UNIVERSITY OF NAIROBI

SCHOOL OF MATHEMATICS

"ACTUARIAL VALUATION OF TEMPERATURE DERIVATIVES"

By

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A Thesis presented to the School of Mathematics in partial fulfillment of the requirements of the degree of Masters of Science in Actuarial Science.

DECLARATION

I hereby declare that this is my original work and it has not been presented for examination for award of degree in any other University

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DEDICATION

I dedicate this project to my parents Julius Kabubi, Faith Njoroge and my sisters Florence and Hellen.

ABSTRACT

Weather affects our daily lives as well as choices. We define the term weather derivative. It is a new class of investment that is yet to gain ground in Africa since the underlying security (weather) is not a trade able asset. In our study we look at 6 different pricing methods for temperature based derivatives. We settle on the one proposed by Alaton and incorporate one of his suggestions, that is, allowing for temperature volatility to be a stochastic process rather than some piecewise constant function. Finally, we use the actuarial method of valuation and find out that the option price greatly depends on our value of the strike price. We conclude that allowing for the mean reverting parameter to also be a stochastic function will greatly improve our option price.

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CHAPTER ONE: INTRODUCTION

Weather derivatives are financial instruments that are used as risk management tools to hedge against losses and volatility of profits due to unfavorable weather. The class of weather derivatives includes weather options, weather futures and forward contracts, weather swaps and weather linked bonds.

Weather derivatives are usually defined by the following characteristics the:

1. The contract period: a starting date and a finishing date
2. A measurement station
3. A weather variable, measured at the measurement station, over the contract period;
4. An index, which collects the weather variable over the contract period in some way
5. A pay-off function, which converts the index into the cash flow that settles the derivative shortly after the end of the contract period.
6. For some kinds of contract, a premium paid from the buyer to the seller at the start of the contract. (Jewson, S. & Brix, A, 2005)

The Weather derivatives market is relatively young. The first transaction in this market took place in the US in 1997(Considine, 1999). It was executed between Aquila Energy and Consolidated Edison. In its few years of existence the market has recorded an impressive growth. The weather derivatives market grew by 20% in 2010-2011 to $11.8 billion. (Weather Risk Management Association, 2011). In comparison to other derivatives market, this amount seems small. Nonetheless, the strong growth highlights the realization that we can no longer afford to ignore weather risk. Currently, the countries that trade in weather derivatives are US, Japan, Canada, Norway and Sweden. In Africa, we only have South Africa and Morocco as the only two countries that actively participate in the weather derivatives market.

The potential impacts of adverse weather include loss of life and livelihoods, destruction of property and infrastructure and stagnation in terms of economic development due to lost revenue from Agriculture, Tourism and other related industries. In Kenya, Tourism, Agricultural and Forestry activities account for more than 30% of our GDP (Kenya National Bureau of Statistics, 2009). These adverse weather events however are not exclusive to Kenya. They are a global phenomenon. Take the US for example. The US Department of Energy estimates that a seventh of its economy is affected by weather (John C. Hull, 2008). Travis L. Jones (2007) noted that the onset of the El Nino events of 1997 made companies in the US realize the importance of hedging their seasonal weather risk.

Kenya experiences various types of extreme weather events. These include flooding, droughts, landslides (mudslides) and thunderstorms. Muthama *et al* (2002) outlined the adverse effects that floods and drought have on Kenya's agricultural sector, water sectors, and horticultural and tourism industry. Persistent crop failure being experienced by most farmers throughout the country is mainly attributable to reduced rainfall and increased temperature. Hydro power generation is now proving an unreliable source of energy due to erratic rainfall. Wildlife and Tourism is also under threat from these poor weather conditions. From this, it is somewhat clear that our economy is largely weather dependent.

Weather derivatives are not the same as weather insurance. Though both try to eliminate weather risk, weather derivatives cover low risk high probability events whereas weather insurance covers high risk, low probability events. For example, a farmers Sacco in Nyeri may use a weather derivative contract to hedge against temperatures that meteorologists predict may be 3° Celsius higher than expected. They may also opt to buy a weather insurance policy to safeguard against any losses they may incur due to flooding. It is quite plausible for Nyeri to experience dry weather (a high probability event) but not flooding (a low probability event). Other differences between derivatives and insurance may include the frequency of revaluation of the contracts, tax liabilities involved and other contractual details.

Several weather variables are used as the underlying 'asset' of the weather derivative. Among these are temperature, wind, precipitation, snow and fog. The most commonly used variable is temperature and the indices mostly used for temperature based contracts are degree day (DD) indices, average temperature indices, cumulative average temperature indices and event indices.

Various models have been proposed by different authors to describe the evolution of temperature with time. Previous literature point to two methodologies used in constructing the temperature models. The first approach is discrete and favors the use of a time-series model. The second approach is continuous. The proponents prefer to use a continuous financial process and then discretize it. This study is interested in comparing some of the different temperature forecasting models proposed for pricing weather derivatives. To assist in our endeavor, we shall use temperature data from the weather station in Kisumu.

The following is an outline of the rest of the study. The second chapter is a literature review of some six different temperature models that have been proposed by various authors. We discuss these models and the conclusions drawn about them. Chapter three is divided into 2 sections. In section 1, we state the model we are going to use to describe the temperature process. We also state the methods we will use to estimate our parameter values and simulate the temperature dynamics. Section 2 dwells on the model set-up. This gives a detailed analysis of how we come up with our temperature model and we summarize the assumptions we make throughout. In the fourth chapter we reveal our parameter values and compare our derivative prices. Finally, we give our conclusions and recommendations based on our findings in the fifth chapter.

1.1 STATEMENT OF THE PROBLEM

A lot of research has been done on the pricing of weather derivatives. The common denominator in these researches is the lack of an effective pricing model. The weather derivatives market lacks a universal starting point like the Black Scholes model for the option markets. Though many models have been proposed, no consensus has been reached as to which model is acceptable across the market. The result of this delay has been a large bid/ask spread (Cao and Wei, 2003). The difficulty in pricing weather derivatives is largely due to the fact that the weather market is incomplete and the underlying weather variable is not a trade-able asset. Dichel, B. (1998a), Mark Garman et al (2000) and Campbell and Diebold (2002) give reasons why the Black-Scholes model is inappropriate. The lack of an effective pricing method has restricted further growth of the market. Further compounding the problem, major weather derivatives market player are reluctant to share the models they have developed with other participants due to the strong financial incentive to keep it secret. Market players in the industry do not communicate in a common language.

Insofar as air temperature is the underlying weather variable, prior studies have set about constructing temperature models using two methods: The time series approach and the financial approach. This research intends to construct a temperature model using one of the approaches and analyze their strengths and/or weaknesses in valuing weather derivatives. Of particular interest will be finding out if the parameter values of our model are significant. The weather derivative to be evaluated using this model will be a generic call option. Our underlying weather variable will be daily minimum and maximum temperature measurements from the Kisumu weather station covering a period of 12 years.

1.2 OBJECTIVES OF THE STUDY

1. Compare the various temperature-based stochastic models that have been constructed and used in evaluating weather derivatives.

2. Construct a model that best describes the temperature dynamics in Kenya.

3. Find out if our model can accurately price weather options by comparing it with the Black Scholes model.

1.3 SIGNIFICANCE OF THE STUDY

1. Meteorologists and financial players/regulators in the market will appreciate the need to work together to mitigate weather risk.
2. Spur interest in Kenya on the benefits of using weather derivatives as risk management tools.

CHAPTER TWO: LITERATURE REVIEW

In this chapter, we review in detail some of the proposed models that have been used to describe temperature dynamics. Previous studies have shown that there are two distinct modelling approaches. The first approach relies on the time series approach. This approach has been used by Cao and Wei (2001), Campbell & Diebold (2002) and Roustant et al (2003). Others include Caballero, Jewson & Brix (2002) and Caballero & Jewson (2003). The second approach incorporates the statistical features of temperature into financial diffusion processes. Dichel, B (1998a, 1998b) pioneered this, basing his study on interest rate derivative models developed by Hull and White, (1990). Alaton, Djehiche, and Stillberger (2002), and Benth and Šaltytė-Benth (2005, 2007) also preferred to first use the financial approach, and then discretize it into a time series model.

Cao and Wei (2001) stated the seven desirable features a daily temperature model should possess. These features were: its ability to capture seasonal cyclic patterns and incorporate the autoregressive property in temperature change. The other feature was that the daily variations in temperature should be around some average normal temperature. Forecasting should play a key role in projecting temperature paths in the future and that this projected temperature path should be bound within the normal range of the temperature for each future projection. The final aspect of the model is that it should reflect the global warming trend. Whilst agreeing that a mean – reverting diffusion process can accommodate most of these features, Cao and Wei (2001) chose to model temperature using the time series approach. They rationalized that a one factor diffusion process could not incorporate autocorrelations in temperature advances for lags beyond one. They proposed to use a discrete autoregressive model. They define the de-meaned and de-trended residual of the daily temperature as $\boldsymbol{U_{yr,t}}$

$$\boldsymbol{U_{yr,t}} = \boldsymbol{Y_{yr,t}} - \boldsymbol{\breve{Y}_{yr,t}}\ \forall \boldsymbol{yr} = \mathbf{1, 2, \dots n}\ \&\ t = 1,2, \dots 365 \qquad \mathbf{(2.1)}$$

where $\boldsymbol{Y_{yr,t}}$ represents the temperature on date $\boldsymbol{t}$ **(*t*=1, 2, 3....365)** in year ***yr*** **(*yr*=1, 2...*n*)** and $\boldsymbol{\breve{Y}_{yr,t}}$ is the adjusted average temperature.

$(\bar{Y}_t)$ (below), denotes the mean temperature and ψ_t the standard deviation on date t where

$$\bar{Y}_t = \frac{1}{n}\sum_{yr=1}^{n} Y_{yr,t} \quad \& \psi_t = \sqrt{\frac{1}{n}\sum_{yr=1}^{n}\left(Y_{yr,t}-\bar{Y}_t\right)^2}, \ \forall t = 1,2,\dots 365 \qquad (2.2)$$

Cao and Wei (2001) assume the temperature residual $U_{yr,t}$ follows a k-lag autocorrelation system of the form:

$$U_{yr,t} = \sum_{i=1}^{k} \rho_i U_{yr,t-i} + \sigma_{yr,t} \cdot \xi_{yr,t} \qquad (2.3)$$

$$\sigma_{yr,t} = \sigma_0 - \sigma_1 \left|\sin\left(\pi t/365 + \phi\right)\right| \qquad (2.4)$$

$$\xi_{yr,t} \sim i.i.d\ N(0,1)$$

$$\forall\ yr = 1,2 \dots n\ \&\ t = 1,2,3 \dots .365$$

where $\xi_{yr,t}$ models the randomness in temperature change and ρ_i is the ACF at lag i $\forall$ i= *1,2....k*

The data used covered 20 years and the historical temperatures were recorded in the cities of Atlanta, Chicago, Dallas, New York and Philadelphia. The parameter ϕ captures the proper starting point of the sine wave. The daily temperature volatility specification using the sine wave reflects the aspect that the extent of variation must be bigger in the winter and smaller in the summer. In order to determine ***k***, the number of lags, Cao and Wei (2001) estimate the system sequentially for ***k =1, 2*** ....and perform maximum likelihood ratio tests ***(χ^2tests)***. They ceased at ***k=3***, i.e when the maximum likelihood value ceases to improve.

