethiopia	Ethiopia	TPLF		1982
70	Ethiopia	Ethiopia	EPDM, TPLF		1983
70	Ethiopia	Ethiopia	EPDM, EPRP, TPLF		1984
70	Ethiopia	Ethiopia	EPDM, EPRP, TPLF		1985
70	Ethiopia	Ethiopia	EPRP, TPLF		1986
70	Ethiopia	Ethiopia	EPRP, TPLF		1987
70	Ethiopia	Ethiopia	TPLF		1988
70	Ethiopia	Ethiopia	EPRDF, Military faction		1989
70	Ethiopia	Ethiopia	EPRDF		1990
70	Ethiopia	Ethiopia	EPRDF		1991
78	Ethiopia	Ethiopia	ELF	Eritrea	1964
78	Ethiopia	Ethiopia	ELF	Eritrea	1965
78	Ethiopia	Ethiopia	ELF	Eritrea	1966
78	Ethiopia	Ethiopia	ELF	Eritrea	1967
78	Ethiopia	Ethiopia	ELF	Eritrea	1968
78	Ethiopia	Ethiopia	ELF	Eritrea	1969
78	Ethiopia	Ethiopia	ELF	Eritrea	1970
78	Ethiopia	Ethiopia	ELF	Eritrea	1971
78	Ethiopia	Ethiopia	ELF	Eritrea	1972
78	Ethiopia	Ethiopia	ELF, EPLF	Eritrea	1973
78	Ethiopia	Ethiopia	ELF	Eritrea	1974
78	Ethiopia	Ethiopia	ELF, EPLF	Eritrea	1975
78	Ethiopia	Ethiopia	ELF, EPLF	Eritrea	1976
78	Ethiopia	Ethiopia	ELF, ELF–PLF, EPLF	Eritrea	1977
78	Ethiopia	Ethiopia	ELF, EPLF	Eritrea	1978
78	Ethiopia	Ethiopia	ELF, EPLF	Eritrea	1979
78	Ethiopia	Ethiopia	ELF, EPLF	Eritrea	1980
78	Ethiopia	Ethiopia	EPLF	Eritrea	1981
78	Ethiopia	Ethiopia	EPLF	Eritrea	1982
78	Ethiopia	Ethiopia	EPLF	Eritrea	1983
78	Ethiopia	Ethiopia	EPLF	Eritrea	1984
78	Ethiopia	Ethiopia	EPLF	Eritrea	1985
78	Ethiopia	Ethiopia	EPLF	Eritrea	1986
78	Ethiopia	Ethiopia	EPLF	Eritrea	1987
78	Ethiopia	Ethiopia	EPLF	Eritrea	1988
78	Ethiopia	Ethiopia	EPLF	Eritrea	1989
78	Ethiopia	Ethiopia	EPLF	Eritrea	1990
78	Ethiopia	Ethiopia	EPLF	Eritrea	1991
133	Ethiopia	Ethiopia	WSLF	Ogaden	1976
133	Ethiopia	Ethiopia	WSLF	Ogaden	1977
133	Ethiopia	Ethiopia	WSLF	Ogaden	1978
133	Ethiopia	Ethiopia	WSLF	Ogaden	1979
133	Ethiopia	Ethiopia	WSLF	Ogaden	1980
133	Ethiopia	Ethiopia	WSLF	Ogaden	1981
133	Ethiopia	Ethiopia	WSLF	Ogaden	1982
133	Ethiopia	Ethiopia	WSLF	Ogaden	1983
133	Ethiopia	Ethiopia	ONLF	Ogaden	1994

133	Ethiopia	Ethiopia	ONLF	Ogaden	1996
133	Ethiopia	Ethiopia	ONLF	Ogaden	1999
133	Ethiopia	Ethiopia	ONLF	Ogaden	2000
133	Ethiopia	Ethiopia	ONLF	Ogaden	2001
133	Ethiopia	Ethiopia	ONLF	Ogaden	2002
133	Ethiopia	Ethiopia	ONLF	Ogaden	2004
133	Ethiopia	Ethiopia	ONLF	Ogaden	2005
133	Ethiopia	Ethiopia	ONLF	Ogaden	2006
133	Ethiopia	Ethiopia	ONLF	Ogaden	2007
133	Ethiopia	Ethiopia	ONLF	Ogaden	2008
168	Ethiopia	Ethiopia	ALF	Afar	1975
168	Ethiopia	Ethiopia	ALF	Afar	1976
168	Ethiopia	Ethiopia	ALF	Afar	1989
168	Ethiopia	Ethiopia	ALF	Afar	1990
168	Ethiopia	Ethiopia	ALF	Afar	1991
168	Ethiopia	Ethiopia	ARDUF	Afar	1996
211	Ethiopia	Ethiopia	al-Itahad al-Islami	Somali	1995
211	Ethiopia	Ethiopia	al-Itahad al-Islami	Somali	1996
211	Ethiopia	Ethiopia	al-Itahad al-Islami	Somali	1999
219	Ethiopia	Ethiopia	OLF	Oromiya	1977
