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Research Report in Actuarial Science, Number 028, 2020

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Submitted to the School of Mathematics in partial fulfilment for a degree in Post Graduate Diploma in Actuarial Science

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Post Graduate Diploma Project

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Abstract

Options are a popular type of investment in the financial derivatives market. The intention of buying or selling options is to make profits. However, investors might end up making losses if the market goes against their anticipation. The Greeks describe different dimensions of risk involved when taking an option's position. The Greeks are Delta, Gamma, Rho, Kappa and Theta. The main aim of this study is achieved by creating a linear program that maximizes a calculated theoretical profit. The profit is calculated by subtracting the real market price of Netflix options from prices calculated using the Black-Scholes formula. We observe that the theoretical prices are higher than the market price. The constraints in our linear program are Greek neutralities. The neutrality of each Greek is achieved by equating the sum of the positional Greeks to zero. The results of the discussion show that Linear Programming can be applied to options to hedge against a combination of all risks that are experienced in the financial derivatives market. The case study reviewed how the profit related to options changes when we include all Greeks and when we reduce the number of Greeks. It was observed that profit is lowest when we include all Greeks and is highest when we use one Greek. The number of shares to buy and sell in order to achieve an optimal strategy for our portfolio are also derived. The effect of shadow price on the risks experienced in the market were also observed.

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Declaration and Approval

I the undersigned declare that this dissertation is my original work and to the best of my knowledge, it has not been submitted in support of an award of a degree in any other university or institution of learning.

Signature   Date

MAINA NDUNG'U SAMMY
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In my capacity as a supervisor of the candidate's dissertation, I certify that this dissertation has my approval for submission.

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Dedication

I dedicate this project to Almighty God and I acknowledge that He is able. I would also like to dedicate this project to my parents Mr and Mrs Joshua Maina Mwangi and my siblings.

Contents

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I wish to thank the Almighty God for granting me the good health and determination necessary to complete this project. I am greatly indebted to my supervisors Dr. Caroline. A. Ogutu, Dr. Nelson Owuor and Prof. Ivivi Mwaniki for their intellectual and moral support throughout the course of the project. Many thanks to the University of Nairobi for according me the requisite resources without which this project could not be successful; The library, internet services and study rooms. Lastly, I take this opportunity to appreciate my parents, my siblings, my niece Ella and nephew Bravyl who gave me reasons to smile throughout the study and the school of Mathematics fraternity for their support, both financial, intellectual and prayers. Thank you all ....

Maina Ndung'u Sammy

Nairobi, 2017.

1 Introduction

1.1 Background

1.1.1 Options

Options are financial derivative instruments that provide the holder the right but not the obligation to trade a specified quantity of underlying asset at a fixed price called the exercise price, at or before the expiration date of the option. Selling an option is also called taking a short position on the option while as buying an option is also called taking a long position on the option. Options can be categorized into different categories. when we consider either buying or selling options, they can be categorized into call and put options. Put options grants the owner the right to sell the underlying asset at an agreed exercise price by the expiration date while as the holder of the call option has the right to buy the underlying asset at an agreed strike price by the expiration date. Options can also be categorized as per their expiration date. The two categories are American option and European option. American options can be exercised on or before expiration. European options are exercised on the date of expiration itself. Another example is the exotic options which include; Chooser, Compound, barrier, Binary, Bermuda, Quantity-Adjusting, Look-Back, Asian, Basket, Extendible, Spread, Shout and Range options. Currency options is also another example.

1.1.2 Options Pricing

The value of options can be categorised into two aspects

Intrinsic Value

This is the value of options assuming that the options contract expires immediately instead of sometime later.
Let the current stock price=A and strike price=B. For a call option, the intrinsic value is

$$Intrinsic value = max(A-B,0) \tag{1}$$

The following terms are used to define the intrinsic value of the call option
in-the-money - when $(A > B)$
at-the-money - when $(A = B)$

out-of-the-money - when $(A < B)$
For a put option, the intrinsic value at time c is;

$$Intrinsicvalue = max(B - A, 0) \tag{2}$$

The following terms are used to define the intrinsic value of the put option
in-the-money - when $(B > A)$
at-the-money - when $(B = A)$
out-of-the-money - when $(B < A)$

Time Value

The time value of an option is the excess option's premium over the intrinsic value of the option.

$$Timevalue = OptionPremium - Intrinsicvalue \tag{3}$$

Option pricing is the process of using mathematical models to value options. The mathematical models use the following parameters which affect their prices; underlying share price, strike price, time to expiration, risk free interest rate, volatility of the underlying share. The following shows how the above parameters affect the values of options prices.

Underlying Share price

A call option with a high stock price implies a high value in the intrinsic value of the option which implies a high option premium. A call option with a low stock price implies a low value in the intrinsic value of the option and thus a low option premium. A put option with a high stock price implies a low intrinsic value and thus a low premium on the option. A high stock price on put options implies a high intrinsic value and thus the option premium is also high.

Strike Price

A call option with a high strike price implies a low intrinsic value and thus a low premium on the option. A call option with a low strike price implies a high intrinsic value and thus a high option's premium. A high strike price on put options would give rise to a high intrinsic value and thus a high premium of the option. The vice-versa holds true too.

Time to expiration

When the time to expiration is long, there is a high possibility that the underlying share price might shift in favour of the option holder just before expiration. Thus the value of the option increases with increase in time to maturity. However, this condition might not hold true for European put options which is deeply in the money. This is because the sooner you exercise the option the higher the possibility of gaining more than waiting for the market to move in favour of you when the time to expiration is long.

Risk free interest rate

A high risk free interest rate gives a higher value of the call option. A high risk free interest rate implies a low value in the put option.

Volatility of the Underlying asset

Volatility shows the variability of the price of the underlying asset in the market. High volatility implies a high value of options too. This is because there is a high chance that the prices of the underlying asset might move in favour of options by expiration. The vice-versa holds true too.
Historically, researchers have established numerous mathematical models that are used to estimate the theoretical values of options. The assumptions made while creating the models do not exist in the real market. This in return gives values that are not equal to the market values because the market values are defined by demand and supply of the options. In our study we shall deal with European options. In order for the European call option holders to make profits, they buy the option when the underlying asset has a low stock price and sell the underlying asset when the stock price is higher than the strike price, at or before the expiration date. If by the expiration date of the option the stock price is lower than the strike price, then the call option holder suffers a loss of the premium he had paid for the that option. If by expiration, the stock price of the underlying asset is equal to the strike price, then the holder of the European call option does not earn any profit or loss. The put option holder will buy the option if they predict that the price of the option will move downwards.

1.1.3 Hedging Options

The financial market experiences many risks which is the case in every other market. Greeks describe the dimensions of risks faced in the derivatives market. There are six greeks namely; delta, gamma, Vega, rho, lambda and theta. Hedging Options is the process of protecting options against risks experienced in the derivatives market in order to maximize profits. Risks include losses caused by; changes in future prices of the Underlying assets,changes in return for a given security, changes in interest rate and changes in the time value for money . Traders might hedge their options using strategies that are decided in advance. The strategies are based on the future expectations of the investors towards the price of the underlying assets. Bull spread, buy- and sell ratio and vertical put spread, are examples of hedging strategies that are decided in advance. Hedging options using Greeks involves creating a "Delta-Vega" , "Delta-Neutral" or "Gamma-Neutral" portfolios in order to obtain high profits.

1.1.4 Linear Programming

Linear Programming is a mathematical process that maximizes or minimizes the values of the parameters involved. A linear program is composed of:

(i) Decision variables which are the quantities to be determined

(ii) Objective function which defines the quantity to be maximized or minimized.

(iii) Constraints which represents how the decision variables use resources that are available in limited quantities

(iv) Data which quantifies the relationships shown in the objective function and the constraints.

We can formulate a linear program for maximizing profits of options. The constraints are calculated neutrality of our greeks and our objective function will be theoretical values of options prices.

1.1.5 Problem Statement

The main challenge experienced by option traders and investors is how to maximize their profits and reduce the risks in the market at the same time. When using Greeks to hedge option prices, traders base their strategy using the Delta neutral, and gamma-neutral strategies which considers only one risk in the market and ignores the other risks which also affect the price of options. Thus the strategies are inefficient and there is need for a strategy that can consider all the Greeks derivatives at the same time.

1.1.6 Significance of study

In order for an option investor to be sure of making a profit , he has to reduce the amount of risks experienced in the market. However reducing all the risks concurrently is complicated. This is because every change in the value of a greek causes a corresponding change in the value of another greek. Thus one can hedge one risk but increase the level of another risk. This implies that there is a need for a strategy that can handle all the risks in the market. Linear Programming is known for handling constraints that are related. Using a linear program, an investor can reduce the risks in the market at the same time.

1.1.7 Objectives of the study

Throughout this project, we will seek to accomplish the following:

General Objective

To create a linear program that hedges a portfolio against all risks in the derivatives market

Specific Objectives

(i) To Calculate values of options and Greeks that will be used to create a linear program.

(ii) To show the options to buy or sell

(iii) To Estimate the effect of shadow prices on our profit.

(iv) To calculate the effects on profit when all Greeks are present and when one or more Greeks are not present in the portfolio.

the outline of the thesis is as follows:

Chapter 1:
In chapter two we have described option pricing using linear programming and discussed studies carried out by other researchers concerning hedging options
Chapter 3:

In chapter three, we have formed a linear program that hedges against risks in the financial market

Chapter 4:

Chapter 4 discusses results found in chapter three and finally chapter 5 contains conclusions.

2 Literature Review

This section analyses the literature on determining the use of Greeks as the hedging tools in the derivatives market. Hull and White (1987) carried out a study to decide which was the best hedging strategy among Delta hedging, Delta-Vega hedging and Delta-Gamma hedging, and they established that the Delta-Vega was the best among the three. Using an option pricing context, Ogden (1987) compared the price behaviour and risk characteristics of a yield curve note model which paid interest rate inversely to rate of short term models and another one with a fixed-rate model. He found out that the risk rate on the model with the fixed rate was approximately half as great as the risk on the yield curve note model with the same maturity. The yield curve note was used in immunization strategies as liabilities by financial institutions. Delta hedging has been vastly applied by investors and hedgers who have positions of long or short options in their portfolio to hedge risks from the constantly changing prices of an option. Delta hedging is widely applied in financial engineering, thus there is a vast literature on the topic. Hull (2003) discussed an introduction of hedging strategies including delta hedging and found out that a long position may be delta hedged by using a short position. Jarrow and Turnbull (1999) provided a well explained lesson on how to replicate portfolios in order to attain a delta-neutral position and implementation of dynamic delta hedging.The options traders can reduce risks faced in the derivatives markets if they have a vast knowledge of the Greeks associated with the options. This project focuses on providing the knowledge of the Greeks and their application in risk management. The knowledge of the effect of hedging Greeks using different strategies will help us to determine how much risk is related to each Greek and potential reward when we neutralize the risk. The impact of the Greeks is shown using a linear programming model. This research will focus on maximizing a theoretical profit using different hedging strategies. This will assist the investor to determine the risk-return-ratio, prior to entry in the trade. Reducing risks faced in the financial market using Greeks can be taken to a new level with the help of a linear program. This project will enhance the existing knowledge on reducing risks faced by options and will assist in determining the best strategy strategy has the greatest impact on profits.