Cao & Wei (2001) proposed the implementation of their model because it allowed for easy estimation (the maximum likelihood method) and incorporated the key features of the daily temperature dynamics such as seasonal cycles and uneven variations throughout the year.

The dataset Campbell and Diebold (2002) used exhibited seasonality. They noted that a seasonal component would be essential in any time series model fit to daily average temperature. In order to model the seasonality, they opted to use a low-ordered, Fourier series as opposed to daily dummies. There were two main advantages of doing this. Firstly, the use of a low-ordered Fourier series produced a smooth seasonal pattern unlike the discontinuous pattern that had been proposed by Cao & Wei (2001). Secondly, the Fourier approximation greatly reduced the number of parameters that were to be estimated, thus significantly reducing computing time and

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Campbell & Diebold (2002) proposed the following model:

$$T_{yr,t} = \beta_0 + \beta_1 d_t + \sum_{p=1}^{P} \left(\delta_{c,p} \cos\left(\frac{2\pi p}{365} t\right) + \delta_{s,p} \sin\left(\frac{2\pi p}{365} t\right)\right)$$

$$+ \sum_{k=1}^{K} \rho_k T_{yrt-k} + \sigma_{yrt} \epsilon_{yrt} \qquad (2.5)$$

$$\sigma^2_{yr,t} = \sum_{q=1}^{Q} \left(\gamma_{c,q} \cos\left(\frac{2\pi q}{365} t\right) + \gamma_{s,q} \sin\left(\frac{2\pi q}{365} t\right)\right) + \sum_{r=1}^{R} \alpha_r \epsilon^2_{yr,t-r}) \qquad (2.6)$$

$$\epsilon_{yrt} \sim i.i.d\, N(0,1)$$

This model was estimated by the ordinary least squares, regressing average temperature on constant trend, Fourier and lagged average temperature terms, using twenty-five autoregressive lags *(K=25)* and three Fourier sine and cosine terms ***(P=3)***. They used the Akaike and Schwarz information criteria to set ***K=25*** and ***P=3***.

Campbell & Diebold (2002) further stated that the addition of the conditional variance equation *(2.6)* allowed for two types of volatility dynamics that were relevant in time-series contexts. These are the volatility seasonality and autoregressive effects in the conditional variance movements. They proposed to approximate volatility seasonality in the conditional variance equation through a Fourier series of order **Q**. On the enduring effects of shocks to the conditional variance, the authors explain that this is accommodated by incorporating $\boldsymbol{R}$ autoregressive lags of squared residual following Engle (1982) (ARCH models). Just as before, the parameters of Q and $\boldsymbol{R}$ are chosen in the same way as those of $\boldsymbol{K}$ and $\boldsymbol{P}$. The optimum values for Q and $\boldsymbol{R}$ were 2 and 1.

By using Engle's (1982) two-step approach, they estimated the model in the following way. Campbell and Diebold (2002) first estimated the conditional mean equation by ordinary least squares, regressing average temperature on constant, trend, Fourier and lagged average temperature terms. They then proceeded to estimate the variance equation by regressing the squared residuals from the conditional mean equation on constant, Fourier and lagged squared residual terms and they used the square root of the inverse fitted values $\widehat{\sigma_{yrt}^{\frac{1}{2}}}$ as weights in a weighted least squares re-estimation of the conditional mean equation.

Roustant et al (2003) chose a simple ARMA model that would capture the major characteristics of temperature: seasonality of the values, seasonality of the dispersion, quick reversion to the mean and correlations between today's temperature and tomorrow. The proposed model was:

$$X_t - m_t + s_t + \sigma_t . Z_t \qquad (2.7)$$

Where:

- m_t represents the trend;
- s_t is the seasonal component;
- σ_t is a deterministic and periodic process with an annual periodicity representing the standard deviation of ; we assume that
- Z_t is an ARMA process with variance 1:

$$Z_t = \phi_1 . Z_{t-1} + \cdots + \phi_p Z_{t-p} + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \cdots + \theta_q \varepsilon_{t-q} \qquad (2.8)$$

where (ε_t) is a Gaussian white noise. Furthermore, we assume that:

$$m_t = d.t + e \tag{2.9}$$

$$s_t = \sum_{i=1}^{N_f} (a_i \cos(i.\omega.t) + b_i \sin(i.\omega.t)) \tag{2.10}$$

$$\sigma_t = a + b\cos(\omega.t) + c\sin(\omega.t) \tag{2.11}$$

$$s_t = \sum_{i=1}^{2} (a_i \cos(i.\omega.t) + b_i \sin(i.\omega.t)) \tag{2.12}$$

with $\omega = {}^{2\pi}/_{365}$

This allowed for easy computation of the maximum likelihood estimator. The choice of frequencies for the seasonal component is achieved by means of a preliminary spectral analysis of the normal temperature of each series. For the Paris data, the seasonal component retained the form:

$$s_t = \sum_{i=1}^{2} (a_i \cos(i.\omega.t) + b_i \sin(i.\omega.t)) \tag{2.12}$$

The orders of the ***ARMA***, *p=3* and *q=0*, were selected after the estimation of m_t, s_t and σ_t.

Roustant et al (2003) proceeded to compute the price of a weather derivatives contract using the parameters defined in the temperature model. They observed that the model had defects. The model showed great uncertainty towards option prices. They concluded that the trend and seasonality parameters were responsible for the uncertainty. To improve on the existing models, the authors suggested that new models should allow for the modelling of trend and seasonality.

Alaton et al (2002) used the financial approach. They used historical data from Bromma Airport in Sweden covering 40years to build a stochastic process that described temperature movements. Starting at $T_s = x$ the model they put forward was:

$$dT_t = \left\{\frac{dT_t^m}{dt} + a(T_t^m - T_t)\right\}dt + \sigma_t dW_t, \quad t > s \tag{2.13}$$

whose solution is

$$T_t = (x - T_s^m)e^{-a(t-s)} + T_t^m + \int_s^t e^{-a(t-\tau)}\sigma_\tau dW_\tau, \qquad (2.14)$$

where

$$T_t^m = A + Bt + C\sin(\omega t - \varphi) \qquad (2.15)$$

and

$$\frac{dT_t^m}{dt} = B + \omega C\cos(\omega t + \varphi) \qquad (2.16)$$

Their model assumes that σ_t is a piecewise constant function with a constant value each month. The use of the Weiner term in the model is to underscore the fact that temperature is not deterministic.

The Ornstein-Uhlenbeck process is a solution to the above Stochastic Differential Equation. This mean-reverting process was proposed by Dornier and Queruel (2000). This temperature model fitted quite well into their dataset. They however admit that their model was a simplification of the real world and suggested the use of a model; sophisticated enough to capture the driving noise process. The other improvements they proposed were the inclusion of stochastic volatility process in the model and the use of larger models where temperature would be one of many different variables.

Benth and Šaltytė-Benth (2005) proposed the following model:

$$dT(t) = ds(t) + \kappa(T(t) - s(t))dt + \sigma(t)dL(t) \qquad (2.17)$$

where

$s(t) = A + Bt + C\sin(\omega t + \phi)$ describes the mean seasonal variation.

κ (a constant) is the speed at which the temperature reverts to its mean

$L(t)$ is the Lévy noise

$T(t)$ is the temperature at time $0 \leq t < \infty$

Applying Ito's formula leads to the following explicit solution to *(2.17)*:

$$T(t) = s(t) + (T(0) - s(0))e^{\kappa t} + \int_0^t \sigma(u)e^{\kappa(t-u)}dL(u) \qquad (2.18)$$

As they pointed out, the difference between their model and the one suggested by Dornier & Queruel (2000) and Alaton et al. (2002) was that their model included a Lévy noise rather than Brownian motion. In addition, Benth and Šaltytė-Benth (2005) criticized Alaton et al. (2002) for not providing normality tests to justify the use of the Weiner process as the driving noise in the Ornstein- Uhlenbeck process. Benth and Šaltytė-Benth (2005) argument for using the class of generalized hyperbolic Lévy processes was; it enabled them to capture the semi-heavy tails and skewness observed on Norwegian temperature data and also allowed for modelling of the dynamics of the squared residuals from daily observed temperature variances. They also claimed that a simple regression model for the deseasonalized temperature in conjunction with a time-dependent variance function σ_t could better explain the fractionality observed for the Norwegian temperature data rather than introducing a fractional Brownian motion. Benth and Šaltytė-Benth (2005) proceed to derive a time–discrete version of *(2.17)* which they use to analyze their Norwegian temperature data. The model is:

$$T_t - s_t = (1+\kappa)(T_{t-1} - s_{t-1}) + \sigma_t \varepsilon_t, t = 1,2 \ldots. \qquad (2.19)$$

They reconstituted *(2.19)* into an additive time series of the following form:

$$T_t = s_t + c_t + \check{\varepsilon}_t, t = 0, 1, 2, \ldots \ldots \qquad (2.20)$$

$$s_t = a_0 + a_1 \cos\left(\frac{2\pi}{365}(t - t_0)\right) \qquad (2.21)$$

$$c_t = \alpha(T_{t-1} - S_{t-1}) \text{ where } \alpha = 1 + \kappa \qquad (2.22)$$

$$\check{\varepsilon}_t - \sigma_t \varepsilon_t \qquad (2.23)$$

s_t denotes the seasonality component, c_t the cyclical component and the residual.

To estimate the parameters of the seasonality component, Benth and Šaltytė-Benth (2005) used the *nlinfit* function in MATLAB. We model the cyclical component by regressing today's deseasonalized temperature against the deseasonalized temperature recorded the previous day. The residuals were estimated using a multiplicative time series model.

They concluded that their model could not reveal why the residual noise (from the daily average temperature) had a positive correlation at lag 1 and a negative correlation at lag 2. They suggested the use of either a moving-average time series model or a GARCH- model to explain this effect.

Benth and Šaltytė-Benth (2005) followed up their study with another publication in 2007. They proposed an Ornstein-Uhlenbeck model to show the evolution of temperature with time:

$$dT(t) = ds(t) + \kappa(T(t) - s(t))dt + \sigma(t)dB(t) \qquad (2.24)$$

Where ***T (t)*** is the daily average temperature, ***B(t)*** is a standard Brownian motion, ***s(t)*** is a deterministic function modelling the trend and seasonality of daily average temperature and $\sigma(t)$ is the daily volatility of temperature variations. They use the Brownian motion instead of a Levy process to drive the temperature dynamics because their main interest was a model where analytical pricing is possible. The explicit solution for the model is given by Ito's formula (Tomas Björk, 2003)

$$T(t) = s(t) + (T(0) - s(0))e^{\kappa t} + \int_0^t \sigma(u)e^{\kappa(t-u)}dB(u) \qquad (2.25)$$

In the temperature model, both ***s (t)*** and $\sigma^2(t)$ are modeled as a truncated Fourier series

$$s(t) = a + bt + a_0 + \sum_{i=1}^{I_1} a_i \sin\left(\frac{2i\pi(t - f_i)}{365}\right) + \sum_{j=1}^{J_1} b_j \cos\left(\frac{2j\pi(t - g_j)}{365}\right) \qquad (2.26)$$

$$\sigma^2(t) = c + \sum_{i=1}^{I_2} c_i \sin\left(\frac{2i\pi t}{365}\right) + \sum_{j=1}^{J_2} d_j \cos\left(\frac{2j\pi t}{365}\right) \qquad (2.27)$$

They discretize the model and use the following time series model to analyze Swedish temperature data:

$$\tilde{T}_{t+1} = \alpha \tilde{T}_t + \tilde{\sigma}(t)e_t \qquad (2.28)$$

where:

$\varepsilon_t \sim iid\ N(0,1)$

$\alpha = e^{-\kappa}$

$\tilde{\sigma}(t) = \alpha\sigma(t)$

Benth and Šaltytė-Benth (2007) concluded that though the model was simple, it was powerful enough to describe seasonality and mean-reversion in temperature data. The simplicity of their model allowed for explicit calculation of futures prices for HDD/CDD, CAT and PRIM futures quoted on the Chicago Mercantile Exchange.