219	Ethiopia	Ethiopia	OLF	Oromiya	1978
219	Ethiopia	Ethiopia	OLF	Oromiya	1980
219	Ethiopia	Ethiopia	OLF	Oromiya	1981
219	Ethiopia	Ethiopia	OLF	Oromiya	1983
219	Ethiopia	Ethiopia	OLF	Oromiya	1984
219	Ethiopia	Ethiopia	OLF	Oromiya	1985
219	Ethiopia	Ethiopia	OLF	Oromiya	1987
219	Ethiopia	Ethiopia	OLF	Oromiya	1988
219	Ethiopia	Ethiopia	OLF	Oromiya	1989
219	Ethiopia	Ethiopia	OLF	Oromiya	1990
219	Ethiopia	Ethiopia	OLF	Oromiya	1991
219	Ethiopia	Ethiopia	OLF	Oromiya	1992
219	Ethiopia	Ethiopia	OLF	Oromiya	1994
219	Ethiopia	Ethiopia	OLF	Oromiya	1995
219	Ethiopia	Ethiopia	OLF	Oromiya	1998
219	Ethiopia	Ethiopia	OLF	Oromiya	1999
219	Ethiopia	Ethiopia	OLF	Oromiya	2000
219	Ethiopia	Ethiopia	OLF	Oromiya	2001
219	Ethiopia	Ethiopia	OLF	Oromiya	2002
219	Ethiopia	Ethiopia	OLF	Oromiya	2003
219	Ethiopia	Ethiopia	OLF	Oromiya	2004
219	Ethiopia	Ethiopia	OLF	Oromiya	2005
219	Ethiopia	Ethiopia	OLF	Oromiya	2006
219	Ethiopia	Ethiopia	OLF	Oromiya	2007
219	Ethiopia	Ethiopia	OLF	Oromiya	2008
71	Ethiopia, Somalia	Ethiopia	Somalia	Ogaden	1960
71	Ethiopia, Somalia	Ethiopia	Somalia	Ogaden	1964
71	Ethiopia, Somalia	Ethiopia	Somalia	Ogaden	1973

71	Ethiopia, Somalia	Ethiopia	Somalia	Ogaden	1983
71	Ethiopia, Somalia	Ethiopia	Somalia	Ogaden	1987
29	India	India	CPI		1948
29	India	India	CPI		1949
29	India	India	CPI		1950
29	India	India	CPI		1951
29	India	India	CPI–ML		1969
29	India	India	CPI–ML		1970
29	India	India	CPI–ML		1971
29	India	India	PWG		1990
29	India	India	PWG		1991
29	India	India	MCC, PWG		1992
29	India	India	MCC, PWG		1993
29	India	India	PWG		1994
29	India	India	MCC, PWG		1996
29	India	India	PWG		1997
29	India	India	MCC, PWG		1998
29	India	India	MCC, PWG		1999
29	India	India	MCC, PWG		2000
29	India	India	MCC, PWG		2001
29	India	India	MCC, PWG		2002
29	India	India	MCC, PWG		2003
29	India	India	MCC, PWG		2004
29	India	India	CPI–M		2005
29	India	India	CPI–M		2006
29	India	India	CPI–M		2007
29	India	India	CPI–M		2008
54	India	India	NNC	Nagaland	1956
54	India	India	NNC	Nagaland	1957
54	India	India	NNC	Nagaland	1958
54	India	India	NNC	Nagaland	1959
54	India	India	NNC	Nagaland	1961
54	India	India	NNC	Nagaland	1962
54	India	India	NNC	Nagaland	1963
54	India	India	NNC	Nagaland	1964
54	India	India	NNC	Nagaland	1965
54	India	India	NNC	Nagaland	1966
54	India	India	NNC	Nagaland	1967
54	India	India	NNC	Nagaland	1968
54	India	India	NSCN – IM	Nagaland	1992
54	India	India	NSCN – IM	Nagaland	1993
54	India	India	NSCN – IM	Nagaland	1994
54	India	India	NSCN – IM	Nagaland	1995
54	India	India	NSCN – IM	Nagaland	1996
54	India	India	NSCN – IM	Nagaland	1997
54	India	India	NSCN – IM	Nagaland	2000
54	India	India	NSCN – K	Nagaland	2005
54	India	India	NSCN – K	Nagaland	2006