3 Methodology

3.1 The Black-Scholes Model

The Black-Scholes model was developed in 1973. The model is used to generate a theoretical value of European Options. Pricing of American Options is not possible while using Black-Scholes model because the model only allows a specified time to expiration. Our research will involve a European option because we will use Black-Scholes to calculate the prices that are used in the linear programming model that is introduced later.
Assumptions of the Black-Scholes Formula:

(i) The price of the underlying share follows a geometric Brownian motion i.e. the share price changes continuously through time according to the SDE(stochastic differential equation):

(ii) Taxes and transaction costs are not included.There are no taxes or transaction costs.

(iii) There is no change in the risk-free interest rate. It is always the same for all maturities and the same for borrowing and lending.The risk-free rate of interest is constant, the same for all maturities and the same for borrowing or lending.

(iv) There are no risk-free arbitrage opportunities.

(v) The underlying asset can be traded continuously and with infinitely small size of units.

(vi) Unlimited short selling is allowed. Under these assumptions, the value of the option will depend on the risk-free interest rate, the price of the stock, the strike price of the asset associated with the option, volatility and time to expiration.
For the European option, Let, f be the price of the call option and g the price of the put option at time c.

(i) A is the stock price of the underlying asset.

(ii) ε is the time to expiration date of the option.

(iii) B is the strike price of the underlying asset.

Then, for such a call option:

$$f = AN(h_1) - B\exp^{-d\varepsilon} N(h_2) \tag{4}$$

The N is the probability that a normally distributed stochastic variable Y will be less than or equal to h. The values are read from normal distribution tables. For a put option we also have:

$$g = B\exp^{-d\varepsilon} N(-h_2) - AN(-h_1) \tag{5}$$

The call and put equations are valid if no dividends are paid over this period and the option is of European type.

3.2 Greek parameters of the Black-Scholes Model

3.2.1 Introduction

The Greeks are used to assess risks faced by options portfolios in derivative markets. The risk level of an options portfolio can be hedged by using greek neutralities namely; Theta-neutral, Delta-neutral, gamma-neutral, Vega-neutral, rho-neutral and kappa-neutral strategies. Let us take a deeper look at the named strategies.

3.2.2 Delta

Delta measures how the value of an option varies with change in the price of the underlying asset. Given that f is the call option , g is the put option price price and B is the stock price,then for a call option

$$\Delta = \frac{\partial f}{\partial B} \tag{6}$$

For a put option,

$$\Delta = \frac{\partial g}{\partial B} \tag{7}$$

Thus, the delta is the slope of a graph of an option against stock price. From Black-Scholes formula, we know that the price of a non-dividend paying call option can be written as

$$f = AN(h_1) - B\exp^{-d\varepsilon} N(h_2) \tag{8}$$

The put price of a non-dividend paying stock can be written as

$$g = B\exp^{-d\varepsilon} N(-h_2) - AN(-h_1) \tag{9}$$

where,

$$h_1 = \frac{\ln\left(\frac{A}{B}\right) + \left(d + \frac{\omega_a^2}{2}\right)\varepsilon}{\omega_a\sqrt{\varepsilon}} \tag{10}$$

and

$$h_2 = \frac{\ln\left(\frac{A}{B}\right) + \left(d - \frac{\omega_a^2}{2}\right)\varepsilon}{\omega_a\sqrt{\varepsilon}} \tag{11}$$

From the equations, it's clear that

$$h_2 = h_1 - \omega_a\sqrt{\varepsilon}$$

$N(.)$ is the cumulative density function of normal distribution. Thus $N(h_1)$ can be shown as follows:

$$N(h_1) = \int_{-\infty}^{h_1} f(s)ds = N(h_1) = \int_{-\infty}^{h_1} \frac{1}{\sqrt{2\pi}} \exp^{\frac{-s^2}{2}} ds \tag{12}$$

First we calculate

$$N'(h_1) = \frac{\partial N(h_1)}{\partial h_1} = \frac{1}{\sqrt{2\pi}} \exp^{\frac{-h_1^2}{2}} \tag{13}$$

Then,

$$N'(h_2) = \frac{\partial N(h_2)}{\partial h_2} = \frac{1}{\sqrt{2\pi}} \exp^{\frac{-h_2^2}{2}} \tag{14}$$

Replacing $h_2 = h_1 - \omega_a\sqrt{\varepsilon}$, we get:

$$= \frac{1}{\sqrt{2\pi}} \exp^{\frac{-(h_1 - \omega_a\sqrt{\varepsilon})^2}{2}} \tag{15}$$

$$= \frac{1}{2\pi} \exp^{\frac{-h_1^2}{2}} \cdot \exp^{h_1 \omega_a \sqrt{\varepsilon}} \cdot \exp^{\frac{-\omega_a^2 \varepsilon}{2}} \tag{16}$$

$$= \frac{1}{2\pi} \exp^{\frac{-h_1^2}{2}} \cdot \exp^{\ln\left(\frac{A}{B}\right) + \left(d + \frac{\omega_a^2}{2}\right)\varepsilon} \cdot \exp^{\frac{-\omega_a^2 \varepsilon}{2}} \tag{17}$$

$$= \frac{1}{\sqrt{2\pi}} \exp^{\frac{-h_1^2}{2}} \cdot \frac{A}{B} \cdot \exp^{d\varepsilon} \tag{18}$$

For a European call option where the underlying asset is a non-dividend paying stock, the equation of delta is shown below

$$\Delta = N(h_1) \tag{19}$$

The derivation of the equation above is shown next

Proof.

$$\Delta = \frac{\partial f}{\partial A} = N(h_1) + A\frac{\partial N(h_1)}{\partial A} B\exp^{-d\varepsilon}\frac{\partial N(h_2)}{\partial A} \tag{20}$$

$$= N(h_1) + A\frac{\partial N(h_1)}{\partial h_1}\frac{\partial h_1}{\partial A} - B\exp^{-d\varepsilon}\frac{\partial N(h_2)}{\partial h_2}\frac{\partial h_2}{\partial A} \tag{21}$$

$$= N(h_1) + A\frac{1}{\sqrt{2\pi}}\exp^{\frac{-h_1^2}{2}} \cdot \frac{1}{A\omega_a\sqrt{\varepsilon}} - B\exp -d\varepsilon\frac{1}{\sqrt{2\pi}}\exp^{\frac{-h_1^2}{2}} \cdot \frac{A}{B} \cdot \exp^{d\varepsilon} \cdot \frac{1}{A\omega_a\sqrt{\varepsilon}} \tag{22}$$

$$= N(h_1) + A\frac{1}{A\omega_a\sqrt{2\pi\varepsilon}}\exp^{\frac{-h_1^2}{2}} - A\frac{1}{A\omega_a\sqrt{2\pi\varepsilon}}\exp^{\frac{-h_1^2}{2}} \tag{23}$$

$$= N(h_1).$$

□

For a European put option with a non-dividend paying stock, delta can be shown as

$$\Delta = N(h_1) - 1 \tag{24}$$

The derivation of this equation is as shown below

Proof.

$$\Delta = \frac{\partial g}{\partial A} = B\exp^{-d\varepsilon}\frac{\partial N(-h_2)}{\partial A} - N(-h_1) - A\frac{\partial N(-h_1)}{\partial A} \tag{25}$$

$$= B\exp^{-d\varepsilon}\frac{\partial(1-N(h_2))}{\partial h_2}\frac{\partial h_2}{\partial A} - (1-N(h_1)) - A\frac{\partial(1-N(h_1))}{\partial h_1}\frac{\partial h_1}{\partial A} \tag{26}$$

$$= -B\exp^{-d\varepsilon}\frac{1}{2\pi}\exp^{\frac{-h_1^2}{2}} \cdot \frac{A}{B} \cdot \exp^{d\varepsilon} \cdot \frac{1}{A\omega_a\sqrt{\varepsilon}} - (1-N(h_1)) + A\frac{1}{\sqrt{2\pi}}\exp^{\frac{-h_1^2}{2}} \cdot \frac{1}{A\omega_a\sqrt{\varepsilon}} \tag{27}$$

$$= -A\frac{1}{A\omega_a\sqrt{2\pi\varepsilon}}\exp^{\frac{-h_1}{2}} - N(h_1) - 1 + A\frac{1}{A\omega_a\sqrt{2\pi\varepsilon}}\exp^{\frac{-h_1}{2}} \tag{28}$$

$$= N(h_1) - 1.$$

□

Application of Delta

A delta neutral position hedges the risk imposed by delta. Creating a delta neutral portfolio involves combining multiple positions with positive and negative deltas of the options to finally give the overall delta of the underlying assets involved equal to zero. A delta-neutral portfolio equals out the response to market movements for a certain range to bring the net change of the position to zero. The positional Greeks in the delta neutral portfolio are not linear thus they change with change in the value of the underlying asset. The positions shift between being negative, neutral and positive. A seller of an option can calculate a delta ratio to achieve the neutral position. The following example shows how delta neutrality is achieved. Take an example of an option which has a premium of $8, the stock price of the underlying asset is $90 and delta is 0.5. If the seller sold 5 options to the buyer, this means that the buyer has the right to purchase 500 shares at time of maturity. The seller constructs a delta neutral position by buying $(0.5 \times 500 = 250)$ shares of stock. Suppose that the price goes up to $1, the price of the option goes up by $0.5. In this situation, the seller has a gain of $250 in stock position and a loss of $250 in the option position i.e (0.5×500). The seller has a pay off of zero on that transaction. If the stock price decreases by $1 the option price will also decrease by $0.5. The total pay off of the seller is also 0 on that transaction Our model involved adding the positional Greeks on all transactions and equating them to zero.

3.2.3 Theta

The value of an option has both the stock value and time value. Theta is a Greek that measures the rate at which options lose their value as they near their expiration date. It is also called the option's time decay. If every assumption of the Black-Scholes model is held constant, the value of the option diminishes as we approach the time to maturity of our option. Since time always moves in the same direction, Theta is always negative. The following is the equation for theta for a call option.

$$\Theta = \frac{\partial f}{\partial c} = \frac{\partial f}{\partial \varepsilon}\frac{\partial \varepsilon}{\partial c} = -1\frac{\partial f}{\partial \varepsilon} \tag{29}$$

where

$$\varepsilon = C - c$$

. The general equation for theta of a put option is

$$\Theta = \frac{\partial g}{\partial c} = \frac{\partial g}{\partial \varepsilon}\frac{\partial \varepsilon}{\partial c} = -1\frac{\partial g}{\partial \varepsilon} \tag{30}$$

where

$$\varepsilon = C - c$$

. The equation of theta of European call option in a non-dividend paying stock is as follows

$$\Theta = \frac{-A\omega_a}{2\sqrt{\varepsilon}} \cdot N'(h_1) - dB \cdot \exp^{-d\varepsilon} N(h_2) \tag{31}$$

Proof.