In the next chapter we construct a temperature model using the financial approach in the same way as has previously been done by Alaton et al. (2002). We choose his model because of its simplicity. However, as an improvement to his original model, we assume temperature volatility to be a stochastic process rather than some piecewise function with constant volatility every month. We suppose that this will make the model more accurate and mathematically tractable.

CHAPTER THREE: METHODOLOGY

SECTION 1: METHODOLOGY

We have a dataset comprising of 12 years of temperature data from the Kisumu Meteorological Station. Our dataset has 12 missing observations. (around 0.3% of the total dataset). We replace the missing values by averaging the preceding 14 consecutive days. We use the Root Mean Squared Error (RMSE) statistic to test the efficacy of the method described above.

$$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(T_{obs} - T_{pred}\right)^2} \tag{3.1}$$

Once we have filled in our missing values, we proceed to develop the temperature model, similar to Alaton's model. The continuous equation for the temperature process is given as:

$$dT_t = a(\theta_t - T_t)dt + \gamma_t dW_t \tag{3.2}$$

The equation for the temperature seasonality is given by

$$\theta_t = A + Bt + C\sin(\omega t + \varphi) \tag{3.3}$$

We use the ordinary least squares (OLS) regression methods to estimate the parameters (**A, B, C & φ**) of seasonality. To estimate the mean reversion parameter a, we use the martingale estimation functions method of Bibby and Sørensen (1995):

$$\hat{a} = -\log\left(\frac{\sum_{i=1}^{n}\frac{(T_{i-1}-\theta_{i-1})(T_i-\theta_i)}{\gamma_{i-1}^2}}{\sum_{i=1}^{n}\frac{(T_{i-1}-\theta_{i-1})^2}{\gamma_{i-1}^2}}\right) \tag{3.4}$$

We assume that volatility is also a mean reverting process given by:

$$d\gamma_t = a_\gamma(\gamma_{trend} - \gamma_t)dt + \sigma_\gamma dW_t \tag{3.5}$$

We estimate a_γ and σ_γ using the formula below:

$$\hat{\sigma}_\gamma^2 = \frac{1}{N_\mu}\sum_{j=0}^{N_\mu - 1}\left(\gamma_{j+1} - \gamma_j\right)^2 \tag{3.6}$$

$$a_\gamma = -\log\left(\frac{\sum_{i=1}^{n}\frac{(\gamma_{trend}-\gamma_{t-1})(\gamma_t-\gamma_{trend})}{\sigma_\gamma^2}}{\sum_{i=1}^{n}\frac{(\gamma_{trend}-\gamma_{t-1})(\gamma_{t-1}-\gamma_{trend})}{\sigma_\gamma^2}}\right) \quad (3.7)$$

In order to simulate the temperature process and the volatility, we use Euler's method to obtain the following discrete sets of equations:

$$T_{t+1} = T_t + a(\theta_t - T_t) + \theta'_t + \gamma_n Z_1 \quad (3.8)$$

$$\gamma_n = \gamma_{n-1} + a_\gamma(\gamma_{trend} - \gamma_{n-1}) + \sigma_\gamma Z_2 \quad (3.9)$$

where $Z_1, Z_2 \sim N(0,1)$ and θ'_t is the differentiated parameter for seasonality.

We also make the following assumptions in constructing the temperature model

1. The missing values in our dataset have no significant impact on our parameter values.
2. The temperature process follows a predictable pattern around some seasonal mean. (Mean reverting property).
3. The temperature seasonality follows some sinusoidal function.
4. The daily temperature deviations are normally distributed & temperature dynamics are driven by some Brownian motion.
5. The volatility is also a mean reverting stochastic process.
6. There are no leap years in our dataset, i.e. we omit February 29th temperature values so that we have 365 temperature readings annually.

SECTION 2: MODEL SET-UP

The purpose of this thesis is:

- To find a stochastic model that describes temperature movements.
- To use the model above to evaluate weather derivatives (options).

We begin by defining some terminologies with regards to weather derivatives. Then we perform quality tests for our dataset and also state the attributes of a generic call option. We finish this section by showing how to construct a temperature model, estimate its parameters and use it to evaluate a call option price.

DEFINITIONS

The dataset we have collected includes temperature observations at the Kisumu Weather station. These observations are the daily minimum and maximum temperatures (in degrees Celsius).

Definition 3.1: *The average daily temperature for a specified weather station is defined as the average of the daily minimum and the daily maximum in day* i.

$$T_i = \frac{T_i^{max} + T_i^{min}}{2} \qquad (3.1.1)$$

where T_i^{max} *denotes the maximum daily temperature in day* i *and* T_i^{min} *denotes the minimum daily temperature in day* i.

Definition 3.2: *A Cooling Degree Day (CDD) is measured in the summer and is defined by the quantity*

$$CDD_i = max\{T_i - T_{ref}, 0\} \qquad (3.1.2)$$

Definition 3.3: *A Heating Degree Day (HDD) is measured in the winter and is defined by the quantity*

$$HDD_i = max\{T_{ref} - T_i, 0\} \qquad (3.1.3)$$

Where T_{ref} *is the reference temperature.*

It is the industry standard in the US to set the reference temperature at 65° Fahrenheit (or 18° Celsius). This is because when the temperature drops below 18°C, people use more energy to heat their homes and when the temperature rises above 18°C, people start using their air conditioners for cooling. (Alaton et al, 2002).

A generic weather option can be expressed if the following parameters are specified:

- The contract type (call or put)
- The contract period
- The underlying index (HDD or CDD)
- An official weather station from which the temperature data are obtained
- The strike level
- The tick size
- The maximum payout (if there is any). (Alaton et al, 2002).

To formulate the payout of an option, let K denote the strike level and λ the tick size. The tick size refers to the amount of money that a writer of a weather derivative is exposed to for each unit of the underlying variable (in our case it is the temperature) that is in the money. Let the contract period consist of n days. Then the number of HDDs and CDDs for that period is given by:

$$H_n = \sum_{i=1}^{n} HDD_i, \qquad and \qquad C_n = \sum_{i=1}^{n} CDD_i \qquad (3.1.4)$$

Now the payout function of an uncapped HDD option is:

$$\chi = \lambda * max\{H_n - K, 0\} \qquad (3.1.5)$$

Our sample period extends from 1st January 1991 to 31st December 2002 resulting in a total of 4380 observations. We have some missing observations in our data series which we have simply replaced with the average temperature of the consecutive 14 days preceding our missing value. We employ this naïve approach because our missing observations are 12 (less than 0.3% of our total observations). We can safely assume that this will not significantly alter our findings.

The table below shows the root mean squared error (RMSE) of some random observations taken during the year of our missing values and replaced by the method we've described above:

Date	Observed Value	Predicted Value	Deviation	Squared deviations
7/29/1992	27.40	28.10	-0.70	0.49
3/12/1993	29.50	31.40	-1.90	3.61
7/28/1993	30.80	29.40	1.40	1.96
6/20/1994	26.80	27.30	-0.50	0.25
4/3/1997	29.40	32.10	-2.70	7.29
8/31/1997	32.20	31.00	1.20	1.44
2/23/1998	30.90	28.80	2.10	4.41
8/18/1998	30.50	30.40	0.10	0.01
11/8/1998	29.00	29.50	-0.50	0.25
1/30/1999	33.20	29.90	3.30	10.89
5/22/2001	28.20	28.90	-0.70	0.49
9/15/2002	31.50	31.20	0.30	0.09

Table 3.1: RMSE

RMSE=1.611934655

Where the RMSE is given by $= \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(T_{obs} - T_{pred}\right)^2}$. Though the standard deviation of 1.62°C is significant, given the few number of missing observations in our data, its impact is negligible.

Now, we proceed to explain how we construct the temperature model.

3.1 CONSTRUCTION OF THE TEMPERATURE MODEL

We begin by plotting the evolution of daily temperature against time. As we can observe, the historic data shows some cyclical nature. The seasonal variation is between *39°C* and *23°C*. Interesting to note is that unlike previous studies, our data suggests a very weak cooling trendline. Using EXCEL, we are able to evaluate and find out that our trendline equation is:

$y = -0.00006x + 29.96$

where y represents the average temperature at a particular date and x the date of the observation Our model should recognize that over time, temperature tends to get drawn back to some average level. We assume therefore that the temperature process is a mean reverting process. Our temperature data seems to be oscillating around some average mean level as can be observed in **figure 1.**

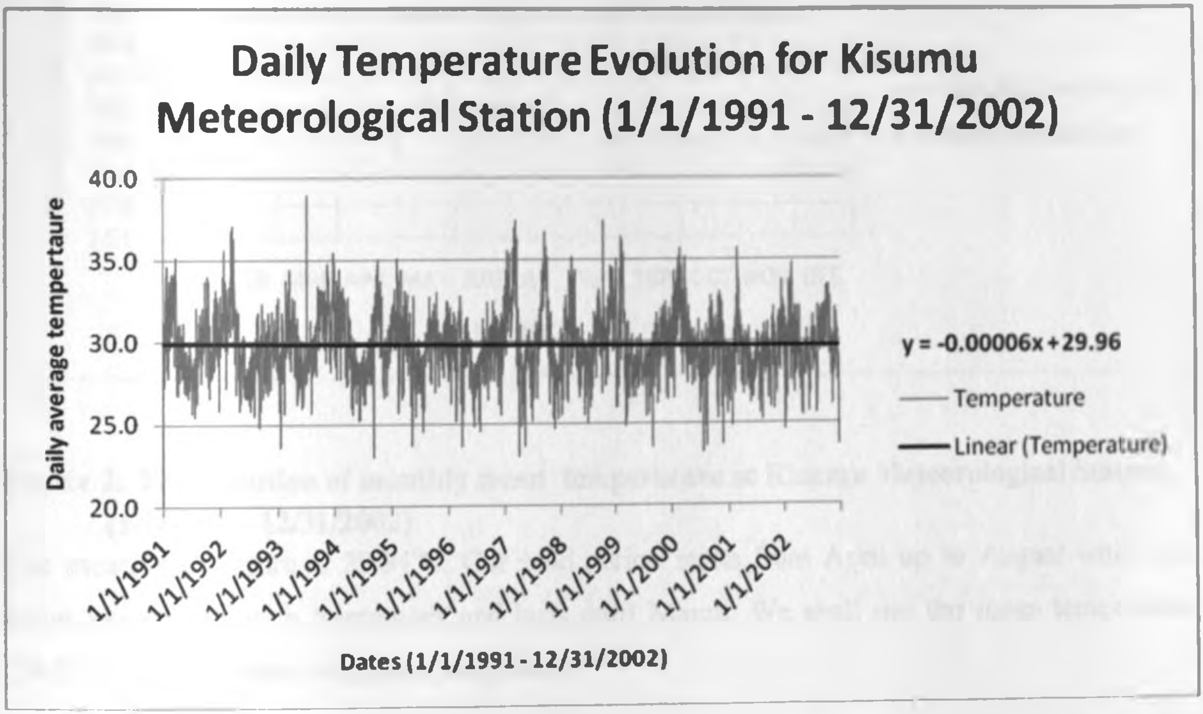

FIGURE 1: Daily evolution of temperature at Kisumu meteorological station (1/1/1991 – 12/31/2002)

In order to create a HDD and CDD Index, we need to determine the monthly mean temperature for the total observations and classify our weather into warm periods and cold periods. With the aid of figure 2, we can visually identify these periods.