54	India	India	NSCN – K	Nagaland	2007
99	India	India	MNF	Mizoram	1966
99	India	India	MNF	Mizoram	1967
99	India	India	MNF	Mizoram	1968
139	India	India	TNV	Tripura	1978
139	India	India	TNV	Tripura	1979
139	India	India	TNV	Tripura	1980
139	India	India	TNV	Tripura	1981
139	India	India	TNV	Tripura	1982
139	India	India	TNV	Tripura	1983
139	India	India	TNV	Tripura	1984
139	India	India	TNV	Tripura	1985
139	India	India	TNV	Tripura	1986
139	India	India	TNV	Tripura	1987
139	India	India	TNV	Tripura	1988
139	India	India	ATTF	Tripura	1992
139	India	India	ATTF	Tripura	1993
139	India	India	NLFT	Tripura	1995
139	India	India	ATTF, NLFT	Tripura	1997
139	India	India	ATTF, NLFT	Tripura	1998
139	India	India	ATTF, NLFT	Tripura	1999
139	India	India	NLFT	Tripura	2000
139	India	India	NLFT	Tripura	2001
139	India	India	NLFT	Tripura	2002
139	India	India	NLFT	Tripura	2003
139	India	India	NLFT	Tripura	2004
152	India	India	NLFT	Tripura	2006
152	India	India	PLA	Manipur	1982
152	India	India	PLA	Manipur	1983
152	India	India	PLA	Manipur	1984
152	India	India	PLA	Manipur	1985
152	India	India	PLA	Manipur	1986
152	India	India	PLA	Manipur	1987
152	India	India	PLA	Manipur	1988
152	India	India	PLA	Manipur	1992
152	India	India	PLA	Manipur	1993
152	India	India	UNLF	Manipur	1994
152	India	India	PLA	Manipur	1995
152	India	India	PLA	Manipur	1996
152	India	India	KNF, UNLF	Manipur	1997
152	India	India	PLA	Manipur	1998
152	India	India	UNLF	Manipur	1999
152	India	India	PLA	Manipur	2000
152	India	India	UNLF	Manipur	2003
152	India	India	PLA, UNLF	Manipur	2004
152	India	India	UNLF	Manipur	2005
152	India	India	PLA, UNLF	Manipur	2006
152	India	India	UNLF	Manipur	2007

152	India	India	KCP, PREPAK	Manipur	2008
156	India	India	Sikh insurgents	Punjab/Khalistan	1983
156	India	India	Sikh insurgents	Punjab/Khalistan	1984
156	India	India	Sikh insurgents	Punjab/Khalistan	1985
156	India	India	Sikh insurgents	Punjab/Khalistan	1986
156	India	India	Sikh insurgents	Punjab/Khalistan	1987
156	India	India	Sikh insurgents	Punjab/Khalistan	1988
156	India	India	Sikh insurgents	Punjab/Khalistan	1989
156	India	India	Sikh insurgents	Punjab/Khalistan	1990
156	India	India	Sikh insurgents	Punjab/Khalistan	1991
156	India	India	Sikh insurgents	Punjab/Khalistan	1992
156	India	India	Sikh insurgents	Punjab/Khalistan	1993
169	India	India	Kashmir Insurgents	Kashmir	1989
169	India	India	Kashmir Insurgents	Kashmir	1990
169	India	India	Kashmir Insurgents	Kashmir	1991
169	India	India	Kashmir Insurgents	Kashmir	1992
169	India	India	Kashmir Insurgents	Kashmir	1993
169	India	India	Kashmir Insurgents	Kashmir	1994
169	India	India	Kashmir Insurgents	Kashmir	1995
169	India	India	Kashmir Insurgents	Kashmir	1996
169	India	India	Kashmir Insurgents	Kashmir	1997