$$\Theta = -\frac{\partial f}{\partial \varepsilon} = -A\frac{\partial N(h_1)}{\partial \varepsilon} + (-d)\cdot B\cdot \exp^{-d\varepsilon} N(h_2) + B\exp^{-d\varepsilon}\frac{\partial N(h_2)}{\partial \varepsilon} \tag{32}$$

$$= -A\frac{\partial N(h_1)}{\partial h_1}\frac{\partial h_1}{\partial \varepsilon} - dB\cdot \exp^{-d\varepsilon} N(h_2) + B\exp^{-d\varepsilon}\frac{\partial N(h_2)}{\partial h_2}\frac{\partial h_2}{\partial \varepsilon} \tag{33}$$

$$= -A\frac{1}{\sqrt{2\pi}}\exp^{\frac{-h_1^2}{2}}\left(\frac{d+\frac{\omega_a^2}{2}}{\omega_a\sqrt{\varepsilon}} - \frac{\ln\frac{A}{B}}{2\omega_a\varepsilon^{\frac{3}{2}}} - \frac{d+\frac{\omega_a^2}{2}}{2\omega_a\sqrt{\varepsilon}}\right) - dB\cdot\exp - d\varepsilon N(h_2) \tag{34}$$

$$+ B\exp^{-d\varepsilon}\cdot\left(\frac{1}{2\pi}\exp^{\frac{-h_1^2}{2}}\cdot\frac{A}{B}\cdot\exp^{d\varepsilon}\right)\cdot\left(\frac{d}{\omega_a\sqrt{\varepsilon}} - \frac{\ln\frac{A}{B}}{2\omega_a\varepsilon^{\frac{3}{2}}} - \frac{d+\omega_a^2}{2\omega_a\sqrt{\varepsilon}}\right) \tag{35}$$

$$- dB\cdot\exp^{-d\varepsilon} N(h_2) + A\frac{1}{\sqrt{2\pi}}\exp^{\frac{-h_1^2}{2}}\cdot\left(\frac{d}{\omega_a\varepsilon^{\frac{3}{2}}} - \frac{d+\frac{\omega_a^2}{2}}{2\omega_a\sqrt{\varepsilon}}\right) \tag{36}$$

$$= -A\frac{1}{\sqrt{2\pi}}\exp^{\frac{-h_1^2}{2}}\cdot\left(\frac{\frac{\omega_a^2}{2}}{\omega_a\sqrt{\varepsilon}}\right) - dB\cdot\exp^{-d\varepsilon} N(h_2) \tag{37}$$

$$= \frac{-A\omega_a}{2\sqrt{\varepsilon}}\cdot N'(h_1) + dB\exp^{-d\varepsilon} N(h_2). \qquad \square$$

The equation of a put option with a non-dividend paying stock is shown below

$$\Theta = -\frac{A\omega_a}{2\sqrt{\varepsilon}}\cdot N'(h-1) + dB\exp^{-d\varepsilon} n(-h_2) \tag{38}$$

The proof of the equation is shown next

Proof.

$$\Theta = -\frac{\partial g}{\partial \varepsilon} = -(-d)\cdot B\cdot \exp^{-d\varepsilon} N(-h_2) - B\exp^{-d\varepsilon}\frac{\partial N(-h_2)}{\partial \varepsilon} \tag{39}$$

$$+A\frac{\partial - h_1}{\partial \varepsilon} \tag{40}$$

$$= -(-d)B\cdot \exp^{-d\varepsilon}(1-N(h_2)) - B\exp^{-d\varepsilon}\frac{\partial(1-N(h_2))}{\partial h_2}\frac{\partial h_2}{\partial \varepsilon} \tag{41}$$

$$+A\frac{\partial(1-N(h_1))}{\partial h_1}\frac{\partial h_1}{\partial \varepsilon} \tag{42}$$

$$= -(-d)B\cdot \exp^{-d\varepsilon}(1-N(h_2)) + B\exp^{-d\varepsilon}\left(\frac{1}{\sqrt{2\pi}}\exp^{\frac{-h_1^2}{2}}\cdot\frac{A}{B}\cdot\exp^{d\varepsilon}\right) \tag{43}$$

$$\cdot\left(\frac{d}{\omega_a\sqrt{\varepsilon}} - \frac{\ln\frac{A}{B}}{2\omega_a\varepsilon^{\frac{3}{2}}} - \frac{d+\frac{\omega_a^2}{2}}{2\omega_a\sqrt{\varepsilon}}\right) - A\frac{1}{\sqrt{2\pi}}\exp^{\frac{-h_1^2}{2}}\left(\frac{d+\frac{\omega_a^2}{2}}{\omega_a\sqrt{\varepsilon}} - \frac{\ln\frac{A}{B}}{2\omega_a\varepsilon^{\frac{3}{2}}} - \frac{d+\frac{\omega_a^2}{2}}{2\omega_a\sqrt{\varepsilon}}\right) \tag{44}$$

$$= dB\exp^{-d\varepsilon}(1-N(h_2)) + A\frac{1}{\sqrt{2\pi}}\exp\frac{-h_1^2}{2}\left(\frac{d+\frac{\omega_a^2}{2}}{\omega_a\sqrt{\varepsilon}} - \frac{\ln\frac{A}{B}}{2\omega_a\varepsilon^{\frac{3}{2}}} - \frac{d+\frac{\omega_a^2}{2}}{2\omega_a\sqrt{\varepsilon}}\right) \tag{45}$$

$$-A\frac{1}{\sqrt{2\pi}}\exp^{\frac{-h_1^2}{2}}\cdot\left(\frac{d+\frac{\omega_a^2}{2}}{\omega_a\sqrt{\varepsilon}} - \frac{\ln\left(\frac{A}{B}\right)}{2\omega_a\varepsilon^{\frac{3}{2}}} - \frac{d+\frac{\omega_a^2}{2}}{2\omega_a\sqrt{\varepsilon}}\right) \tag{46}$$

$$= dB\cdot\exp^{-d\varepsilon}(1-N(h_2)) - A\frac{1}{\sqrt{2\pi}}\exp^{\frac{-h_1^2}{2}}\cdot\left(\frac{\frac{\omega_a^2}{2}}{\omega_a\sqrt{\varepsilon}}\right) \tag{47}$$

$$= dB\cdot\exp^{-d\varepsilon}(1-N(h_2)) - \frac{A\omega_a}{2\sqrt{\varepsilon}}\cdot N'(h_1) \tag{48}$$

$$= dB\cdot\exp^{-d\varepsilon}N(-h_2) - \frac{A\omega_a}{2\sqrt{\varepsilon}}\cdot N'(h_1) \tag{49}$$

□

Application of Theta

The passage of time of an option is guaranteed. Thus it is not compulsory to make a hedge on theta. However, theta is used to approximate gamma in a delta neutral portfolio. Theta Neutrality is achieved when the sum of all positional thetas in a portfolio adds up to zero.

3.2.4 Gamma

The gamma measures the rate of change of its delta. Neutralizing gamma is a method usedfor managing risk in options trading. This is the general formula for gamma of a call option

$$\Gamma = \frac{\partial \Delta}{\partial A} = \frac{\partial^2 f}{\partial A^2} \tag{50}$$

This is the general formula for gamma of a call option

$$\Gamma = \frac{\partial \Delta}{\partial A} = \frac{\partial^2 g}{\partial A^2} \tag{51}$$

For a European Call Option which has a non-dividend paying stock, Gamma takes the following formula

$$\Gamma = \frac{1}{A\omega_a\sqrt{\varepsilon}}N'(h_1) \tag{52}$$

The derivation of the formula is as follows

Proof.

$$\Gamma = \frac{\partial^2 f}{\partial A^2} = \frac{\partial\left(\frac{\partial f}{\partial A}\right)}{\partial A} \tag{53}$$

$$= \frac{\partial N(h_1)}{\partial h_1} \cdot \frac{\partial h_1}{\partial A} \tag{54}$$

$$= N'(h_1) \cdot \frac{\frac{1}{A}}{\omega_a\sqrt{\varepsilon}} \tag{55}$$

$$= \frac{1}{A\omega_a\sqrt{\varepsilon}}N'(h_1). \qquad \square$$

For a European Put Option that has no dividends paid on the underlying asset, the equation is as shown below:

$$\Gamma = \frac{1}{A\omega_a\sqrt{\varepsilon}}N'(h_1) \tag{56}$$

Proof.

$$\Gamma = \frac{\partial^2 g}{\partial A^2} = \frac{\partial\left(\frac{\partial g_c}{\partial A}\right)}{\partial A} \tag{57}$$

$$= \frac{\partial N(h_1)}{\partial h_1} \cdot \frac{\partial h_1}{\partial A} \tag{58}$$

$$= N'(h_1) \cdot \frac{\frac{1}{A}}{\omega_a \sqrt{\varepsilon}} \tag{59}$$

$$= \frac{1}{A\omega_a \sqrt{\varepsilon}} N'(h_1). \qquad \square$$

The first step towards achieving gamma neutrality is establishing a portfolio with the rate of change of the delta being equal to zero. A good gamma-neutral position hedges the option against risks associated with volatility. Gamma neutral options strategies are used to generate new security positions or to modify an existing one. When establishing a gamma neutral strategy we combine different gamma positions of the option, making the total gamma value equal to zero. When the value of the gamma is zero or near zero, the delta's value movement should be very minimal in response to the price of the underlying security moves.

Application Of Gamma

Delta and gamma are used together to calculate the change in stock value of an option. The following equation approximates the relation between the two greeks.
$ChangeinStockvalue \approx \Delta \times changeinStockprice + \frac{1}{2} \times \gamma \times (Changeinstockprice)^2$
It is clearly shown from the equation that gamma is used to correct the fact that delta is a linear function of stock price. The approximation shown above suggests a Taylor series expansion. Let us show the Taylor series. For a call option price f, A be the stock price, A_0 be the initial stock price, then the Taylor series expansion around A_0 yields the following:
$f(A) \approx f(A_0) + \frac{\partial P(A_0)}{\partial A}(A - A_0) + \frac{1}{2!}\frac{\partial A_o}{\partial A^2}(A - A_0) + \cdot + \frac{1}{2!}\frac{\partial^n f(A_0)}{\partial A^n}(A - A_0)^n$
For a put option price g, A be the stock price, A_0 be the initial stock price, then the Taylor series expansion around A_0 yields the following:
$g(A) \approx g(A_0) + \frac{\partial P(A_0)}{\partial A}(A - A_0) + \frac{1}{2!}\frac{\partial A_0}{\partial A^2}(A - A_0) + \cdot + \frac{1}{2!}\frac{\partial^n g(A_0)}{\partial A^n}(A - A_0)^n$
Considering the first three terms, the approximation of the call delta can be written as
$f(A) - f(A_0) \approx \frac{\partial f(A_0)}{\partial A}(A - A_0) + \frac{1}{2!}\frac{\partial^2 f(A_0)}{\partial A^2}(A - A_0)^2$
Considering the first three terms, the approximation of the put delta can be written as
$g(A) - g(A_0) \approx \frac{\partial g(A_0)}{\partial A}(A - A_0) + \frac{1}{2!}\frac{\partial^2 g(A_0)}{\partial A^2}(A - A_0)^2$
Take an example of a portfolio of call options with a delta equal of \$5000 and gamma equal to \$2500. The change in the value of the portfolio if the stock price drop to \$18 from

\$19 is approximately
$f(A) - f(A_0) \approx (\$5000) \times (\$18 - \$19) + \frac{1}{2} \times (\$2500) \times (\$18 - \$19)^2$
The above approximation can also be used to measure Modified Duration and Convexity as risk measures corresponding to delta and gamma. Modified duration measures the portfolio value which is a result of a percentage change in interest rate.
Modified duration = $\frac{\Delta \times changeininterestrate}{P}$
Modified duration = $(-Duration \times P) \times$ change in interest rate
Modified duration is similar to delta in that it only shows the first order approximation of the changes in the linear relation between the value of the portfolio and interest rate. Convexity is the interest rate gamma divided by price.
$Convexity = \frac{\Gamma}{P}$ It is used to capture the non-linear change in the changes of prices due to change in interest rate. Combining modified duration and convexity make the two work as delta and gamma.
Change in portfolio=$-Duration \times P \times (changeinrate) + \frac{1}{2} + Convexity \times P \times (changeinrate)^2$
The delta and gamma can be used in measuring risk in interest rate related portfolios. Consider the gamma of a delta-neutral portfolio to be Γ, the gamma of an option in this portfolio will be Γ_0 and the number of options added to the delta neutral portfolio will be ϕ_0. Gamma of this new portfolio will be:
$\phi_0 \Gamma_0 + \Gamma$.
A gamma-neutral portfolio is achieved if we trade
$\phi_0 = \frac{-\Gamma}{\Gamma_0}$ options
The position of options changes thus the new portfolio is not delta-neutral. Thus we change the position of the underlying asset to make it delta-neutral. Taking the delta and gamma of a call option to be 0.5 and 1.6 respectively, a delta-neutral portfolio has a gamma of -1360. To make the portfolio both delta-neutral and gamma-neutral, we buy $\frac{1360}{1.6}$ = 850 shares and sell 850×0.5 = 425 shares in the original portfolio.