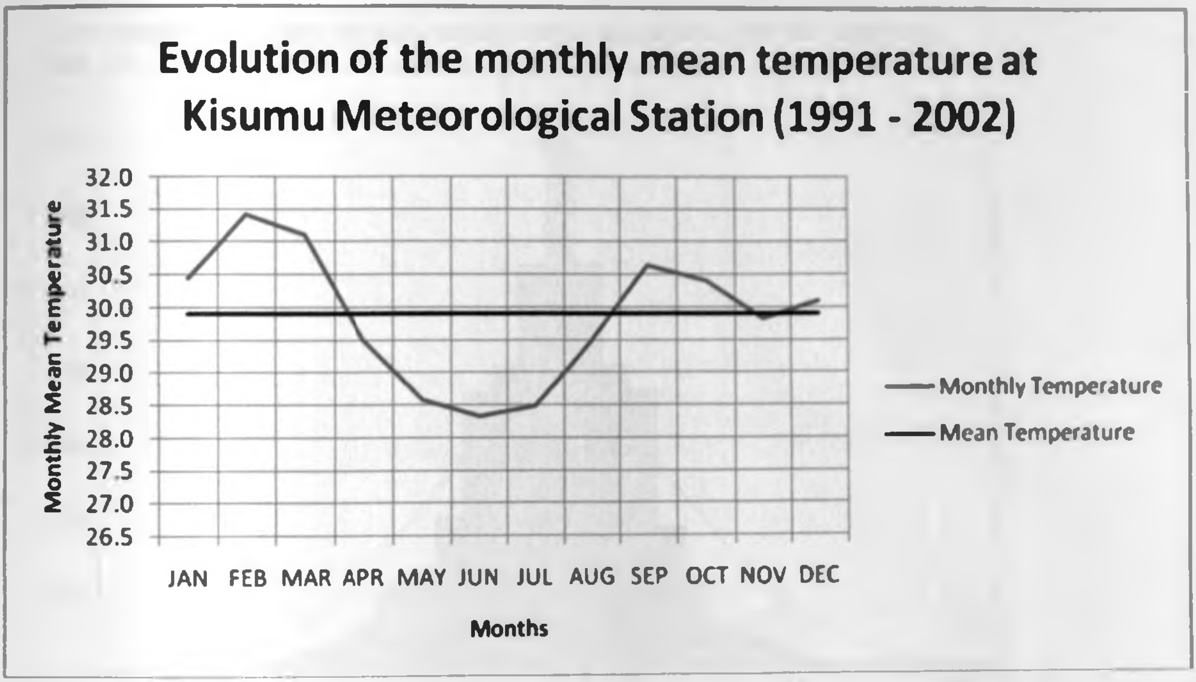

Figure 2: The evolution of monthly mean temperature at Kisumu Meteorological Station (1/1/1991 – 12/31/2002)

The mean temperature is 29.94°C. Our cold period starts from April up to August while our warm period begins in September and lasts until March. We shall use the mean temperature (29.9°C) as our standard reference temperature.

We now plot a histogram of the frequency (number of observations) against the daily temperature deviations. We then use STATISTICA to fit a normal distribution curve to the histogram.

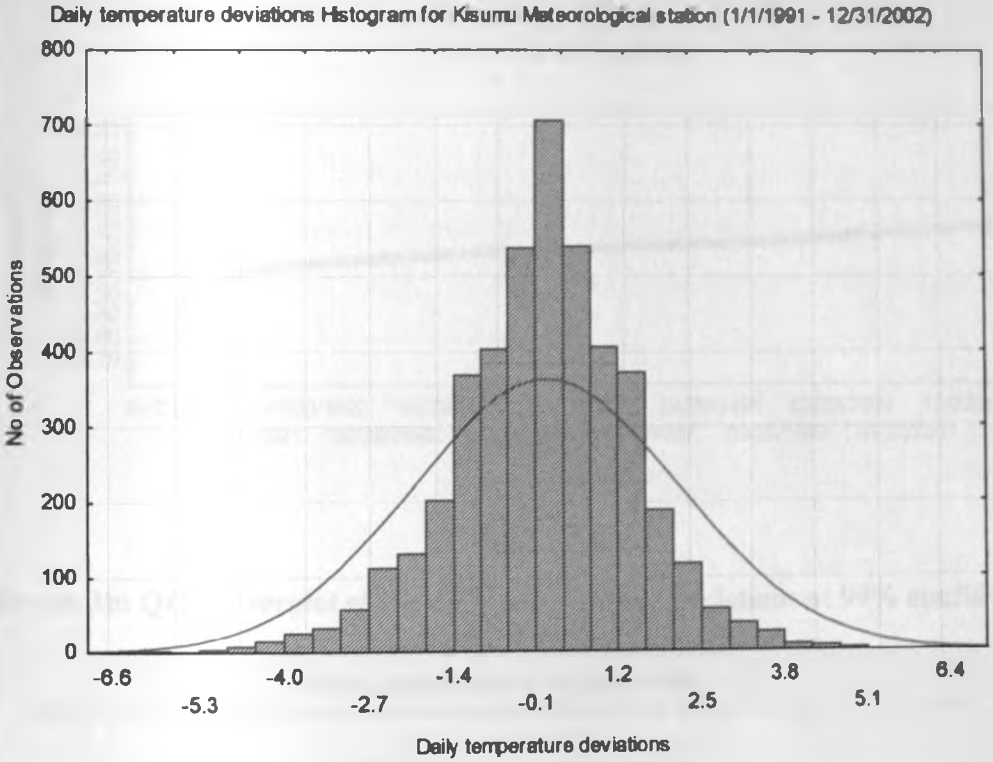

Figure 3: Daily temperature deviations Histogram for Kisumu Meteorological Station (1/1/1991 – 12/31/2002)

The histogram plotted indicates some form of normal distribution driven by some noise. To further investigate the normality assumption, we carry out a QQ test. This test will help us figure out if the temperature deviations follow a normal distribution.

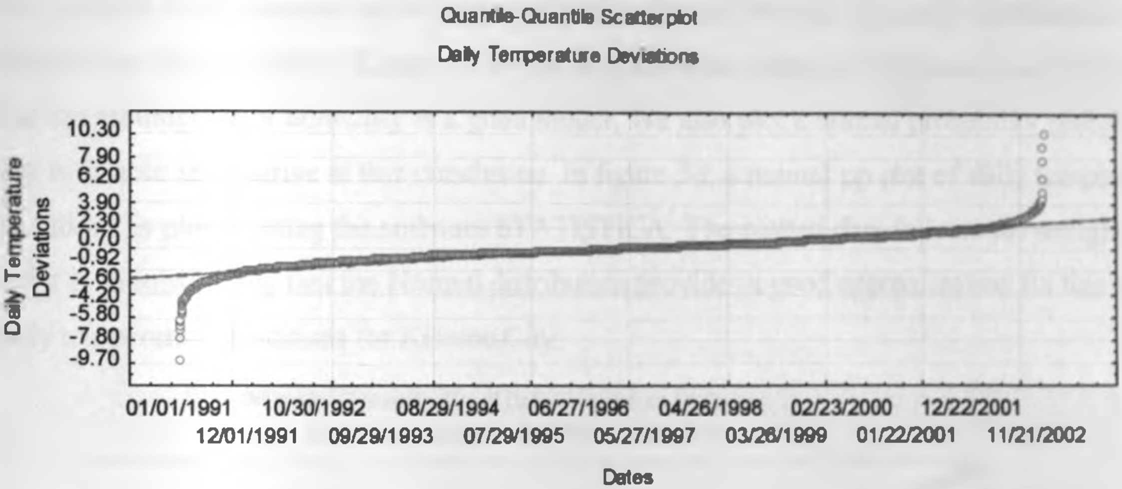

Figure 3b: QQ scatterplot of the daily temperature deviations at 99% confidence interval.

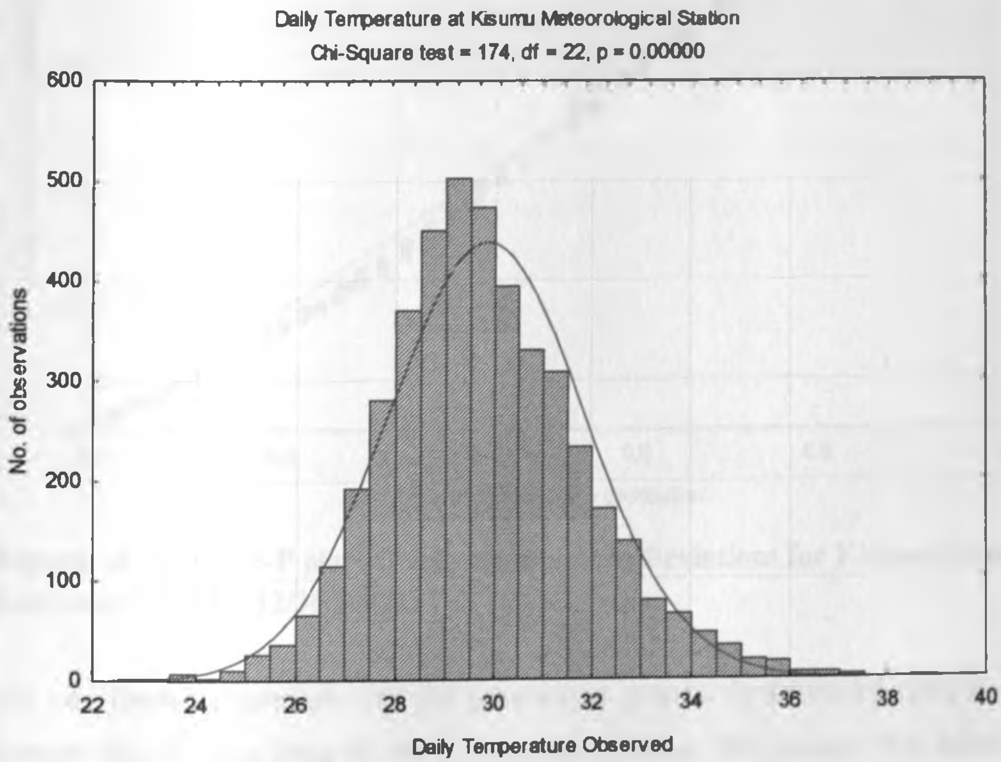

Figure 3c: A histogram of daily temperatures (blue) superimposed with a normal probability density curve (red).

The straight line represents what data would like if it is perfectly normally distributed. It is evident that the residuals of Kisumu City fall approximately along the reference line, indicating that the assumption of normality is a good model. We also plot a normal probability probability plot to enable us to arrive at this conclusion. In figure 3d, a normal pp plot of daily temperature deviations is plotted using the software STATISTICA. The plotted data follows the straight line and it is rational to say that the Normal distribution provides a good approximation for this set of daily temperature deviations for Kisumu City.

