UNIVERSITY OF NAIROBI

SCHOOL OF MATHEMATICS

STRUCTURAL CREDIT RISK MODELLING AND VALUATION BASED ON THE MERTON MODEL

BY

AYWAK BRENDA OWINO

REG. NUMBER - I56/32293/2019

A project submitted in partial fulfilment of the requirements for the award of the degree of Master of Science in Actuarial Science, Faculty of Science and Technology, University of Nairobi

MARCH 2021

Declaration and Approval

I, the undersigned declare that this dissertation is my original work and to the best of my knowledge, it has not been submitted in support of an award of a degree in any other university or institution of learning.

	22/11/2021
Signature	Date

AYWAK BRENDA OWINO
Reg No. I56/32293/2019

In my capacity as a supervisor of the candidate's dissertation, I certify that this dissertation has my approval for submission.

	22.11.2021
Signature	Date

Prof. P.G.O. Weke
School of Mathematics,
University of Nairobi,
Box 30197, 00100 Nairobi, Kenya.
E-mail: pweke@uonbi.ac.ke

	22 November 2021
Signature	Date

Prof. J. I. Mwaniki
School of Mathematics
University of Nairobi,
Box 30197, 00100 Nairobi, Kenya.
E-mail: jimwaniki@uonbi.ac.ke

Acknowledgments

First, I would like to thank God for giving me the strength and will to do this project. I would like to express deep gratitude to everyone who played a role in my academic accomplishments. I am eternally grateful to my project supervisors Prof P.G.O. Weke and Prof J.I. Mwaniki for their indispensable guidance, insightful ideas, constant motivation and impeccable support throughout the period I spent working on my project. Furthermore, I am thankful to every member of my family, my dear father Sir Charles, my beloved mother Margaret and my brothers, David, Erick and Brian, for their constant, immense love, their moral and Intellectual support and their prayers and encouragement throughout this period. Finally, This work would not have been possible without the support of members of the School of Mathematics, University of Nairobi. I am especially indebted to all the lecturers who taught me, the vast knowledge and mentorship I have acquired from them are more than I could ever give credit for. To my classmates, I am grateful for all the opportunities we got to solve complex problems together, laugh and share brilliant ideas, for the wonderful contributions in class, discussion groups and finally this project.
....

Aywak Brenda Owino

Nairobi, 2021.

Dedication

To all tenacious doers and motivated dreamers.

Abstract

The main aim of this research is to model and analyze the probability of default of corporate entities on their bonds using the Merton Model and subsequently evaluate each of the entity's credit spread. Financial companies have a risk of becoming insolvent due to poor credit risk assessment and management. Bankruptcy is a major issue in the financial sector as it leads to a decrease in economic growth rate. The structural credit risk model used has been effective in the determination of the probability of default. The Merton Model uses a company's capital structure and its asset volatility to analyze the default probability at a given maturity time. The data analysis shows that default probabilities are directly proportional to the company's liabilities, whereas the credit spread is directly proportional to the risk of default but inversely proportional to the credit quality of that particular company. This research is a comprehensive guide to the assessment, analysis and management of credit risk.

Master Thesis in Mathematics at the University of Nairobi, Kenya.
ISSN 2410-1397: Research Report in Mathematics

DISTRIBUTOR: School of Mathematics, University of Nairobi, Kenya

Contents

Figures and Tables

Figures

Tables

1 INTRODUCTION

1.1 Background

1.1.1 Global Perspective

Over the past decades, companies, banks and Corporate entities at large have devoted vast resources for the sole purpose of the development of Credit risk models, to better understand the aspect of credit risk with a goal to do away with corporate debt and have an In-dept insight into the interaction between bondholders and shareholders and the nature and risk of default. Credit risk can be defined as the degree of fluctuation in debt instrument and derivatives due to changes in the underlying credit quality of borrowers and counter parties (Chen and Pan, 2012). It is the distribution of financial losses due to unexpected changes in the credit quality of a counter party in a financial agreement (Giesecke, 2004). The creditworthiness of a borrower is of great importance as it affects a company's decision to lend, its credit spread, cost of capital and price of its derivatives.

Due to the essence of credit risk management, there has been extensive and growing literature on credit risk modelling. The measurement of Credit risk has focused majorly on the company's probability of default. There are several approaches used to measure credit risk with the three main ones being; the structural approach, the reduced-form approach and the incomplete information approach. The Structural approach uses structural models for entities which issue equity and debt. These are simple models that can't ideally be used for credit risk pricing but studying them is essential as they clearly look into the nature of default as they make explicit assumptions about a corporate entity's capital structure, its assets and debt. Default occurs when a corporate entity's total value of assets is less that the actual promised debt repayment. A good example of a structural model is the Merton Model (Merton, 1974).