3.2.5 Vega

Vega measures the sensitivity of the value of an option with respect to volatility of the underlying asset. The general formula for call Vega is as shown below

$$\nu = \frac{\partial f}{\partial \omega} \tag{60}$$

The general formula for put Vega is as shown below

$$\nu = \frac{\partial g}{\partial \omega} \tag{61}$$

Now, let us look at derivation of various kinds of options starting with call options which have a non-dividend paying underlying asset

$$\nu = A\sqrt{\varepsilon} \cdot N'(h_1) \tag{62}$$

Proof. The equation is derived as shown below

$$\begin{aligned}
\nu = \frac{\partial f}{\partial \omega_a} &= A\frac{\partial N(h_1)}{\partial \omega_a} - B\exp^{-d\varepsilon}\frac{\partial N(h_2)}{\partial \omega_a} && (63)\\
&= A\frac{\partial N(h_1)}{\partial h_1}\frac{\partial h_1}{\partial \omega_a} - B\exp^{-d\varepsilon}\frac{\partial N(h_2)}{h_2}\frac{\partial h_2}{\omega_a} && (64)\\
&= A\frac{1}{\sqrt{2\pi}}\exp^{\frac{-h_1^2}{2}}\cdot\left(\omega_a^2\varepsilon^{\frac{3}{2}} - \frac{\left[\ln\frac{A}{B} + \left(d + \frac{\omega_a^2}{2}\right)\varepsilon\right]\cdot\varepsilon^{\frac{1}{2}}}{\omega_a^2\varepsilon}\right) && (65)\\
&- B\exp^{-d\varepsilon}\left(\frac{1}{\sqrt{2\pi}}\exp^{\frac{-h_1^2}{2}}\cdot\frac{A}{B}\cdot\exp^{d\varepsilon}\right)\cdot\left(\frac{-\left[\ln\frac{A}{B} + \left(d + \frac{\omega_a^2}{2}\right)\varepsilon\right]}{\omega_a^2\varepsilon}\right) && (66)\\
&= A\frac{1}{\sqrt{2\pi}}\exp^{\frac{-h_1^2}{2}}\cdot\left(\frac{\omega_a^2\varepsilon^{\frac{3}{2}} - \left[\ln\frac{A}{B} + \left(d + \frac{\omega_a^2}{2}\right)\varepsilon\right]\cdot\varepsilon^{\frac{1}{2}}}{\omega_a^2\varepsilon}\right) && (67)\\
&- A\frac{1}{2\pi}\exp^{\frac{-h_1^2}{2}}\left(\frac{-\left[\ln\frac{A}{B} + \left(d + \frac{\omega_a^2}{2}\right)\varepsilon\right]\cdot\varepsilon^{\frac{1}{2}}}{\omega_a^2\varepsilon}\right) && (68)\\
&= A\frac{1}{\sqrt{2\pi}}\exp^{\frac{-h_1^2}{2}}\cdot\left(\frac{\omega_a^2\varepsilon^{\frac{3}{2}}}{\omega_a^2\varepsilon}\right) && (69)\\
&= A\sqrt{\varepsilon}\cdot N'(h_1).
\end{aligned}$$

□

Vega of a put option that has a non-dividend paying underlying asset takes the following equation

$$\nu = A\sqrt{\varepsilon}\cdot N'(h_1) \tag{70}$$

The proof of the above equation is shown below

Proof.

$$\nu = \frac{\partial g}{\partial \omega_a} = B\exp^{-d\varepsilon}\frac{\partial N(-h_2)}{\partial \omega_a} - A\frac{\partial N(-h_1)}{\partial \omega_a} \tag{71}$$

$$= B\exp^{-d\varepsilon}\frac{\partial(1-N(h_2))}{\partial h_2}\frac{\partial h_2}{\partial \omega_a} - A\frac{\partial(1-N(-h_1))}{\partial h_1}\frac{\partial h_1}{\partial \omega_a} \tag{72}$$

$$= -B\exp^{-d\varepsilon}\left(\frac{1}{\sqrt{2\pi}}\exp^{\frac{-h_1^2}{2}}\cdot\frac{A}{B}\cdot\exp^{d\varepsilon}\right)\cdot\left(\frac{-\left[\ln\frac{A}{B}+\left(d+\frac{\omega_a^2}{2}\right)\varepsilon\right]\varepsilon^{\frac{1}{2}}}{\omega_a^2\varepsilon}\right) \tag{73}$$

$$+A\frac{1}{\sqrt{2\pi}}\exp^{\frac{-h_1^2}{2}}\cdot\left(\frac{\omega_a^2\varepsilon^{\frac{3}{2}}-\left[\ln\frac{A}{B}+\left(d+\frac{\omega_a^2}{2}\right)\varepsilon\right]\cdot\varepsilon^{\frac{1}{2}}}{\omega_a^2\varepsilon}\right) \tag{74}$$

$$= -A\frac{1}{\sqrt{2\pi}}\exp^{\frac{-h_1^2}{2}}\cdot\left(\frac{-\left[\ln\frac{A}{B}+\left(d+\frac{\omega_a^2}{2}\right)\varepsilon\right]\cdot\varepsilon^{\frac{1}{2}}}{\omega_a^2\varepsilon}\right) \tag{75}$$

$$= +A\frac{1}{\sqrt{2\pi}}\exp^{\frac{-h_1^2}{2}}\cdot\left(\frac{\omega_a^2\varepsilon^{\frac{3}{2}}-\left[\ln\frac{A}{B}+\left(a+\frac{\omega_a^2}{2}\right)\varepsilon\right]\cdot\varepsilon^{\frac{1}{2}}}{\omega_a^2\varepsilon}\right) \tag{76}$$

$$= A\frac{1}{\sqrt{2\pi}}\exp^{\frac{-h_1^2}{2}}\cdot\left(\frac{\omega_a^2\varepsilon^{\frac{3}{2}}}{\omega_a^2\varepsilon}\right).$$

□

Application of Vega

Vega is the Greek that relates with the Black-Scholes price factor for volatility, but it signifies the sensitivity of the price of an option to volatility and not volatility itself. Options traders have always used a Vega-neutral strategy when they believe that volatility presents a risk to the profits. Calculating the Vega-neutrality of an options portfolio, involves summing up the Vegas of all the positions involved. In a Vega-neutral portfolio, total Vega of all the positions will be zero. The following example illustrates vega-neutrality Consider a portfolio that is both delta-neutral and gamma-neutral. Given that the portfolio has a vega equal to V and the vega of an option in the portfolio is V_0. We add a position of $\frac{-V}{V_0}$ in an option to make the portfolio vega neutral. For a portfolio to be vega-neutral we ought to consider taking at least two kinds of options on the same underlying asset in the portfolio. Take an example of a portfolio that has two options H and I and an underlying asset. Both options are delta-neutral and gamma-neutral. Let gamma and vega be 1600 and 1550 respectively. Option H has a delta of 0.5, gamma of 1.6, and vega of 1.8. Option I has a delta of 0.3, gamma of 1.4 and vega of 0.9. The new portfolio will be both vega neutral and gamma neutral when adding ϕ_1 of option H and ϕ_2 of option I into the original portfolio.

$Gammaneutral: -1600+1.6\phi_1+1.8\phi_2=0$

$Veganeutral: -1550+1.4\phi_1+0.9\phi_2=0$

Solving the equation gives $\phi_1 = 750$, $\phi_2 = 150$.
The delta of new portfolio is $750 \times 0.5 + 150 \times 0.3 = 420$
To maintain delta neutral we sell shares of the underlying asset.

3.2.6 Rho

Rho measures the rate at which the value of the option changes with respect to change of the risk-free interest rate. The equation of Rho is as shown below

$$\rho = B\varepsilon \cdot \exp^{-d\varepsilon} N(h_2) \tag{77}$$

Proof.

$$\rho = \frac{\partial f}{\partial d} = A\frac{\partial N(h_1)}{\partial d} - (-\varepsilon)\cdot B \cdot \exp^{-d\varepsilon} N(h_2) - B\exp^{-d\varepsilon}\frac{\partial N(h_2)}{\partial d} \tag{78}$$

$$= A\frac{\partial N(h_1)}{\partial h_1}\frac{\partial h_1}{\partial d} + B\varepsilon \cdot \exp^{-d\varepsilon} N(h_2) - B\exp^{-d\varepsilon}\frac{\partial N(h_2)}{\partial h_2}\frac{\partial h_2}{\partial d} \tag{79}$$

$$= A\frac{1}{\sqrt{2\pi}}\exp^{\frac{-h_1^2}{2}} \cdot \left(\frac{\sqrt{\varepsilon}}{\omega_a}\right) + B\varepsilon \cdot \exp^{-d\varepsilon} N(h_2) - B\exp^{-d\varepsilon}\frac{\partial N(h_2)}{\partial h_2}\frac{\partial h_2}{\partial d} \tag{80}$$

$$= A\frac{1}{\sqrt{2\pi}}\exp^{\frac{-h_1^2}{2}} \cdot \left(\frac{\sqrt{\varepsilon}}{\omega_a}\right) + B\varepsilon \cdot \exp^{-d\varepsilon} N(h_2) - A\frac{1}{\sqrt{2\pi}}\exp^{\frac{-h_1^2}{2}} \cdot \left(\frac{\sqrt{\varepsilon}}{\omega_a}\right) \tag{81}$$

$$= B\varepsilon \cdot \exp^{-d\varepsilon} N(h_2). \qquad \square$$

The rho of a put option of a non-dividend paying stock is shown below

$$\rho = \frac{\partial g}{\partial d} = (-\varepsilon)\cdot B \cdot \exp^{-d\varepsilon} N(-h_2) + B\exp^{-d\varepsilon}\frac{\partial N(-h_2)}{\partial d} - A\frac{\partial N(-h_1)}{\partial d} \tag{82}$$

Proof.