169	India	India	Kashmir Insurgents	Kashmir	1998
169	India	India	Kashmir Insurgents	Kashmir	1999
169	India	India	Kashmir Insurgents	Kashmir	2000
169	India	India	Kashmir Insurgents	Kashmir	2001
169	India	India	Kashmir Insurgents	Kashmir	2002
169	India	India	Kashmir Insurgents	Kashmir	2003
169	India	India	Kashmir Insurgents	Kashmir	2004
169	India	India	Kashmir Insurgents	Kashmir	2005
169	India	India	Kashmir Insurgents	Kashmir	2006
169	India	India	Kashmir Insurgents	Kashmir	2007
170	India	India	Kashmir Insurgents	Kashmir	2008
170	India	India	ULFA	Assam	1990
170	India	India	ULFA	Assam	1991
170	India	India	ULFA	Assam	1994
170	India	India	ULFA	Assam	1995
170	India	India	ULFA	Assam	1996
170	India	India	ULFA	Assam	1997
170	India	India	ULFA	Assam	1998
170	India	India	ULFA	Assam	1999
170	India	India	ULFA	Assam	2000
170	India	India	ULFA	Assam	2001
170	India	India	ULFA	Assam	2002
170	India	India	ULFA	Assam	2003
170	India	India	ULFA	Assam	2004
170	India	India	ULFA	Assam	2005
170	India	India	ULFA	Assam	2006
		India	ULFA	Assam	2007

170	India	India	ULFA	Assam	2008
227	India	India	ABSU	Bodoland	1989
227	India	India	ABSU	Bodoland	1990
227	India	India	NDFB	Bodoland	1993
227	India	India	NDFB	Bodoland	1994
227	India	India	NDFB	Bodoland	1995
227	India	India	NDFB	Bodoland	1996
227	India	India	NDFB	Bodoland	1997
227	India	India	NDFB	Bodoland	1998
227	India	India	NDFB	Bodoland	1999
227	India	India	NDFB	Bodoland	2000
227	India	India	NDFB	Bodoland	2001
227	India	India	NDFB	Bodoland	2002
227	India	India	NDFB	Bodoland	2003
227	India	India	NDFB	Bodoland	2004
258	India	India	DHD – BW	Dimaraji	2008
259	India	India	PULF	Islamic State	2008
20	India, Pakistan	India	Pakistan	Kashmir	1948
20	India, Pakistan	India	Pakistan	Kashmir	1964
20	India, Pakistan	India	Pakistan	Kashmir	1965
20	India, Pakistan	India	Pakistan	Kashmir	1971
20	India, Pakistan	India	Pakistan	Kashmir	1984
20	India, Pakistan	India	Pakistan	Kashmir	1987
20	India, Pakistan	India	Pakistan	Kashmir	1989
20	India, Pakistan	India	Pakistan	Kashmir	1990
20	India, Pakistan	India	Pakistan	Kashmir	1991
20	India, Pakistan	India	Pakistan	Kashmir	1992
20	India, Pakistan	India	Pakistan	Kashmir	1996
20	India, Pakistan	India	Pakistan	Kashmir	1997
20	India, Pakistan	India	Pakistan	Kashmir	1998
20	India, Pakistan	India	Pakistan	Kashmir	1999
20	India, Pakistan	India	Pakistan	Kashmir	2000
20	India, Pakistan	India	Pakistan	Kashmir	2001
20	India, Pakistan	India	Pakistan	Kashmir	2002
20	India, Pakistan	India	Pakistan	Kashmir	2003
46	Indonesia	Indonesia	Darul Islam		1953
46	Indonesia	Indonesia	PRRI		1958
46	Indonesia	Indonesia	PRRI		1959
46	Indonesia	Indonesia	PRRI		1960
46	Indonesia	Indonesia	PRRI		1961