The reduced-form approach uses reduced-form models that use market statistics instead of particular data relating to a specific issuing organization. These market statistics are majorly credit ratings from credit rating agencies such as Poor's and Moody's. Default in this case is analyzed using a default rate which can be used to price credit securities by calculating a risk free rate of interest which is modified by the intensity. An example of such a model is the Jarrow-Lando-Turnbull Model which uses multiple credit ratings. The Incomplete information approach is a combination of the reduced-form models and the structural models. The incomplete information models uses the intuitive nature and tractability/empirical fit respectively of the two models mentioned above.

Credit risk is measured and managed in many different ways by different companies. In the past credit risk was analyzed subjectively in the attempt to evaluate the credit worthiness of borrowers. Experts used judgments that relied on reputation, volatility, capital and collateral where borrowing and lending was driven by internal rate risk (Valášková et al., 2014). The problem with this method of analysis is that there is a lack of an objective and uniform rating algorithm. In addition, companies incurred high costs maintaining experts whose opinions may lead to adverse losses because of underestimation or overestimation of risk, therefore with time most companies opted to rely on credit rating agencies. Rating agencies were used way before the development of mathematical models and are still used to date. John Moody and Company in 1900 published a book known as "Moody's Manual of Industrial and Miscellaneous Securities" which gave insightful information and statistics about bonds, shares and stocks of various industries and consequently led to introduction of bond ratings in the US in 1909. John Moody and Henry Varnum Poor are among the pioneers of credit rating agencies. Among the first credit rating companies was a company called Standard and Poor's, established in 1922 and whose main objective was rating of bonds.

The Fitch publishing company introduced the AAA through D rating systems in 1924. The industry has since then certified and used this method of analysis as a basis for ratings. Fast forward, Beaver W.H. came up with the first scoring mathematical model used to predict credit default by use of financial ratios (Beaver, 1966). In 1968, Edward Altman added to Beaver's concept and came up with the Altman's Z-score model which used a combination of five financial ratios that were weighted by coefficients to predict the probability of bankruptcy. The first structural form model was the Merton Model (Merton, 1974) which was based on the Black and Scholes (1973) option pricing theory. Merton's Model assesses a firm's debt by relating the firm's capital structure to its credit risk (Hull et al., 2004). The Merton Model was modified by Geske (1977) who introduced discrete interest payments on risky bonds . Black and Cox (1976) later worked on the same problem and came up with a model that analyzed the instant where default happens immediately when the value of a company's assets falls under a positive threshold.

Jarrow, Lando and Turnbull came up with the first reduced-form credit model in 1997. They added the idea of term structure of credit risk spread and incorporated credit ratings into the model they built which was used to estimate corporate debt (Jarrow et al., 1997). The reduced-form models use risk rates instead of the probability of default and the default time is influenced by a stochastic process that is independent of the characteristic of the company. Credit ratings have allowed experts to draw conclusions about the financial well-being of corporate entities without the need to know the market value of such organizations. These reduced-form models have also been useful in pricing of credit derivatives.

In the late 1990's some financial companies and banks started using credit value-at-risk models to valuate credit risk. A Value-at-risk model assesses loss by estimating the likelihood of a company under-performing by measuring downside risk. Some of the essential Credit Value-at-risk models are the credit portfolio view, credit Metrics, KMV's credit portfolio manager and the Credit Risk+. Credit Metrics uses both structural and reduced access measurement of value-at-risk to calculate the credit risk of a portfolio. The KMV credit portfolio manager Model doesn't focus on estimation of debt of a company but rather looks at the likelihood of default of that particular company. The credit Risk+ only focuses on defaults and therefore requires default probabilities. The model gives default rates and default rate volatility to assess and estimate the level of default. The Credit Portfolio view is a rating-based portfolio model that also looks at default probabilities but incorporates economic migration likelihood (Valášková et al., 2014). Gordy (2000) has done an in-depth comparison of these models and he concludes that they all have the same mathematical structures with the differences being in their assumptions and functional forms.

1.1.2 Kenyan Perspective

The capital market in Kenya is not highly developed compared to those in developed countries and therefore credit facilitation is a major problem. Credit risk management is a crucial area of interest because of the constant inflation and deflation of prices of shares, stocks and bonds in the financial market due to the deteriorating country's economy. There is also an issue of creditworthiness of prospective borrowers. The Central Reference Bureau (CRB) is among the founding organizations instituted by the Central Bank of Kenya (CBK) whose objective is to collect and analyze credit data from different financial organizations and facilitate credit lending and sharing of information among corporate entities. In 2013, a significant number of borrowers defaulted on there loans due to the economic depression and inflation of interest rates that was linked to the General elections held earlier that year. This led to a five-year period of bad loans in Kenya. The Central Bank of Kenya reported that there were more that 70 billion Kenyan shillings worth of non-performing loans held by commercial banks that year. This was due to declined spending by the private sector and the government at large because of the adverse effects of the economic instability brought about by the general elections (Wanjagi, 2018). A report by the central bank of Kenya indicates that there was an increase from 4.7% to 5.2% of the ratio of non-performing loans to gross loans which later increased to 5.6% in the subsequent year 2014. The Capital adequacy, measured by ratio of total capital to total risky assets was also reported to have decreased in that particular year (Muriithi et al., 2016).