$$= B\varepsilon \cdot \exp -d\varepsilon\,(1 - N(h_2)) + B\exp^{-d\varepsilon}\frac{\partial(1 - N(h_2))}{\partial h_2} - A\frac{\partial(1 - N(h_1))}{\partial h_1}\frac{\partial h_1}{\partial d} \tag{83}$$

$$= B\varepsilon \cdot \exp -d\varepsilon\,(1 - N(h_2)) - B\exp^{-d\varepsilon} \cdot \left(\frac{1}{\sqrt{2\pi}}\exp^{\frac{-h_1^2}{2}} \cdot \frac{A}{B} \cdot \exp^{d\varepsilon}\right) \cdot \left(\frac{\sqrt{\varepsilon}}{\omega_a}\right) \tag{84}$$

$$+ A\frac{1}{\sqrt{2\pi}}\exp^{\frac{-h_1^2}{2}} \cdot \left(\frac{\sqrt{\varepsilon}}{\omega_a}\right) \tag{85}$$

$$= B\varepsilon \cdot \exp -d\varepsilon\,(1 - N(h_2)) - A\frac{1}{\sqrt{2\pi}}\exp^{\frac{-h_1}{2}} \cdot \left(\frac{\sqrt{\varepsilon}}{\omega_a}\right) + A\frac{1}{\sqrt{2\pi}}\exp^{\frac{-h_1^2}{2}} \cdot \left(\frac{\sqrt{\varepsilon}}{\omega_a}\right) \tag{86}$$

$$= -B\varepsilon \cdot \exp^{-d\varepsilon} N(-h_2). \qquad \square$$

Application of Rho

Let A=$40 and B=$32, d= 4 per annum and is 27 per annum. The rho of a 6 months put option can be calculated as

$$\rho = B\varepsilon \cdot \exp^{-d\varepsilon}\left(1-N\left(h_2\right)\right) = (32)\,(0.27)\exp^{-0.04\times 0.27} N\frac{\ln\frac{40}{32}+\left[0.04-\frac{1}{2}\left(0.3^2\right)\right](0.5)}{0.27\sqrt{0.5}} \approx 7.496 \tag{87}$$

The rho neutrality of an option portfolio is achieved when their respective rho positions are equal to zero.

3.3 Linear Programming Model

We now want to minimize all the risks in the financial derivative market. However combining all Greek neutralities would require so much work because it would involve adjusting the position of other neutralities every time you neutralize one Greek. Linear programming is known to combine constraints to provide a solution for really complex problems. In this study we construct a standard linear program which maximizes profit while reducing the risks in the market altogether.
The following notations will apply to the constraints and variables
Variables
W- Buy a call option
X- Sell a call option
Y- Buy a put option
Z-Sell a put Option.
Constraints
f and g are the call and put options (This is value attained after subtracting the option premium from options black-scholes price)
Δ_W - Buy Call Delta.
Γ_X - Sell Call Gamma.
ν_Y - Buy Put Vega.
ρ_Z - Sell Put Rho.
Θ_W - Buy Call Theta.

$$\begin{aligned} \text{maximize} \quad & fW + fX + gY + gZ \\ \text{subject to} \quad & \Delta_W + \Delta_X + \Delta_Y + \Delta_Z = 0 \\ & \Gamma_W + \Gamma_X + \Gamma_Y + \Gamma_Z = 0 \\ & \nu_W + \nu_X + \nu_Y + \nu_Z = 0 \\ & \rho_W + \rho_X + \rho_Y + \rho_Z = 0 \\ & \Theta_W + \Theta_X + \Theta_Y + \Theta_Z = 0 \end{aligned} \tag{88}$$

such that
$f = (f_1, \ldots, f_8)$
$g = (g_1, \ldots, g_8)$
$W = (W_1, \ldots, W_8)$
$X = (X_1, \ldots, X_8)$
$Y = (Y_1, \ldots, Y_8)$
$Z = (Z_1, \ldots, Z_8)$
Thus
$fW = (f_1W_1, \ldots, f_8W_8)$, $\Delta_W = (\Delta_{W_1}, \ldots, \Delta_{W_8})$,$\Gamma_X = (\Gamma_{X_1}, \ldots, \Gamma_{X_8})$, etc
This Linear programming model used in this study applies assumptions like the ones used by Christos and is applied to Netflix options. The following are the assumptions that were made for the linear program.
(a) No a priori strategies are considered or all combinations are possible.
(b) All transaction costs are not considered. Hull argues that this assumption holds true for options with big portfolios because the transaction costs are insignificantly small. Thus this model can be applied to large options portfolios
(c)An arbitrage profit is possible if one takes a position now and an opposite position upon the option expiration date. In our case upon expiration of the option. The values of the Greeks were also calculated and plugged in the linear program.

3.3.1 Data Collection

The data was obtained from the website finance.yahoo.com. We focused on the adjusted closing stock prices of 8 call and 8 put Netflix options. Netflix is one of the best performers in the 500 index. The range of data was a full trading year, from 22/10/2018-22/10/2019. The market premium of the options was observed from www.nasdaq.com on their expiration date.

3.3.2 Excel Calculations that were used to calculate values found in the linear program

The following calculations were used to find values of our Greek constraint parameters and theoretical prices in the objective function.

Netfix Historical Volatility

This type of volatility relies on past price movements of an asset. In our case, this will equal a 21- day standard deviation of daily returns and a 31-day standard deviation both expressed in yearly terms.These computations were done using Excel with the steps out-

lined below;
(i) First the log returns were calculated using the formula $H = \ln\left(\frac{X_c}{X_c - 1}\right)$ where X_t is the adjusted close of the current day and $X_c - 1$ the adjusted close of the previous day.

(ii) The sample standard deviation of a rolling block of 21 days and 31 days respectively was then calculated and each of the results annualized by multiplying with 252.
The graphs of annual historical volatility against time are found in the appendix.

Netfix Options Prices and Greeks

The Black-Scholes formula was used to calculate the put and call options prices using excel. The Greeks were calculated using excel too. The values of the Greeks were used as the parameters in the constraints of our linear program. The Excel tables are shown below. we start with a table for 21 days then 31 days. Then calculations of options prices and Greeks.

The table below is for options prices with expiration date of 21 days, a risk free interest rate of 0.019 and historical volatility of 0.3640

Table 1. Table of Call Option Prices with expiration date of 21 days

Strike price	d1	d2	N(d1)	N(d2)	Call value
260	0.179665408	0.074564854	0.571292374	0.529719531	12.78641516
262.5	0.088614975	-0.016485579	0.535306047	0.493423503	11.50956203
265	-0.001572408	-0.106672962	0.4993727	0.457524217	10.32287023

The table below shows values of Greeks with expiration date of 21 days, risk free interest rate of 0.019 and historical volatility of 0.3640

Table 2. Table of Call Greeks with expiration date of 21 days

Stock Price	Strike price	Call Delta	Gamma	Call Theta	Vega	Call Rho
263.08	260	0.571292	0.014197366	-65.13493144	0.298124616	0.114590986
263.08	262.5	0.535306	0.014371829	-65.93459021	0.301788094	0.107765627
263.08	265	0.499373	0.014428351	-66.19323403	0.302974962	0.10087675

The next table is for call options prices with expiration date of 31 days, a risk free interest rate of 0.019 and historical volatility of 0.3290

Table 3. Table of Call Option prices with expiration date of 31 days

Stock price	Strike price	d1	d2	N(d1)	N(d2)	Call value
263.08	260	0.180001515	0.064597735	0.571424311	0.525752856	13.95369116
263.08	262.5	0.097080045	-0.018323734	0.538668586	0.492690297	12.68366184
263.08	265	0.014944573	-0.100459207	0.5059618	0.459989884	11.49568932
263.08	267.5	-0.066419662	-0.181823442	0.473521858	0.427860643	10.38860649
263.08	270	-0.147027011	-0.262430791	0.44155535	0.396494668	9.360745346

The table below shows values of Greeks with expiration date of 31 days, risk free interest rate of 0.019 and historical volatility of 0.3290

Table 4. Table of Call Greeks with expiration date of 31 days

Stock price	Strike price	Call Delta	Gamma	Call Theta	Vega	Call Rho
263.08	260	0.571424	0.012929047	-48.448828	0.36219528	0.167764885
263.08	262.5	0.538669	0.013078432	-49.00794489	0.366380173	0.158726483
263.08	265	0.505962	0.013138739	-49.23332611	0.368069624	0.149602975
263.08	267.5	0.473522	0.013111254	-49.1297929	0.36729965	0.140466319
263.08	270	0.441555	0.012998946	-48.70847468	0.364153439	0.131385425

The table below is for put options prices with expiration date of 21 days, a risk free interest rate of 0.019 and historical volatility of 0.3640

Table 5. Table of Put Option Prices with expiration date of 21 days

Stock Price	Strike price	d1	d2	N(-d1)	N(-d2)	Put Value
263.08	260	-0.179665408	-0.074564854	0.428707626	0.470280469	9.295074221
263.08	262.5	-0.088614975	0.016485579	0.464693953	0.506576497	10.5142659
263.08	265	0.001572408	0.106672962	0.5006273	0.542475783	11.82361889

The table below shows values of Greeks with expiration date of 21 days, risk free interest rate of 0.019 and historical volatility of 0.3640

Table 6. Table of Put Greeks with expiration date of 21 days

Stock Price	Strike price	Put Delta	Gamma	Put theta	Vega	Put Rho
263.08	260	-0.428707626	0.014197366	-64.64010489	0.298124616	-0.101732897
263.08	262.5	-0.464693953	0.014371829	-65.39797553	0.301788094	-0.110638293
263.08	265	-0.5006273	0.014428351	-65.61453449	0.302974962	-0.119607207

The table below is for put options prices with expiration date of 31 days, a risk free interest rate of 0.019 and historical volatility of 0.3290

Table 7. Table of Put Option Prices with expiration date of 21 days

Stock price	Strike price	d1	d2	N(-d1)	N(-d2)	Put Value
263.08	260	-0.180001515	-0.064597735	0.428575689	0.474247144	10.26670238
263.08	262.5	-0.097080045	0.018323734	0.461331414	0.507309703	11.49083663
263.08	265	-0.014944573	0.100459207	0.4940382	0.540010116	12.79702768
263.08	267.5	0.066419662	0.181823442	0.526478142	0.572139357	14.18410842
263.08	270	0.147027011	0.262430791	0.55844465	0.603505332	15.65041085

The table below shows values of Greeks with expiration date of 31 days, risk free interest rate of 0.019 and historical volatility of 0.3290

Table 8. Table of Put Greeks with expiration date of 31 days

Stock price	Strike price	put delta	Gamma	Put theta	Vega	Put Rho
263.08	260	-0.428575689	0.012929047	-47.95001714	0.36219528	-0.151329692
263.08	262.5	-0.461331414	0.013078432	-48.47060493	0.366380173	-0.163436312
263.08	265	-0.4940382	0.013138739	-48.65718752	0.368069624	-0.175628036
263.08	267.5	-0.526478142	0.013111254	-48.51481405	0.36729965	-0.187832909
263.08	270	-0.55844465	0.012998946	-48.05483217	0.364153439	-0.19998202

The calculated values were plugged in the linear programming model below, which is part of the complete linear program. The full linear program is found in the appendix
Max
$1.6W_1 + 2.1W_2 + 1.6W_3 + 1.27W_4 + 2.12W_5 + 1.63W_6 + 1.56W_7 + 1.93W_8 - 1.6X_1 - 2.1X_2 - 1.6X_3 - 1.27X_4 - 2.12X_5 - 1.63X_6 - 1.56X_7 - 1.93X_8 + 2.4Y_1 + 2Y_2 + 1.6Y_3 + 1.88Y_4 + 2.09Y_5 + 1.75Y_6 + 2.3Y_7 + 1.63Y_8 - 2.4Z_1 - 2Z_2 - 1.6Z_3 - 1.88Z_4 - 2.09Z_5 - 1.75Z_6 - 2.3YP267.5Z_7 - 1.63Z_8$