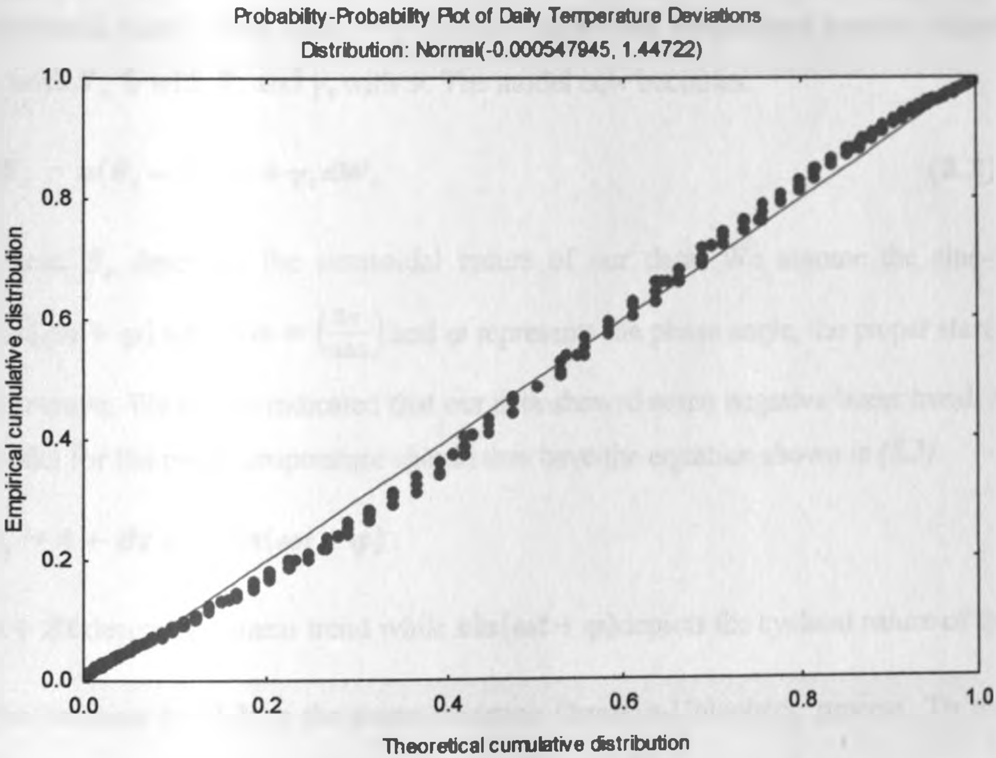

Figure 3d Normal P-P plot of Daily temperature Deviations for Kisumu Meteorological Station (1/1/1991 – 12/31/2002)

We can therefore conclude that the temperature process is driven by some normal noise. We assume this driving force to be a Brownian motion. We assume the driving force of the temperature dynamics should be a Brownian motion rather than a Lévy process because we are interested in a model that can be used to analytically price a weather derivative.

Next, we now consider an equilibrium mean reverting model. The Vasicek model (Vasicek, O. A., 1977) is our natural starting point. Vasicek's model is widely used in interest rate derivatives to derive the risk-neutral process of the short-term interest rate r.

$$dr = a(b - r)dt + \sigma dz \tag{3.1.7}$$

where a, b and σ are constants. The speed of mean reversion is a, the mean to which the short rate r is pulled back to is b, σ is the volatility of the model and dz is a normally distributed stochastic term. In the same way, we can model the temperature process whereby we replace r with T_t, b with θ_t and γ_t with σ. The model now becomes:

$$dT_t - a(\theta_t - T_t)dt + \gamma_t dW_t \tag{3.2}$$

Where θ_t describes the sinusoidal nature of our data. We assume the sine-function to be $\sin(\omega t + \varphi)$ where $\omega = \left(\frac{2\pi}{365}\right)$ and φ represents the phase angle, the proper starting point of the sine wave. We earlier indicated that our data showed some negative linear trend. A deterministic model for the mean temperature should thus have the equation shown in *(3.3)*:

$$\theta_t = A + Bt + C\sin(\omega t + \varphi).$$

$A + Bt$ depicts the linear trend while $\sin(\omega t + \varphi)$depicts the cyclical nature of the historic data.

The solution to *(3.2)* is the mean reverting Ornstein-Uhlenbeck process. To ensure that *(3.2)* reverts back to θ_t, we introduce the term $\frac{d\theta}{dt}$ to *(3.2)*. Starting at some point, say T_s=x, we get following temperature process:

$$dT_t = \left[a(O_t - T_t) + \frac{d\theta}{dt}\right]dt + \gamma_t dW_t, \quad t > s \tag{3.1.0}$$

Trivially, $\frac{d\theta}{dt} = B + \omega\cos(\omega t + \varphi)$.

We now need to estimate the mean parameters (A, B, C andφ), the speed of mean reversion a and the volatility γ_t.

From elementary trigonometry, we know that;

$$\sin(\omega t + \varphi) = \sin(\omega t)\cos(\varphi) + \cos(\omega t)\sin(\varphi) \qquad (3.1.9)$$

We can therefore rewrite *(3.1.8)* as $\theta_t = A + Bt + C[\sin(\omega t)\cos(\varphi) + \cos(\omega t)\sin(\varphi)]$. We can make this look more linear by changing some variables.

$$\theta_t = \eta_1 + \eta_2 t + \eta_3 \sin(\omega t) + \eta_4 \cos(\omega t) \qquad (3.2.0)$$

$$\text{where} \begin{cases} A = \eta_1 \\ B = \eta_2 \\ C = \frac{\eta_1}{\sin(\varphi)} \\ \varphi = \tan^{-1}\left(\frac{\eta_4}{\eta_3}\right) \end{cases}$$

Through regression by ordinary least squares, we can obtain the parameter values of seasonality for Kisumu meteorological station.

To estimate the mean reversion parameter a, we use the martingale estimation functions method (Bibby and Sørensen, 1995) as shown in equation *(3.4)* to obtain:

$$\hat{a} = -\log\left(\frac{\sum_{i=1}^{n} \frac{(T_{i-1} - \theta_{i-1})(T_i - \theta_i)}{\gamma_{i-1}^2}}{\sum_{i=1}^{n} \frac{(T_{i-1} - \theta_{i-1})^2}{\gamma_{i-1}^2}}\right)$$

Alaton et al (2002) assumed that σ_t is a piecewise constant function with a constant value each month, i.e. $\{\sigma_i\}_{i=1}^{12}$. We assume that volatility is a stochastic process; we proceed to model it in the following way;

1. Calculate the standard deviations of each month of our dataset
2. Plot these standard deviations against time.

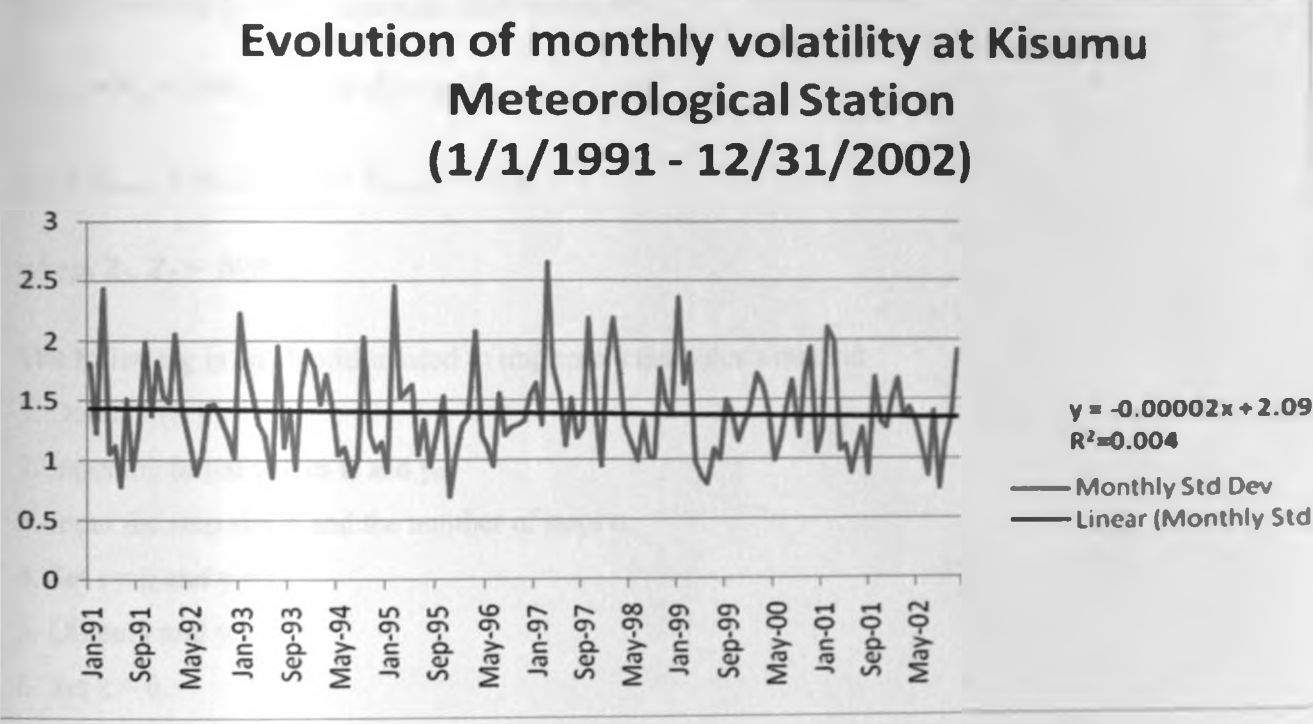

Figure 4: The evolution of monthly volatility at Kisumu Meteorological Station (1/1/1991 – 12/31/2002)

It can be seen that the volatility is also a mean reverting process whose stochastic differential equation (*3.5*) is given by:

$$d\gamma_t = a_\gamma(\gamma_{trend} - \gamma_t)dt + \sigma_\gamma dW_t$$

Since γ_{trend} is constant, we only need to estimate a_γ and σ_γ. The mean reversion parameter will be estimated as shown in *(3.7)*.

$$\hat{\sigma}^2_\gamma = \frac{1}{N_\mu}\sum_{j=0}^{N_\mu-1}(\gamma_{j+1} - \gamma_j)^2 \qquad (3.6)$$

μ – the specific month

N_μ = the number of days in the specific month.

$j = 1, \ldots\ldots, N_\mu$

Now that our parameter values are known, we discretize the temperature process and the volatility process in order to simulate the temperature:

Euler's method gives us equations *(3.8)* and *(3.9)*;

$$T_{t+1} = T_t + \alpha(\theta_t - T_t) + \acute{\theta}_t + \gamma_n Z_1$$

$$\gamma_n = \gamma_{n-1} + a_\gamma(\gamma_{trend} - \gamma_{n-1}) + \sigma_\gamma Z_2$$

where $Z_1, Z_2 \sim N(0,1)$.

The following is an algorithm used to implement the Euler's method:

1. Define f (t, y).
2. Input the initial values t_0 and y_0.
3. Input the step size h and the number of steps n.
4. Set t = t_0 and y = y_0.
5. Output t and y.
6. Set k = 0.
7. If k = n, then end the algorithm; otherwise, move on.
8. Do the following:

(ii) Set y = y + h · f (t, y).