The Increasing level of insufficient loan collateral, faulty loan processing, increasing number of non-performing loans among other things are linked to inadequate and poor credit risk analysis and management (Muriithi et al., 2016). Kithinji (2010) carries out an analysis of how credit risk affects the returns on total assets in commercial banks in Kenya, concluding that the amount of credit and non-performing loans are not the only factors affecting profits generated by commercial banks in Kenya. In 2011, there was a 45% increase in economic crimes due to fraud with commercial banks loosing a tune of 1.7 billion Kenyan shillings between August and October 2010 (Muriithi et al., 2016). Mbua (2017) indicates in his research that there has been an attempt by the Central Bank of Kenya to regulate lending and deposit rates at 4% above and below the 91-day T-bill respectively, from 2001 through to 2016. The Vision 2030 plan, incorporated a medium term plan between 2008 and 2012 to develop a payment system that is safe and reliable in terms of ensuring smooth settlements and transfer of funds between borrowers and lenders (Muriithi et al., 2016). The aim of every corporate entity in Kenya is to make profit which will maintain financial stability and facilitate growth and expansion. However, in the past 2 decades, Kenya has faced a lot of setbacks such as unstable cost of borrowing, increase in non-performing loans, political instability among many others, which have affected the financial fluency of capital markets and which in turn have affected the aspect of lending and borrowing.

Majority of financial institutions in Kenya depend on credit facilitation as the major source of income. Borrowing and lending is a key factor contributing to Kenya's economic framework and therefore policymakers are required to do an in-dept review of creditworthiness of different corporations in order to reduce shortfalls that would adversely affect the progression of the economy. Therefore managing credit risk is a strategic way of controlling uncertainty by developing risk assessment methods that help reduce and mitigate risk in these financial institutions (Wanjagi, 2018). A sound credit risk management involves the establishment and proper implementation of credit policies. Inefficient credit standards and poor credit scoring methods have been a huge challenge faced by many financial institutions in Kenya. Credit risk analysis and valuation acts as a guide for these financial entities to handle and manage its clients assets while meeting the company's objectives.

The main aim of this project is to analyze and model credit risk among financial institutions in Kenya. The probability of default is measured by looking at the market value of the corporate entity's shares on the stock exchange market. The future price and market value of a particular entity can be predicted by modelling its historical data to obtain the market value of its securities. This project will assist companies in decision making regarding the amount of risk premium they should offer to lenders. The project will analyze various companies by looking at their equity and debt in order to calculate its total value of assets and liabilities that will help in predicting their future value.

1.2 Statement of Problem

Credit risk assessment and management is a serious matter of concern in some if not all financial institutions today. With regard to this crucial fact, it is evident that there is need to develop and constantly maintain effective systems to ensure better future performance (Gakure et al., 2012). Selection of a good strategy will lead to good pricing of financial products which consequently leads to profitability. Poor credit risk management is an issue of concern because its a major factor affecting the rate of default. According to Mutonga (2009), loan defaults are increasing in Kenya due to increasing interest rates, high rate of inflation and economic deterioration which directly affects consumer's budget. This problem has led to banks in Kenya receiving a decreasing amount of savings which in turn affects lending and investment. The financial sector is affected negatively when economic activities decrease. This has led to financial institutions, such as banks, to develop methods to cub the effects of the rising loan defaults. For Instance, Barclays Bank has come up with a system that involves working with employers to deduct agreed amounts during pay roll instead of waiting for the borrowers to remit monthly installments for these loan repayments (Mutonga, 2009). A decrease in savings has reduced the capacity of Kenya's banking sector and non-banking sector at large to lend, leading to stagnant, or worse, a decreasing economic growth rate. These corporate entities grant credit facilities and are therefore inevitably exposed to credit risk (Muriithi et al., 2016).