Subject to:
$0.57129W_1 + 0.53531W_2 + 0.49937W_3 + 0.57142W_4 + 0.53867W_5 + 0.50596W_6 + 0.47352W_7 + 0.44156W_8 - 0.57129X_1 - 0.53531X_2 - 0.49937X_3 - 0.57142X_4 - 0.53867X_5 - 0.50596X_6 - 0.47352X_7 - 0.44156X_8 - 0.42871Y_1 - 0.46469Y_2 - 0.50063Y_3 - 0.42858Y_4 - 0.46133Y_5 - 0.49404Y_6 - 0.52648Y_7 - O.55884Y_8 - 0.42871Z_1 + 0.46469Z_2 + 0.50063Z_3 + 0.42858Z_4 + 0.46133Z_5 + 0.49404Z_6 + 0.52648Z_7 + 0.55884Z_8 + K - S = 0$
$\vdots$
$\vdots$
$\vdots$

It was observed that the linear program with the five risk constraints could not observe a unique solution and one had to repeat the same equation ten times in order to achieve ten times the profit thus we added a scale constraint to ensure a unique solution. The scale constraint
$0.57129W_1 + 0.53531W_2 + 0.49937W_3 + 0.57142W_4 + 0.53867W_5 + 0.50596W_6 + 0.47352W_7 + 0.44156W_8 + 0.42871Z_1 + 0.46469Z_2 + 0.50063Z_3 + 0.42858Z_4 + 0.46133Z_5 + 0.49404Z_6 + 0.52648Z_7 + 0.55845Z_8 + K = 1700$
Buying an option at the market price and selling at the Black-Scholes price would lead to a profit and buying the option at black-scholes price and then selling at the market price would lead to a loss.
It was observed that some values in our general equation took a negative sign. The negative sign in the objective function means that the investor has sold the option at the option premium . The values were then ran in excel solver and used a simplex method. The results are shown in the appendix. The complete Linear program containing calculated values is found in the appendix. In order for us to determine the effect of the Greeks on the option, we started deleting one Greek at a time and observed the changes in the results of our model. The adjustment gave rise to five more solutions which are briefly described below:
(1) Solution 1 - Consisted of Delta, Gamma, Kappa, Rho, Theta and the Scale constraints.
(2) Solution 2 - Consisted of Delta, Gamma, Kappa, Rho and the Scale constraints.
(3) Solution 3 - Consisted of Delta, Gamma, Kappa and the Scale constraints.
(4) Solution 4 - Consisted of Delta, Gamma and the Scale constraints.
(5) Solution 5 - Consisted of Delta and the Scale constraints.

4 Data Analysis And Results

4.1 Solution 1

Table 9. Summary of Answer and Sensitivity Reports for Delta, Gamma, Kappa, Rho, Theta and Constant

Solution 1				
Greek Name	Shadow Price	Allowable increase	Transaction name and number of shares	Max Profit
Delta	$-1.58	1700	W_5 - 349.61	$2682.4
Gamma	$-19040.34	0.56	X_3 - 752.48	
Kappa	$325.69	1.02	Y_1 - 3088.89	
Rho	$-3.52	158.3	Z_3 - 2286.97	
Theta	$-2.7	272.5	Z_6 - 348.01	
Constant	$1.58	infinity	K - 194.8	

4.1.1 Answers Report

In solution 1, the Linear Programming model offers the following optimization strategy; buying 3.4961 call options equivalent to 349.61 shares (with strike price of 262.5 and time to maturity of 31 days), selling 7.5248 call options equivalent to 752.48 shares with a strike price of 265 and time to maturity of 21 days), buying 30.8889 put options equivalent to 3088.89 shares with a strike price of is 260 and time to maturity is 21 days, selling 22.8696 put options equivalent to 2286.96 shares with a strike price of 265 and a time to maturity of 21 days), selling 3.48006 put options equivalent to 348.006 shares with a strike price of 267.5 and time to maturity of 31 days), sell 194.82 shares. All constraints were found to be binding thus no slack variables and the maximum profit is $2682.40.

4.1.2 Shadow Price

The Delta shadow price for the solution 1 is -1.578. The shadow price suggests that there is a loss of 1.578 if we increase delta risk by one unit. An example of increasing delta risk is by buying one more share. The gamma constraint has a shadow price of -19040.34 .The price suggests that there will be a loss of 19040.34 if we increase the gamma risk by one unit. The shadow price associated with Kappa constraint is 325.69. This suggests that if we increase kappa risk by one unit, the profit goes down by 325.69. However, the risk has an allowable increase of 0.31 thus the profit can only increase by 32.57. If we increase the rho risk by one unit, then there will be a loss of 3.516 on the maximum profit. Thus its not profitable move. If we increase theta risk by one unit, there will be a loss of 2.701 on the maximum profit. If we increase the constant risk by one unit, the maximum profit will increase by 1.578. The allowable increase is 0.55 giving a maximum increase of 0.8679 on the profit. From the results above, we note that the critical constraint is the kappa constraint.

4.2 solution 2

Table 10. Summary of Answer and Sensitivity Reports for Delta, Gamma, Kappa, Rho and Constant

Solution 2				
Greek Name	Shadow Price	Allowable increase	Transaction name and number of shares	Max Profit
Delta	$-0.86	50.37	X_3 165.7	$3048.24
Gamma	$152.04	0.56	X_4 60.13	
Kappa	$-0.49	9.71	Y_1 3619.41	
Rho	$-0.17	6.9	Y_7 59.29	
Constant	$1.79	infinity	X_3 3395.74	

4.2.1 Answers Report

In solution 2, when the theta constraint is deleted, the Linear Programming model gives the following optimization strategy; selling 1.657 call options equivalent to 165.7 shares with a strike price of $265 and time to maturity of 21 days), sell 6.013 call options equivalent to 60.13 shares with a strike price of $260 and expires after 31 days, buy 36.19 put options equivalent to 3619 shares with a strike price of $260 and expires after 21 days, sell 0.5929 put options equivalent to 59.29 shares with a strike price of $267.5 and time to expiration of 31 days, sell 33.9574 put options equivalent to 3395.74 shares the strike price of the underlying asset is $265 and time to maturity is 21 days

4.2.2 Shadow Price

The Delta shadow price for the models 2 is $-0.8603. The price suggests that there is a loss of $0.8603 change in profit if we increase the delta risk by one unit. The gamma constraint has a shadow price of $152.04. The figure suggests that the maximum profit increases by $152.04 if the gamma risk is increased by one unit. The allowable increase of the constraint is 0.56 which means that the increase on the maximum profit can only get to $85.14. The shadow price associated with Kappa constraint is $-0.48. This suggests that if we increase the kappa risk by one unit, the maximum profit would decrease by $0.48. The shadow price for Rho constraint is $-0.16. This suggests that if we increase the Rho risk by one unit, the maximum profit would suffer a loss of $0.16. The shadow prices for the size constraints in our model is $1.79. The price suggests that if we buy an extra share the profit increases by $1.79. However, the size constraint is a slack variable thus the critical constraint is the gamma constraint.

4.3 Solution 3

Table 11. Summary of Answer and Sensitivity Reports for Delta, Gamma, Kappa and Constant

Solution 3				
Greek Name	Shadow Price	Allowable increase	Transaction name and number of shares	Max Profit
Delta	$-0.92	220.54	X_4 202.22	$3051.01
Gamma	$149.6	0.87	Y_1 3450.99	
Kappa	$-0.39	155.36	Y_7 199.41	
Constant	$1.79	infinity	Z_3 3395.74	

4.3.1 Answer Report

In solution 3, after deleting theta and rho, the optimal solution gave us the following strategy; selling 2.022 call options equivalent to 202.2 shares (the strike price of the underlying asset is 260 and the time to maturity is 31 days), buying 34.51 put options equivalent to 3451 shares (the strike price of the underlying asset is 260 and time to maturity is 21 days), buying 1.994 put options equivalent to 199.4 shares (the strike price of the underlying asset is 265 and time to maturity is 21 days), buying 33.96 put options equivalent to 3396 shares (the strike price of the underlying asset is 267.5 and time to maturity is 21 days).

4.3.2 Shadow Price

The Delta shadow price for solution 3 is $-0.9151. The price suggests that there is a loss of $0.9151 in the profit if we increase the delta risk by one unit. The gamma constraint has a shadow price of $149.6.The figure suggests that the profit increases by $149.6 if the gamma risk was increased by one unit. The allowable increase associated with the shadow price is 0.8721. This gives a maximum increase of $130.2 on the maximized profit. The shadow price associated with Kappa constraint is $-0.3901. This indicates that if we increase the kappa risk by one unit, the maximized profit would decrease by $-0.3901. The shadow prices for the size constraints in our model is $1.794. The critical constraint in this solution is the gamma.

4.4 Solution 4

Table 12. Summary of Answer and Sensitivity Reports for Delta, Gamma and Constant

Solution 4				
Greek Name	Shadow Price	Allowable increase	Transaction name and number of shares	Max Profit
Delta	$-0.95	50.37	X_4 229.29	$3048.24
Gamma	$140.3	0.56	Y_1 3659.79	
Constant	$1.79	infinity	Z_3 3395.74	

4.4.1 Answer Report

In solution 4, after deleting theta, rho and kappa, the optimal solution gave the following strategy; sell 2.2929 call options equivalent to 229.29 shares (where the underlying asset has a strike price of 260 and time to maturity is 31 days), buy 36.5979 put options equivalent to 3659.74 shares (where the underlying asset has a strike price of 260 and time to expiration is 21 days), sell 33.9574 put options equivalent to 3395.74 shares (with the underlying asset of the options having a strike price of 265 and time to expiration date is 21 days).

4.4.2 Shadow Price

The Delta shadow price for the models 3 is $-0.9519. The $-0.9519 shadow price suggests that there is a loss of $0.9519 in the maximized profit if we increase the delta risk value by one unit. The gamma constraint has a shadow price of $140.3.The figure suggests that the maximized profit increased by $140.3 if the gamma risk was increased by one unit. The allowable increase is 7.30. This indicates that the profit can increase upto $1024.19.. The shadow price for the size constraints in our model is $1.799. Thus gamma is our critical constraint.

4.5 Solution 5

4.5.1 Answers Report

In solution 5, after deleting, theta, rho, kappa and gamma constraints the optimal solution took the form; buying 38.5003 call options equivalent to 3850.03 shares with a strike price

Table 13. Summary of Answer and Sensitivity Reports for Delta and Constant

Solution 5				
Greek Name	Shadow Price	Allowable increase	Transaction name and number of shares	Max Profit
Delta	$-5.6	1700	W_8 3850.03	$16947.53
Constant	$9.97	infinity	Y_1 3965.41	

of 270 and time to maturity is 31 days and buying 39.6541 put options equivalent to 3965.41 shares with a strike price of 260 and time to maturity is 21 days.

4.5.2 Shadow Price

The Delta shadow price for the models 5 is $-5.60. The $-5.60 shadow price suggests that there is a loss of $5.60 in the profit if we increase the delta risk by one unit. The constant constraint has a shadow price of $9.969. The price suggests that if we buy an extra share the profit increases by $9.97.