94	Indonesia	Indonesia	OPM	West Papua	1965
94	Indonesia	Indonesia	OPM	West Papua	1967
94	Indonesia	Indonesia	OPM	West Papua	1968
94	Indonesia	Indonesia	OPM	West Papua	1969
94	Indonesia	Indonesia	OPM	West Papua	1976
94	Indonesia	Indonesia	OPM	West Papua	1977
94	Indonesia	Indonesia	OPM	West Papua	1978
134	Indonesia	Indonesia	Fretilin	East Timor	1975

134	Indonesia	Indonesia	Fretilin	East Timor	1976
134	Indonesia	Indonesia	Fretilin	East Timor	1977
134	Indonesia	Indonesia	Fretilin	East Timor	1978
134	Indonesia	Indonesia	Fretilin	East Timor	1979
134	Indonesia	Indonesia	Fretilin	East Timor	1980
134	Indonesia	Indonesia	Fretilin	East Timor	1981
134	Indonesia	Indonesia	Fretilin	East Timor	1982
134	Indonesia	Indonesia	Fretilin	East Timor	1983
134	Indonesia	Indonesia	Fretilin	East Timor	1984
134	Indonesia	Indonesia	Fretilin	East Timor	1985
134	Indonesia	Indonesia	Fretilin	East Timor	1986
134	Indonesia	Indonesia	Fretilin	East Timor	1987
134	Indonesia	Indonesia	Fretilin	East Timor	1988
134	Indonesia	Indonesia	Fretilin	East Timor	1989
134	Indonesia	Indonesia	Fretilin	East Timor	1992
134	Indonesia	Indonesia	Fretilin	East Timor	1997
134	Indonesia	Indonesia	Fretilin	East Timor	1998
171	Indonesia	Indonesia	GAM	Aceh	1990
171	Indonesia	Indonesia	GAM	Aceh	1991
171	Indonesia	Indonesia	GAM	Aceh	1999
171	Indonesia	Indonesia	GAM	Aceh	2000
171	Indonesia	Indonesia	GAM	Aceh	2001
171	Indonesia	Indonesia	GAM	Aceh	2002
171	Indonesia	Indonesia	GAM	Aceh	2003
171	Indonesia	Indonesia	GAM	Aceh	2004
171	Indonesia	Indonesia	GAM	Aceh	2005
177	Mali	Mali	MPA	Azawad	1990
177	Mali	Mali	FIAA	Azawad	1994
177	Mali	Mali	ATNMC	Azawad	2007
177	Mali	Mali	ATNMC	Azawad	2008
72	Nepal	Nepal	Nepali Congress		1960
72	Nepal	Nepal	Nepali Congress		1961
72	Nepal	Nepal	Nepali Congress		1962
72	Nepal	Nepal	CPN–M		1996
72	Nepal	Nepal	CPN–M		1997
72	Nepal	Nepal	CPN–M		1998
72	Nepal	Nepal	CPN–M		1999
72	Nepal	Nepal	CPN–M		2000
72	Nepal	Nepal	CPN–M		2001
72	Nepal	Nepal	CPN–M		2002
72	Nepal	Nepal	CPN–M		2003
72	Nepal	Nepal	CPN–M		2004
72	Nepal	Nepal	CPN–M		2005
72	Nepal	Nepal	CPN–M		2006
178	Niger	Niger	CRA	Air and Azawad	1994
212	Niger	Niger	FDR	Eastern Niger	1996
212	Niger	Niger	FARS	Eastern Niger	1997
255	Niger	Niger	FLAA		1991

255	Niger	Niger	FLAA		1992
255	Niger	Niger	UFRA		1997
255	Niger	Niger	MNJ		2007
255	Niger	Niger	MNJ		2008
249	Nigeria	Nigeria	Ahlul Sunnah Jamaa	Northern Nigeria	2004
250	Nigeria	Nigeria	NDPVF	Niger Delta	2004
210	Cameroon, Nigeria	Cameroon	Nigeria	Bakassi	1996
154	Chad, Nigeria	Chad	Nigeria	Lake Chad	1983
129	Pakistan	Pakistan	Baluchi insurgents	Baluchistan	1974
129	Pakistan	Pakistan	Baluchi insurgents	Baluchistan	1975
129	Pakistan	Pakistan	Baluchi insurgents	Baluchistan	1976