(iii) Set t = t + h.

(iv) Output t and y.

(v) Set k = k + 1.

(v) If k < n, then go to (i); otherwise, move on.

9. End the algorithm.

γ_n is simulated using **(3.9)**. The simulated γ_n is then used to simulate temperature for a whole month. The diagram below shows the simulated temperature for the period 1/1/2003 to 3/1/2005.

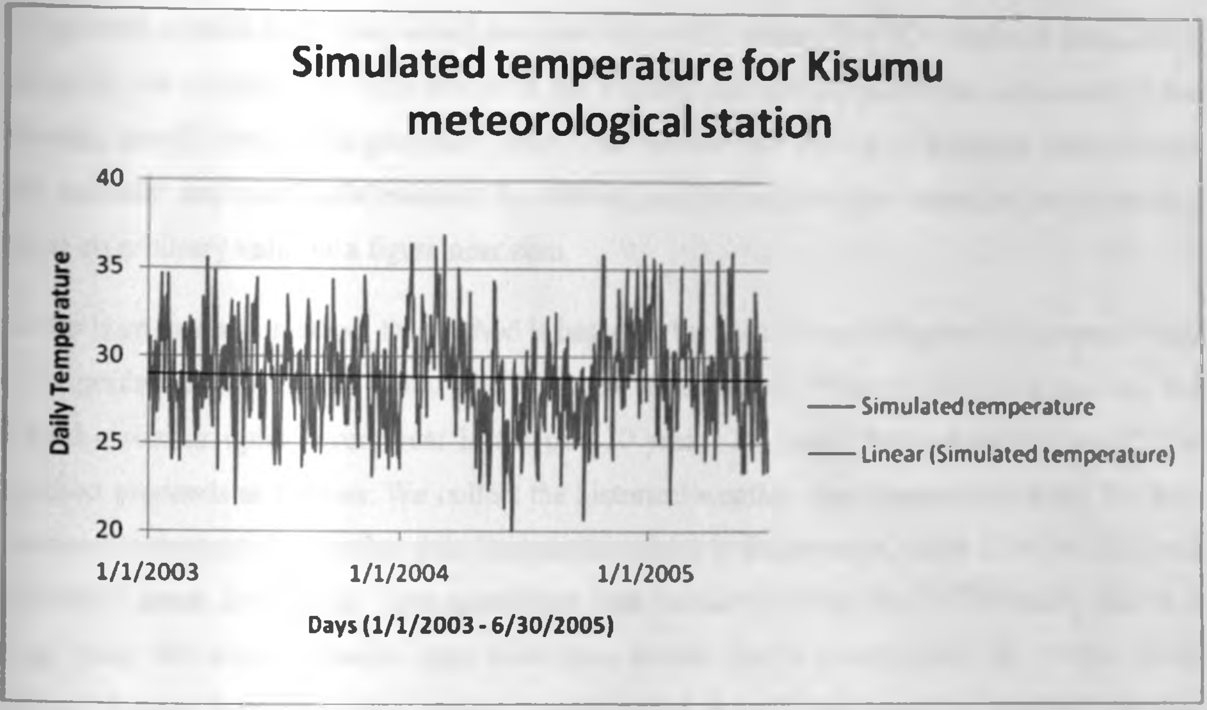

Figure 5: Simulated temperature for Kisumu meteorological station (1/1/2003 – 3/1/2005) using Euler's method.

The weak cooling trend is evident and continues through the simulation two and a half years later.

3.2 THE PRICING FUNCTION OF A TEMPERATURE DERIVATIVE.

Let us now embark on pricing a European call option. A European call option is a contract which gives its holder the right to buy an asset by a certain date for a predefined price. The price of the weather option can be expressed as the expected value of its discounted payouts. A HDD call option is sold to protect the buyer from loss of profits due to cold periods. Various methods have been used to price weather derivatives. These include the actuarial pricing method, the burn analysis method and the index modelling method. Other valuations are based on the models of temperature dynamics.

The actuarial pricing method consists of determining appropriate historical meteorological data and meteorological forecasts. Meteorological historical data needs to be cleaned and corrected of gaps and discontinuities (Jewson, 2004). We then use a model for the daily average temperature to generate a set of paths from which we construct a HDD index. The HDD index is then used to calculate the payoffs. The expectation of the weather derivative price is then equivalent to the average payoff from all the generated paths. If we assume that there is no arbitrage, then we need not consider any martingale measure. In addition, various authors have taken the safety loading to be an arbitrary value or a figure near zero.

In the burn analysis method, the method is based on the idea of evaluating how a contract would have performed in previous years. One might ask the question: "What would I have paid out had I sold a similar option every year in the past 20 years? 30 years? What about 40 years?" The method proceeds as follows: We collect the historical weather data (temperature data). We then proceed to convert the weather data (temperature data) to degree days, either HDD or CDD and we make some corrections. This corrections may include omitting the 29^{th} February date in a leap year, the weather station may have been moved due to construction, the "urban island effect" (where heavy industrial activity, construction or pollution results in gradually warmer weather in that area), the period of historical data that one should consider and the extreme weather patterns that occur in some years, notably El Nino and La Nina. Next, we determine what the option would have paid out for every past year. We then find the average of these payout amounts. Finally, we discount back to the settlement date. As we can observe, we have no need for using Monte Carlo simulations or fitting of distributions.

The main Assumption of this method is that the data values for different years are independent and identically distributed. To price a weather option, we simply calculate the historical pay-offs of the option, then we calculate the mean of the historical pay-offs. This historical mean is an estimate of the historical payoff. The main disadvantage of burn analysis is that we have no idea of the probabilities of events more extreme than those that occurred during the historical period that has been considered.

The Index Modelling approach extends the burn analysis by estimating the distribution of the weather index. If the distribution can be estimated relatively well, the Index Modelling approach yields more stable price estimation than the burn analysis. But questions such as should the distribution be discrete or continuous and parametric or non-parametric still linger. In our opinion, it is better to choose a distribution that is likely to be an accurate representation of the real unknown index distribution. This way, parameter estimation and hypothesis testing becomes much simpler and easier.

In this paper, we will use the actuarial pricing method. Corbally and Henderson (2001) noted that the classic actuarial method was dominant in the early stages of pricing weather derivatives. Brix, Jewson and Ziehmann (2002) and Roustant et al (2003) also proposed using the actuarial pricing method to value weather derivatives. The price of a weather call option at time $t=0$ is calculated as

$$C(0,t) = \lambda e^{-rt}(E(\chi) + \phi\sigma_\chi) \qquad (3.1.9)$$

Where λ is the tick size, r is the risk free interest rate, and t is the time to maturity ϕ is the safety loading which denotes the risk premium (ϕ is a real and positive number), σ_χ is the standard deviation of the payoff and $E(\chi)$ is the expected payoff under the real world probability measure P. (See Milica Latinovic, 2007)

We use the Monte Carlo simulation to calculate the expected value of the derivative. The Monte Carlo simulation tends to be numerically more efficient than other procedures when there are more stochastic variables. This is due to the fact that the time taken to carry out a Monte Carlo simulation increases linearly with the number of variables, whereas the time taken for most other procedures increases exponentially with the number of variables. Another advantage is that it can provide a standard error for the estimates that it makes. Moreover, as an approach, it can accommodate complex payoffs and stochastic processes. The Monte Carlo simulation can also be used when the payoff depends on some function of the whole path followed by a variable, not just its terminal value. (John C. Hull, 2008)

We could also use the following number of variance reduction procedures that significantly lessen our computation time.

3.2.1 ANTITHETIC VARIABLE TECHNIQUE

In this technique, we calculate two values of the derivative. The first value, f_1 is calculated in the usual manner while the second value, f_2 is calculated by changing the sign of all random samples from the standard normal distributions. Denote f as the average of f_1 and f_2 i.e.

$$\bar{f} = \frac{f_1 + f_2}{2}$$

If $\bar{\omega}$ is the standard deviation of the $\bar{f}$'s, and M is the simulation trial, the standard error of the estimate is $\bar{\omega}/\sqrt{M}$

3.2.2 STRATIFIED SAMPLING

Stratified sampling is a probability sampling technique wherein the researcher divides the entire population into different subgroups or strata, then randomly selects the final subjects proportionally from the different strata. It is important to note that the researcher must use simple probability sampling within the different strata.

Suppose we wish to take 5000 samples from a probability distribution. We could divide the distribution into 5000 equally likely intervals and choose a representative value for each interval.

In the case of a standard normal distribution, when there are n intervals, we can calculate the representative value of our ith interval as:

$$N^{-1}\left(\frac{i - 0.5}{n}\right)$$

where N^{-1} is the inverse cumulative normal distribution. The function N^{-1} can be calculated using the NORMSINV function in Excel.

3.2.3 QUASI – RANDOM SEQUENCES

This are also called a low discrepancy sequence. It is a sequence of n-tuples that fills n-space more uniformly than uncorrelated random points. The outputs are constrained by a low-discrepancy requirement that has a net effect of points being generated in a highly correlated manner.

Let us specify the terms of this call option. The underlying "asset" is the Heating Degree Days (HDDs). The Contract period runs from January 1, 2003 to March 31, 2003, i.e. it is a 3 month call option. Our weather station is the Kisumu Meteorological Station while our strike price is 12. The tick amount is Kes 5000 and our reference temperature is 29.9°C.

The HDD & CDD Index is shown in the table below

MONTH	MONTHLY INDEX	HDD/CDD VALUES	STRIKE PRICE
JAN	HDD	3.12	6
FEB	HDD	4.74	9
MAR	HDD	5.41	12
APR	CDD	1.56	6
MAY	CDD	1.37	6
JUN	CDD	0.32	3
JUL	CDD	1.23	3
AUG	CDD	0.19	3
SEP	HDD	9.77	15
OCT	HDD	8.41	12
NOV	HDD	3.91	9
DEC	HDD	3.23	6

Table 3.2: HDD & CDD Index values for the year 2003

We use the daily simulated temperatures covering our contract period of 3 months. We get our HDD measures **3.12, 4.74** and **5.41** for the months of January, February and March respectively. The cumulative HDD is **13.27**. The payoff for this uncapped HDD option thus becomes:

$$= \boldsymbol{Kes}\mathbf{5{,}000\ max(13.27 - 12, 0)}$$

$$= \boldsymbol{Kes}\mathbf{5.000\ (1.27)} = \boldsymbol{Kes}\ \mathbf{6350}.$$

We now price the same option using Black Scholes method, with the following assumptions. The risk free interest rate r is 4.3%. This was the prevailing 90 day Treasury bill rates in January 2003. To accurately get the payoff of the Black Scholes model, we use the formula

$$S_0 - e^{-rt}K$$

$$= Kes\,5{,}000 \max(13.27 - e^{-0.043*0.25} * 12, 0)$$

$$= Kes\,5{,}000\ (1.398) = Kes\,6990.$$

Suppose we now change our strike price to 10. The new option payoff function becomes:

$$= Kes5,000 \max(13.27 - 10, 0)$$

$$= Kes5,000\ (3.27) = Kes\ 16,350$$

as per our model. But according to the Black-Scholes model, the option payoff becomes

$$= Kes\,5{,}000 \max(13.27 - e^{-0.043*0.25} * 10, 0)$$

$$= Kes\,5{,}000\ (3.377) = Kes\,16{,}884.$$

Suppose that now we change our strike price to 8. The new option payoff function becomes

$$- Kes5,000 \max(13.27 - 8, 0)$$

$$= Kes5,000\ (5.27) = Kes\ 26,350$$

as per our model. But according to the Black-Scholes model, the option payoff becomes

$$= Kes\,5{,}000 \max(13.27 - e^{-0.043*0.25} * 8, 0)$$

$$= Kes\,5{,}000\ (5.356) = Kes\,26{,}780$$

It is important to note that our model under prices the call option payoff when we use the Black-Scholes model as our benchmark model. In the first scenario, it underpriced the option payoff by **9.16%,** in the second scenario; it underpriced the option payoff by **3.16%** while in the third scenario: it underpriced the option payoff by **1.61%.** We can therefore conclude that the model simulates the payoff within certain confines pretty well.