4.6 Comment

It was observed that profit was lowest in solution 1 and highest in solution 5. That is when one includes all Greeks the profit reduces and when one considers only one Greek and the size constraint the profit is highest. This means that the investing in a risky option gives more profit than an option that is hedged from all risks in the market.

5 Conclusion

Linear Programming models provide an efficient way for solving complicated problems. The case study in this study is a good example. We found the optimal strategy for hedging a portfolio of options. The solution provided by the Linear Programming model gives the best strategy for optimizing the portfolio compared to delta-neutral, delta-vega, delta-gamma neutral and even a guess work practised by speculative traders. The Linear Program hedges the options portfolio against all risks in the market, something that would have been more complicated due to the fact that hedging one Greek influences the value of another Greek. This study contributed in specifying the number of options to be bought or sold when the linear program is optimized and gave the critical constraint in every solution. The shadow price in the excel solver solutions provide the investor with information on how the profit would be affected by increasing the risk level by one unit. It was observed that the profit was lowest when all five Greeks were included and increased with decrease in number of constraints. linear, yet they are not.

Future Research

The assumption that transaction costs and margins are not included also give profit values that might be higher than the ones experienced in the market. Thus need for more research on this study

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6 Appendix

6.1 Annual Historical Volatility Graphs

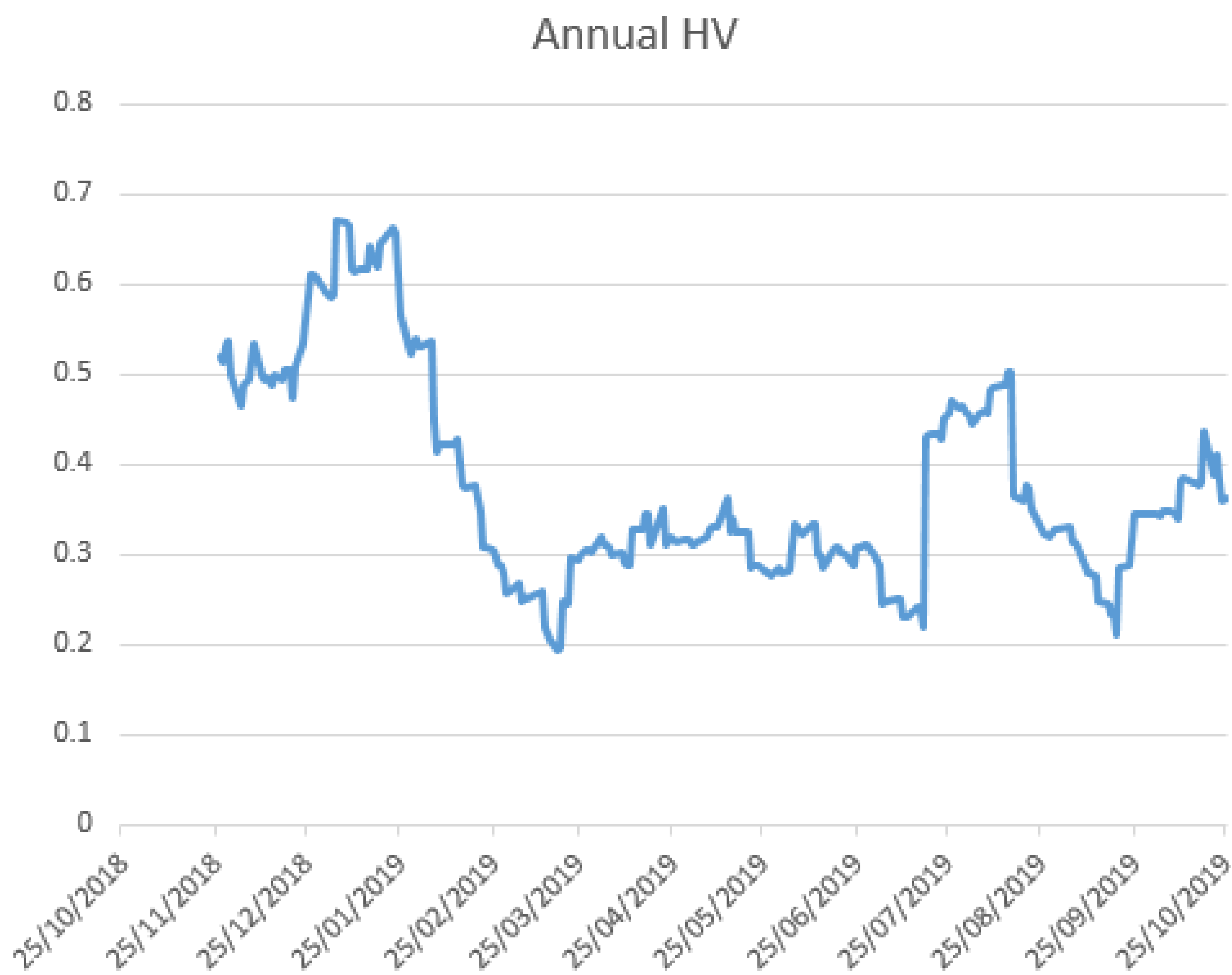

Figure 1. The Historical Volatility for 21 days

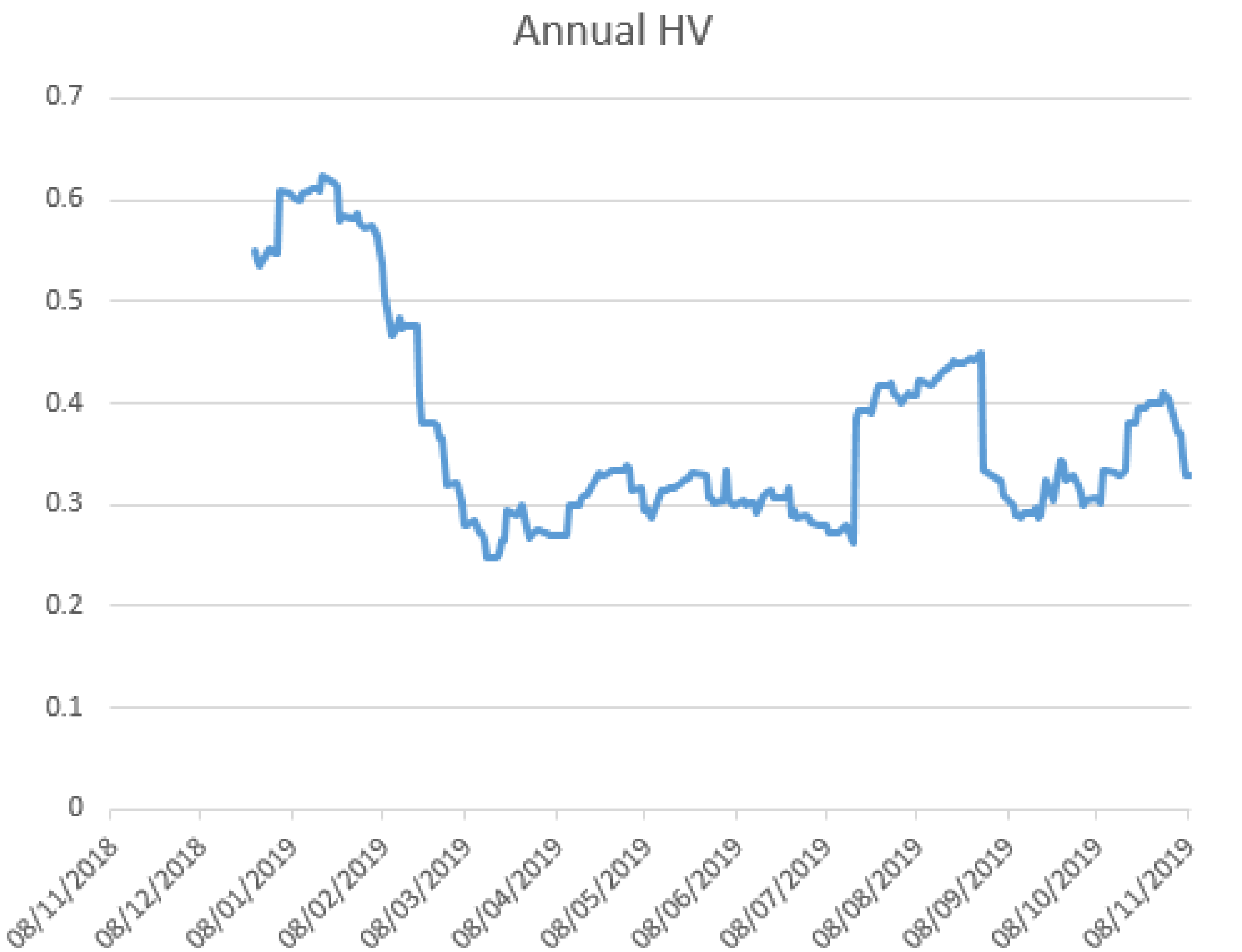

Figure 2. The Historical Volatility for 31 days

Max

$1.6W_1 + 2.1W_2 + 1.6W_3 + 1.27W_4 + 2.12W_5 + 1.63W_6 + 1.56W_7 + 1.93W_8 - 1.6X_1 - 2.1X_2 - 1.6X_3 - 1.27X_4 - 2.12X_5 - 1.63X_6 - 1.56X_7 - 1.93X_8 + 2.4Y_1 + 2Y_2 + 1.6Y_3 + 1.88Y_4 + 2.09Y_5 + 1.75Y_6 + 2.3Y_7 + 1.63Y_8 - 2.4Z_1 - 2Z_2 - 1.6Z_3 - 1.88Z_4 - 2.09Z_5 - 1.75Z_6 - 2.3YP267.5Z_7 - 1.63Z_8$

$0.57129W_1 + 0.53531W_2 + 0.49937W_3 + 0.57142W_4 + 0.53867W_5 + 0.50596W_6 + 0.47352W_7 + 0.44156W_8 - 0.57129X_1 - 0.53531X_2 - 0.49937X_3 - 0.57142X_4 - 0.53867X_5 - 0.50596X_6 - 0.47352X_7 - 0.44156X_8 - 0.42871Y_1 - 0.46469Y_2 - 0.50063Y_3 - 0.42858Y_4 - 0.46133Y_5 - 0.49404Y_6 - 0.52648Y_7 - O.55884Y_8 - 0.42871Z_1 + 0.46469Z_2 + 0.50063Z_3 + 0.42858Z_4 + 0.46133Z_5 + 0.49404Z_6 + 0.52648Z_7 + 0.55884Z_8 + K - S = 0$

$0.0142W_1 + 0.01437W_2 + 0.01443W_3 + 0.01293W_4 + 0.01308W_5 + 0.01314W_6 + 0.01311W_7 + 0.013W_8 - 0.0142X_1 - 0.01437X_2 - 0.01443X_3 - 0.01293X_4 - 0.01308X_5 - 0.01314X_6 - 0.01311X_7 - 0.013X_8 + 0.0142Y_1 + 0.01437Y_2 + 0.01443Y_3 + 0.01293Y_4 + 0.01308Y_5 + 0.01314Y_6 + 0.01311Y_7 + 0.013Y_8 - 0.0142Z_1 - 0.01437Z_2 - 0.01443Z_3 - 0.01293Z_4 - 0.01308Z_5 - 0.01314Z_6 - 0.01311Z_7 - 0.013Z_8 = 0$