129	Pakistan	Pakistan	Baluchi insurgents	Baluchistan	1977
129	Pakistan	Pakistan	BLA	Baluchistan	2004
129	Pakistan	Pakistan	Baluch Ittehad	Baluchistan	2005
129	Pakistan	Pakistan	Baluch Ittehad, BLA	Baluchistan	2006
129	Pakistan	Pakistan	BLA	Baluchistan	2007
129	Pakistan	Pakistan	BLA, BRA	Baluchistan	2008
209	Pakistan	Pakistan	MQM		1990
209	Pakistan	Pakistan	MQM		1995
209	Pakistan	Pakistan	MQM		1996
209	Pakistan	Pakistan	TNSM		2007
209	Pakistan	Pakistan	TTP		2008
95	Peru	Peru	ELN, MIR		1965
95	Peru	Peru	Sendero Luminoso		1982
95	Peru	Peru	Sendero Luminoso		1983
95	Peru	Peru	Sendero Luminoso		1984
95	Peru	Peru	Sendero Luminoso		1985
95	Peru	Peru	Sendero Luminoso		1986
95	Peru	Peru	Sendero Luminoso		1987
95	Peru	Peru	Sendero Luminoso		1988
95	Peru	Peru	MRTA, Sendero Luminoso		1989
95	Peru	Peru	Sendero Luminoso		1990
95	Peru	Peru	MRTA, Sendero Luminoso		1991
95	Peru	Peru	MRTA, Sendero Luminoso		1992
95	Peru	Peru	MRTA, Sendero Luminoso		1993
95	Peru	Peru	Sendero Luminoso		1994
95	Peru	Peru	Sendero Luminoso		1995
95	Peru	Peru	Sendero Luminoso		1996
95	Peru	Peru	Sendero Luminoso		1997
95	Peru	Peru	Sendero Luminoso		1998
95	Peru	Peru	Sendero Luminoso		1999
95	Peru	Peru	Sendero Luminoso		2007
95	Peru	Peru	Sendero Luminoso		2008
43	Thailand	Thailand	Military faction (Navy)		1951
43	Thailand	Thailand	CPT		1974
43	Thailand	Thailand	CPT		1975
43	Thailand	Thailand	CPT		1976
43	Thailand	Thailand	CPT		1977

43	Thailand	Thailand	CPT		1978
43	Thailand	Thailand	CPT		1979
43	Thailand	Thailand	CPT		1980
43	Thailand	Thailand	CPT		1981
43	Thailand	Thailand	CPT		1982
148	Tunisia	Tunisia	Résistance Armée Tunisienne		1980
159	Turkey	Turkey	PKK	Kurdistan	1984
159	Turkey	Turkey	PKK	Kurdistan	1985
159	Turkey	Turkey	PKK	Kurdistan	1986
159	Turkey	Turkey	PKK	Kurdistan	1987
159	Turkey	Turkey	PKK	Kurdistan	1988
159	Turkey	Turkey	PKK	Kurdistan	1989
159	Turkey	Turkey	PKK	Kurdistan	1990
159	Turkey	Turkey	PKK	Kurdistan	1991
159	Turkey	Turkey	PKK	Kurdistan	1992
159	Turkey	Turkey	PKK	Kurdistan	1993
159	Turkey	Turkey	PKK	Kurdistan	1994
159	Turkey	Turkey	PKK	Kurdistan	1995
159	Turkey	Turkey	PKK	Kurdistan	1996
159	Turkey	Turkey	PKK	Kurdistan	1997
159	Turkey	Turkey	PKK	Kurdistan	1998
159	Turkey	Turkey	PKK	Kurdistan	1999
159	Turkey	Turkey	PKK	Kurdistan	2000
159	Turkey	Turkey	PKK	Kurdistan	2001
159	Turkey	Turkey	PKK	Kurdistan	2002
159	Turkey	Turkey	PKK	Kurdistan	2003
159	Turkey	Turkey	PKK	Kurdistan	2004
159	Turkey	Turkey	PKK	Kurdistan	2005
159	Turkey	Turkey	PKK	Kurdistan	2006
159	Turkey	Turkey	PKK	Kurdistan	2007
159	Turkey	Turkey	PKK	Kurdistan	2008
188	Turkey	Turkey	Devrimci Sol		1991
188	Turkey	Turkey	Devrimci Sol		1992
188	Turkey	Turkey	MKP		2005