To get the option prices we use the Monte Carlo simulation method. After performing 40,000 simulations and averaging the values and the standard deviations, we get the following (4) best option prices as per our strike prices.

K=12

A) Our Model Option Price	B) Black – Scholes Option Price	Difference between A & B (%)
6.3590	5.3449	15.94
6.5535	5.5106	15.91
6.3959	5.3825	15.85
6.4414	5.4421	15.52

Table 3.3 Comparative values of the option prices when the strike price K=12

K=10

A) Our Model Option Price	B) Black – Scholes Option Price	Difference between A & B (%)
6.3905	6.9006	7.98
6.3188	6.8015	7.64
6.4375	6.9080	7.31
6.2550	6.6966	7.07

Table 3.4: Comparative values of the option prices when the strike price K=10

K=8

A) Our Model Option Price	B) Black – Scholes Option Price	Difference between A & B (%)
7.2518	7.2300	0.30
7.1606	7.1522	0.12
7.2962	7.3208	-0.34
7.2548	7.3254	-0.96

Table 3.5: Comparative values of the option prices when the strike price K = 8

As it can be seen, the strike price greatly affects our option price. A strike price of **12** results in huge pricing errors of up to **15.94%** whereas a strike price of **10** results in a minor pricing error of up to around **8%**. A strike price of **8** results in pricing errors in the neighborhood of **+ 1%**. We can therefore conclude that a lower the strike price will result in a more accurate option pricing model. It is imperative to note therefore that we should choose a strike price that gives us a lower discrepancy between our option pricing model and the Black – Scholes option pricing model.

CHAPTER FOUR: DATA ANALYSIS AND RESULTS

We begin this chapter by presenting the findings of our study and interpreting their meaning and/or relevance. Our dataset comprised of daily mean temperatures observed at the Kisumu meteorological station over a period of 12 years (01/01/1991 to 12/31/2002). The total number of observations was 4380. The weather station is located in Kisumu city, the western part of Kenya. A table of descriptive statistics associated to the Kisumu weather station is given below.

Kisumu Meteorological Station	
Mean	29.94
Median	29.70
Mode	29.80
Minimum	23.00
Maximum	38.90
Range	15.90
Standard Deviation	1.993249
Skewness	0.368054
Kurtosis	0.545576

Table 4.0 Descriptive statistics for daily average temperature

Our dataset is weakly positively skewed. The kurtosis is also positive. The values of the kurtosis and skewness validate the assumption on the normality of our dataset. **Figure 6** below shows the peakedness of our dataset relative to the normal distribution.

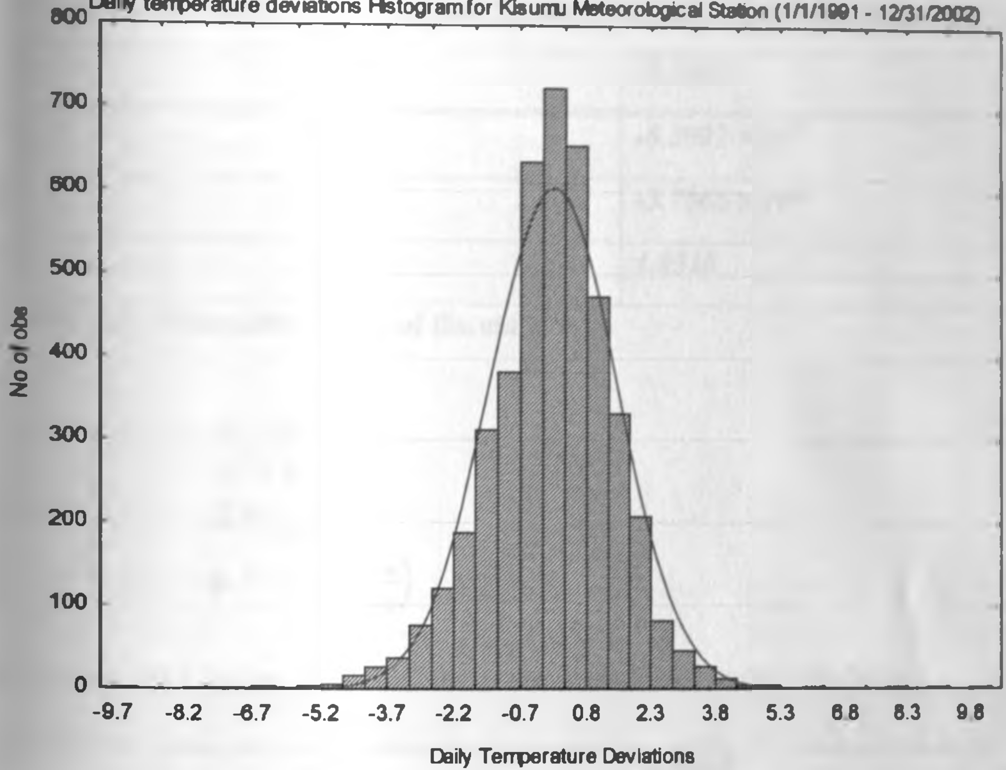

Figure 6: The daily temperature deviation histogram showing the peakedness of our dataset.

We also notice that our values are widespread around the mean. The range between the mean, mode and median is quite small (less than 0.2°C). This explains the peakedness of the graph observed above. The value of our standard deviation also lends credence to our earlier assumption that temperature is a mean reverting process. It tends to oscillate around the mean.

In our data analysis we did not find any significant linear trend (see **figure 1**). The value of R^2 for the trend line was 0.11%. Our dataset extends to over 10 years. When we compare this to the datasets of Alaton et al. (2002) and Campbell and Diebold (2002) they contain temperature data recorded for approximately 40 years. We observed that there is a negative trend in the daily mean temperatures in Kisumu meteorological station. This negative trend is aptly captured in the by the estimated seasonality parameters. We modeled the seasonality using a simple sine function. The parameters were as follows:

η_1	*29.9945*
η_2	*-6.5093 ×10⁻⁵*
η_3	*-3.7066 × 10⁻³*
η_4	*1.0516*

Table 4.1: Parameter values of the eta's

$$\text{where} \begin{cases} A - \eta_1 \\ B = \eta_2 \\ C = \frac{\eta_4}{\sin(\varphi)} \\ \varphi = \tan^{-1}\left(\frac{\eta_4}{\eta_3}\right) \end{cases}$$

The parameter values of *A, B, C* and φ are summarized in the table below:

A	*29.9945*
B	*-6.5093 ×10⁻⁵*
C	*-1.0516*
φ	*-1.5673*

Table 4.2: Parameter values of A, B, C and φ

Parameters *A* and *B* estimate the trend in the seasonal component while *C* and φ estimate the cyclical component of the seasonality parameter. As we can see, the estimate for parameter *B* is negative (illustrating a weak cooling trend). Our equation for seasonality can thus be written as:

$$\theta(t) = 29.995 \quad 6.509X10^{-5}t \quad 1.052\sin(\omega t \quad 1.567)$$

where *t=1,2,.....* refers to the number of days from 01/01/1991

C, the amplitude of the sine function is around *-1.052°C*. Therefore the difference between daily temperature during the cold and warm periods is about *2.1°C*.

We use the values generated for $\theta(t)$ to estimate the speed of mean reversion (a) for our temperature process. By performing the calculation in *(3.4)*, we find that *a=0.366592375*. This is consistent with our dataset since it does not exhibit extreme jumps. The speed of mean reversion for the volatility process is calculated through *(3.5)* and we find out that a_γ**=0.25711152045.** Again we notice that the speed of mean reversion is small. Our temperature process is volatile but only to a small extent.

Using Euler's method, we simulate the future temperature for the period 01/01/2003 up to 03/31/2005. This is done in order to evaluate a 3 month derivative from 01/01/2003 up to 03/31/2003. The simulated temperature is shown in **figure 5.0.** We also evaluate the performance of our model by calculating the error between the actual and modeled temperature. The graphs below show the comparison between the actual and modeled temperature for the period 01/01/2000 – 12/31/2001. We show the best 4 simulations we got. We chose the first simulation (figure 5.1) since it had the least standard deviation of 3.07°C.