$0.29812W_1 + 0.30179W_2 + 0.30298W_3 + 0.36280W_4 + 0.36638W_5 + 0.36807W_6 + 0.3673W_7 + 0.36415W_8 - 0.29812X_1 - 0.30179X_2 - 0.30298X_3 - 0.36280X_4 - 0.36638X_5 - 0.36807X_6 - 0.3673X_7 - 0.36415X_8 + 0.29812Y_1 + 0.30179Y_2 + 0.30298Y_3 + 0.3622Y_4 + 0.36638Y_5 + 0.36807Y_6 + 0.3673Y_7 + 0.36415Y_8 - 0.29812Z_1 - 0.30179Z_2 - 0.30297Z_3 - 0.3622Z_4 - 0.36638Z_5 - 0.36807Z_6 - 0.3673Z_7 - 0.36415Z_8 = 0$

$0.11459W_1 + 0.10777W_2 + 0.10088W_3 + 0.16777W_4 + 0.15873W_5 + 0.1496W_6 + 0.14047W_7 + 0.13139W_8 - 0.11459X_1 - 0.10777X_2 - 0.10088X_3 - 0.16777X_4 - 0.15873X_5 - 0.1496X_6 - 0.14047X_7 - 0.13139X_8 - 0.10173Y_1 - 0.11064Y_2 - 0.11961Y_3 - 0.15133Y_4 - 0.16344Y_5 - 0.17563Y_6 - 0.18783Y_7 - 0.19998Y_8 + 0.10173Z_1 + 0.11064Z_2 + 0.11961Z_3 + 0.15133Z_4 + 0.16344Z_5 + 0.17563Z_6 + 0.18783Z_7 + 0.19998Z_8 = 0$

$-65.135W_1 - 65.935W_2 - 66.193W_3 - 48.449W_4 - 49.008W_5 - 49.233W_6 - 49.13W_7 - 48.709W_8 + 65.135X_1 + 65.935X_2 + 66.193X_3 + 48.449X_4 + 49.008X_5 + 49.233X_6 + 49.13X_7 + 48.708X_8 - 64.64Y_1 - 65.398Y_2 - 65.615Y_3 - 47.95Y_4 - 48.471Y_5 - 48.657Y_6 - 48.515Y_7 - 48.055Y_8 + 64.64Z_1 + 65.398Z_2 + 65.615Z_3 + 47.95Z_4 + 48.471Z_5 + 48.657Z_6 + 48.515Z_7 + 48.055Z_8 = 0$

$0.57129W_1 + 0.53531W_2 + 0.49937W_3 + 0.57142W_4 + 0.53867W_5 + 0.50596W_6 + 0.47352W_7 + 0.44156W_8 + 0.42871Z_1 + 0.46469Z_2 + 0.50063Z_3 + 0.42858Z_4 + 0.46133Z_5 + 0.49404Z_6 + 0.52648Z_7 + 0.55845Z_8 + K = 1700$

6.2 Excel Solver Results

6.2.1 Answers report when Delta, Gamma, Vega, Rho, Theta and Scale Constraints are included

Table 14. Profits

Name	Original Value	Final Value
Total Profit	2682.400607	2682.400607

Table 15. Variables

Name	Original Value	Final Value	Integer
Transaction XC262.5W	349.6107221	349.6107221	Contin
Transaction YC265V	752.4780646	752.4780646	Contin
Transaction XP260V	3088.894836	3088.894836	Contin
Transaction YP265V	2286.966457	2286.966457	Contin
Transaction YP265W	348.005998	348.005998	Contin
Transaction K	194.8294433	194.8294433	Contin

Table 16. Constraints

Name	Cell Value	Status	Slack
Delta	7.95808E-13	Binding	0
Gamma	8.88178E-15	Binding	0
Kappa	5.40012E-13	Binding	0
Rho	2.34479E-13	Binding	0
Theta	-4.36557E-11	Binding	0
Constant	1700	Binding	0

6.2.2 Sensitivity Report when Delta, Gamma, Vega, Rho, Theta and Scale Constraints are included

Table 17. Variables

	Final	Reduced	Objective	Allowable	Allowable
Name	Value	Cost	Coefficient	Increase	Decrease
Transaction XC262.5W	349.6107221	0	2.12	0.117608243	0.335140026
Transaction YC265V	752.4780646	0	-1.6	0.227016961	0.219012049
Transaction XP260V	3088.894836	0	2.4	0.312218142	0.100156887
Transaction YP265V	2286.966457	0	-1.6	0.169455125	0.105776947
Transaction YP265W	348.005998	0	-1.75	0.241925155	0.036643621
Transaction K	194.8294433	0	0	0.554657928	0.896750674

Table 18. Constraints

	Final	Shadow	Constraint	Allowable	Allowable
Name	Value	Price	R.H. Side	Increase	Decrease
Delta	7.95808E-13	-1.57788271	0	1700	220.048188
Gamma	8.88178E-15	-19040.3434	0	0.038404736	0.017477667
Kappa	5.40012E-13	325.6858056	0	1.020667793	2.265715994
Rho	2.34479E-13	-3.515549066	0	158.3282812	44.69541638
Theta	-4.36557E-11	-2.701011651	0	272.5107859	124.7389757
Constant	1700	1.57788271	1700	1E+30	1700

6.2.3 Answers Report When Theta is deleted

Table 19. Profit

Name	Original Value	Final Value
Total profit	2.66454E-15	3048.24471

Table 20. Variables

Name	Original Value	Final Value	Integer
Transaction YC265V	1	165.722755	Contin
Transaction YC260W	1	60.1288673	Contin
Transaction XP260V	1	3619.405759	Contin
Transaction XP267.5W	1	59.29325527	Contin
Transaction YP265V	1	3395.739706	Contin

Table 21. Constraints

Name	Cell Value	Status	Slack
Delta	3.18323E-12	Binding	0
Gamma	7.81597E-14	Binding	0
Kappa	2.27374E-12	Binding	0
Rho	7.95808E-13	Binding	0
Constant	1700	Binding	0

6.2.4 Sensitivity when Theta is deleted

Table 22. Variables

	Final	Reduced	Objective	Allowable	Allowable
Name	**Value**	**Cost**	**Coefficient**	**Increase**	**Decrease**
Transaction YC265V	165.722755	0	-1.6	0.292178051	0.088867813
Transaction YC260W	60.1288673	0	-1.27	0.171102939	0.183769668
Transaction XP260V	3619.405759	0	2.4	0.768105608	0.142715719
Transaction XP267.5W	59.29325527	0	2.3	0.310585379	0.159346168
Transaction YP265V	3395.739706	0	-1.6	8.54664E+13	0.154529944

Table 23. Constraints

	Final	Shadow	Constraint	Allowable	Allowable
Name	**Value**	**Price**	**R.H. Side**	**Increase**	**Decrease**
Delta	3.18323E-12	-0.860334258	0	50.37334771	31.5840249
Gamma	7.81597E-14	152.0408147	0	0.564944112	0.480744419
Kappa	2.27374E-12	-0.485154977	0	9.709871507	12.44292728
Rho	7.95808E-13	-0.169322287	0	6.899595085	16.30423129
Constant	1700	1.793085124	1700	1E+30	1700

6.2.5 Answer report when Theta and Rho are deleted

Table 24. Profit

Name	Original Value	Final Value
Total profit	2.66454E-15	3051.00538

Table 25. Variables

Name	Original Value	Final Value	Integer
Transaction YC260W	1	202.2176345	Contin
Transaction XP260V	1	3450.986774	Contin
Transaction XP267.5W	1	199.4074121	Contin
Transaction YP265V	1	3395.739706	Contin

Table 26. Constraints

Name	Cell Value	Status	Slack
Delta	-2.27374E-13	Binding	0
Gamma	7.10543E-15	Binding	0
Kappa	2.27374E-13	Binding	0
Constant	1700	Binding	0

6.2.6 Sensitivity report when Theta and rho are deleted

Table 27. Variables

	Final	Reduced	Objective	Allowable	Allowable
Name	Value	Cost	Coefficient	Increase	Decrease
Transaction YC260W	202.2176345	0	-1.27	0.18372528	0.019429191
Transaction XP260V	3450.986774	0	2.4	0.016391678	0.153128854
Transaction XP267.5W	199.4074121	0	2.3	0.332216599	0.019703004
Transaction YP265V	3395.739706	0	-1.6	1E+30	0.155620187

Table 28. Constraints

	Final	Shadow	Constraint	Allowable	Allowable
Name	Value	Price	R.H. Side	Increase	Decrease
Delta	-2.27374E-13	-0.915138432	0	220.5356534	3.30678E+17
Gamma	7.10543E-15	149.6037873	0	0.872136955	12.2696653
Kappa	2.27374E-13	-0.390127432	0	155.3629074	20.79713508
Constant	1700	1.794709047	1700	1E+30	1700

6.2.7 Answer report when Theta, Rho and Kappa are deleted

Table 29. Profit

Name	Original Value	Final Value
Total Profit	2.66454E-15	3059.118913

Table 30. Variables

Name	Original Value	Final Value	Integer
Transaction YC260W	1	229.2868207	Contin
Transaction XP260V	1	3659.790294	Contin
Transaction YP265V	1	3395.739706	Contin

Table 31. Constraints

Name	Cell Value	Status	Slack
Delta	4.54747E-13	Binding	0
Gamma	2.13163E-14	Binding	0
Constant	1700	Binding	0

6.2.8 Sensitivity Report when Theta, Rho and Kappa are deleted

Table 32. Variables

	Final	Reduced	Objective	Allowable	Allowable
Name	Value	Cost	Coefficient	Increase	Decrease
Transaction YC260W	229.2868207	0	-1.27	0.190943969	0.05252137
Transaction XP260V	3659.790294	0	2.4	0.389891285	0.038857261
Transaction YP265V	3395.739706	0	-1.6	1E+30	0.159355219

Table 33. Constraints

	Final	Shadow	Constraint	Allowable	Allowable
Name	Value	Price	R.H. Side	Increase	Decrease
Delta	4.54747E-13	-0.951928551	0	220.5356534	1E+30
Gamma	2.13163E-14	140.3007382	0	7.303405076	87.45914544
Constant	1700	1.799481713	1700	1E+30	1700

6.2.9 Answers Report when Theta, Rho, Kappa and Gamma have been deleted

Table 34. Profit

Name	Original Value	Final Value
Total Profit	2.66454E-15	16947.53348

Table 35. Variables

Name	Original Value	Final Value	Integer
Transaction XC270W	1	3850.030008	Contin
Transaction XP260V	1	3965.406486	Contin

Table 36. Constraints

Name	Cell Value	Status	Slack
Constant	1700	Binding	0
Delta	-4.54747E-13	Binding	0

6.2.10 Sensitivity Report when Theta, Rho, Kappa and Gamma have been deleted

Table 37. Variables

	Final	Reduced	Objective	Allowable	Allowable
Name	Value	Cost	Coefficient	Increase	Decrease
Transaction XC270W	3850.030008	0	1.93	1E+30	0.192204434
Transaction XP260V	3965.406486	0	2.4	1E+30	0.457797689

Table 38. Constraints

	Final	Shadow	Constraint	Allowable	Allowable
Name	Value	Price	R.H. Side	Increase	Decrease
Constant	1700	9.969137342	1700	1E+30	1700
Delta	-4.54747E-13	-5.598220921	0	1700	1E+30