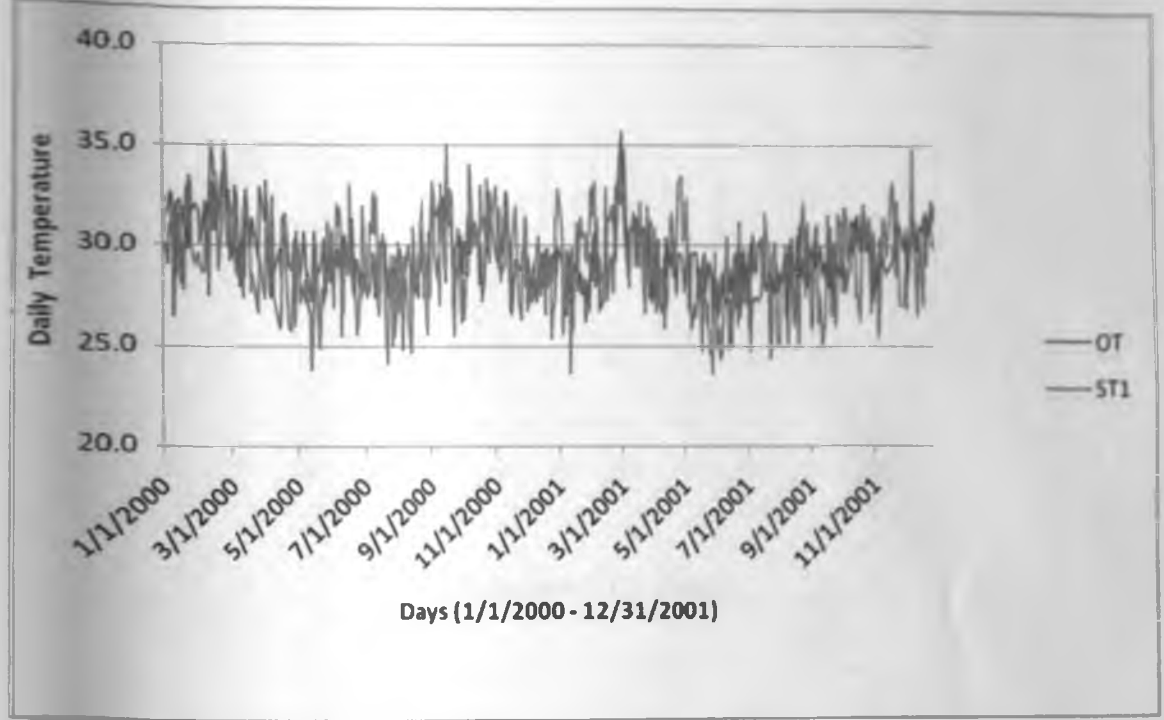

Figure 5.1: Simulation 1, standard deviation 3.070°C

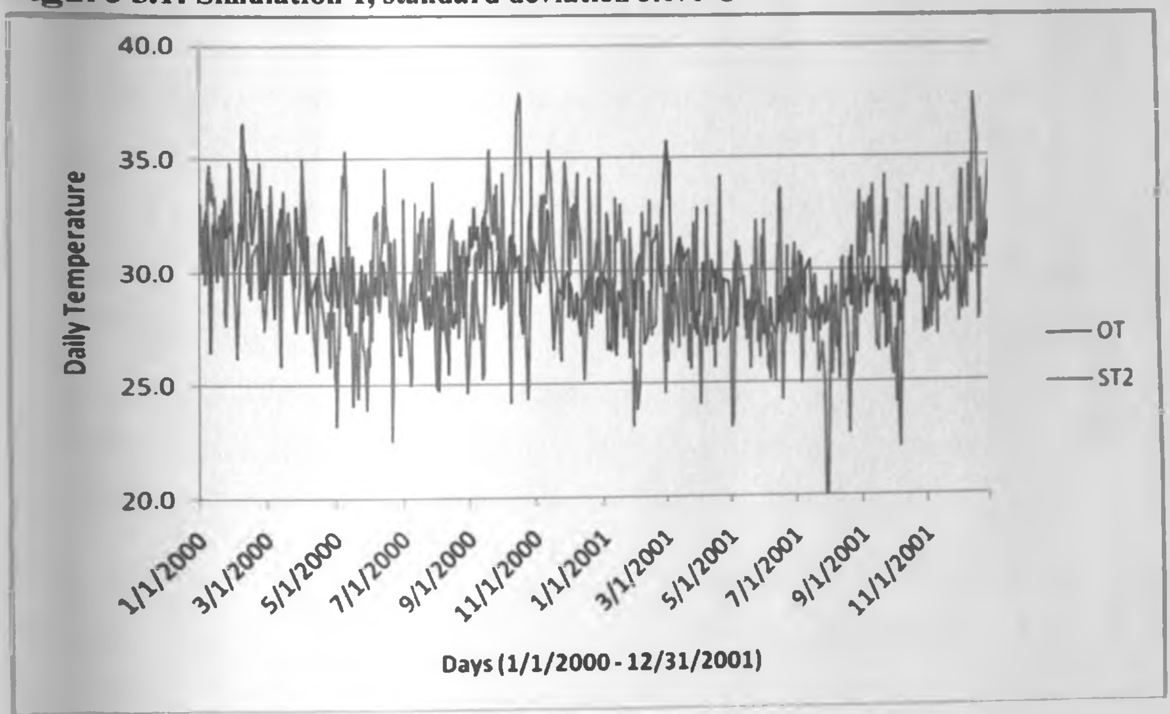

Figure 5.2: Simulation 2, standard deviation 3.116°C

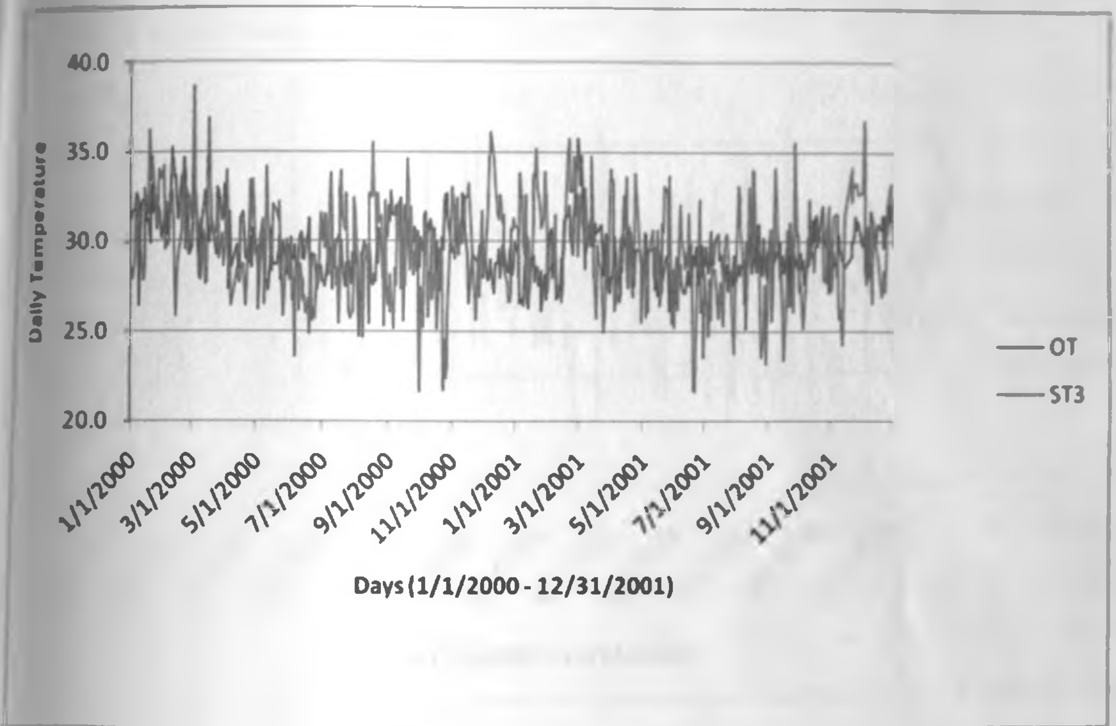

Figure 5.3: Simulation 3, standard deviation 3.084°C

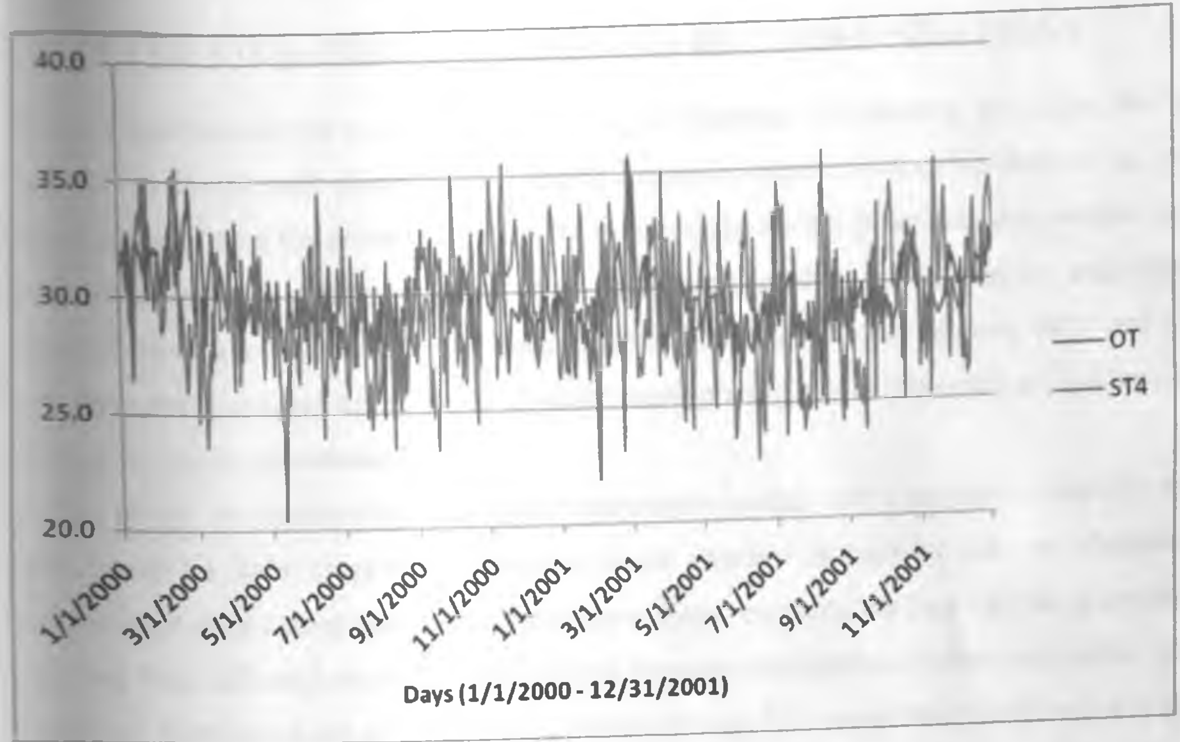

Figure 5.4: Simulation 4, standard deviation 3.093°C

We can observe that our margin of error lies between **[-3.5, 3.5]** and that out of **730** observations we had **517** observations within the margin of error and **213** observations out of error margin. **70.82%** of our values were within the margin of error. We can therefore conclude that our model forecasts temperatures with some degree of accuracy.

In our the methodology section 2, we evaluated a 3 month European call option with a strike price of **12** using the HDD index and also using the Black Scholes model where we chose our risk free interest rate as **4.3%.** This rate is equivalent to the **90** day Treasury bill interest rates in January 2003(CBK Annual Bulletin, June 2003).

The difference in pricing was quite significant. Our temperature model underpriced the option by about **16%** when we used a strike price of **12.** But we showed that this can be remedied using a lower strike price of **8.** This reduced the percentage error on our payoff function, and also gave us an equally good pricing function.

CHAPTER FIVE: CONCLUSIONS AND RECOMMENDATIONS

Weather derivatives are a relatively new class of financial instruments. In Kenya, the use of derivatives is not well documented. Despite immense interest in weather derivatives, pricing issues appear to be the greatest impediment in unlocking its full potential since weather is not a trade able asset. Therefore, it exists in an incomplete market. Meteorologists and interested stakeholders should appreciate that their roles are not independent of each other and should therefore strive to come up with a temperature based pricing model that could act as a benchmark pricing model in our economy.

In this paper, we constructed a stochastic temperature model with a stochastic volatility process that could be used to price temperature based weather derivatives. As we observed, our temperature model consistently overpriced the options, exposing the buyer of the option to great risk and loss. Although our temperature model forecasted temperature within reasonable range of accuracy, it did not do so when it came to option pricing. The overpricing could be due to several factors. We had a limited dataset of 12 years whereas Alaton et al. (2002) and Campbell and Diebold (2002) had data covering more than 40 years. If we had more data, we suppose that our temperature process would yield much better results (in terms of pricing). Secondly, we restricted our simulation to Euler's method. We believe better estimates can be obtained if Milstein's scheme of approximation or Monte Carlo simulation method is used.

The temperature data we analysed showed that over a period of 12 years, temperature levels in Kisumu have been declining, albeit insignificantly. This is unlike other parts of the world that have been experiencing a phenomenon called global warming. We have also seen that temperature in Kisumu can be modeled using well known stochastic mean reverting processes.

FURTHER RESEARCH

As a possible improvement to our temperature model, it would be interesting to find out if modelling the speed of mean reversion instead of assuming it to be a constant as suggested in Zapranis, A. and Alexandridis, A (2008) could significantly change our temperature model.

Our temperature model is simplistic because it fails to capture other factors that affect weather, for example humidity, atmospheric pressure, wind. We believe that a better model should be able to capture these phenomena as our model assumes that temperature can be modeled on its own.

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