As a result of this problem, extensive resources have been devoted to model and analyze credit risk since time immemorial because of the devastating effects it has on financial markets and as a result, measures have been put in place to minimize these effects. For instant, the Central Bank of Kenya (CBK), established by an act of parliament to promote price stability by formulating and implement monitory policies, formulated and implemented credit risk management guidelines in 2013 due to the fact that banks and other financial institutions were increasingly facing credit risk, asserting that loans are the largest but not the only form of risk. The assessment of credit risk has therefore been of great importance because it helps greatly in valuation of debt. There has been a comparative mispricing of debt because the debt market is underdeveloped compared to the equity and shares market mostly due to discrepancies in their liquidity. This has led to proliferation of bankruptcies and creation of credit arbitrage opportunities which in turn has led to extortion of borrowers and lenders at large. Furthermore, it has led to loans having increased competitive margins and there has also been a decrease in the market value of real assets. With regard to the adverse effects of this problem, researchers and academics alike have developed systems to analyze credit scores and intriguing models to price credit risk. This project will provide an incisive and in-depth overview of credit risk modelling and valuation with the main focus being on the Merton Model, with the hope that it will eliminate existing credit arbitrage opportunities and create healthy and thriving competitive financial and capital markets.

1.3 Objectives

The following are the objectives of this research:

1.3.1 General Objective

The main purpose of this research is to use the Merton Model in structural credit risk modelling and valuation.

1.3.2 Specific Objectives

i. Evaluate the credit spread for different corporate entities.

ii. Assess the different methods of credit risk analysis and management.

iii. Model and analyze the probability of default of corporate entities on their bonds using the Merton Model.

1.4 Significance of Study

Credit risk assessment, analysis and management is of great importance to all financial institutions, Investors, regulators, policy makers, the government and the financial industry at large. Therefore, the research findings and the knowledge shared in this work will be essential in the following ways;

Many financial institutions are facing financial problems due to devastating losses brought about by mismanagement of credit risk, to an extent of some of them going bankrupt or insolvent. This project will help financial companies and corporate entities mitigate losses by maintaining its credit risk exposure through setting up of credit risk models.

This project's findings can be used by policy makers to formulate and implement a proper credit risk assessment and management system. Regulators can also use this project's findings to set up a good regulatory framework. The set up and implementation of these systems is significant as it will reduce financial losses and lead to growth in the financial sector and the industry at large.

Companies that have already set up systems to mitigate credit risk can use this project's derivatives to develop and improve their credit policies and debt financing structures. This will help correct faulty credit risk management systems which do not meet the objectives of these financial institutions.

The process of lending and borrowing is crucial as it contributes to the growth of a country's economy through facilitation of credit which makes it possible for people to trade and investors to invest into different projects and businesses. Knowledge from this project can be used by lenders, borrowers and investors to strategically plan ahead and make informed investment decisions.

2 LITERATURE REVIEW

2.1 Introduction

This Chapter comprises of past studies with regard to Credit risk assessment and modelling. It incorporates both the theoretical literature review and the empirical literature review, discussing development of theories and prevailing disparities in research with relation to the consequences, identification and management of credit risk.

2.2 Theoretical Literature Review

2.2.1 Credit Risk Appraisal

Credit risk appraisal is the process by which a financial institution assesses the value or quality of a borrower's creditworthiness in an attempt to avoid bad debt. The main objective of carrying out a credit appraisal is to ensure relative security of capital and interest income by ensuring that lenders lend money to the right individuals or organizations. A proper evaluation of the borrower's financial condition measures the ability of the borrower to repay the loan on time at a specified date in future depending on the agreement between the borrower and the lender. According to Butterworth's (1990), Financial institutions need to focus on professional and effective management of credit risk because that is the key to the development and success of lending institutions. Credit risk management allows corporate entities to provide quality services in the areas of investment and financial services. Thygerson (1995), asserts that adapting to a new financial environment requires a lot of risk taking, cultural change and altering the way things have been done in the past by embracing new ways of thinking and doing business.

Credit risk control strategies consist of guidelines that are used to govern the process of credit facilitation. The strategies outline the repayment arrangements, possible collateral and the obligations of the borrower. An Effective credit risk control strategy will minimize credit risk and expensive bad debts (Mutonga, 2009). A corporate entity's credit risk policy ensures that it adopts a favorable method of assessment and evaluation of credit risk of each prospective borrower. The primary objective of this policy is to increase profit by minimizing non-performing loans and therefore enhance the company's financial stability (Kwagara, 2006). Financial institutions at large normally consider a lot of factors when setting up a lending policy, which ideally should be in line with the company's overall objectives and credit management strategies. The factors considered include: Economic conditions in the country, the political and economic climate, the industry norms, the existing credit policy, among others (Abedi, 2000). Changes in regulatory and statutory should also be included in the Credit risk policy (Karanja, 2010). The risk of default is minimized by providing a sound credit creation procedure through elimination of unsuitable applicants on the outset, which enables a company to avoid the cost of chasing slow payments and incurring bad debts. The adage prevention is better than cure holds in this case (McMenamin, 2002).

Credit Risk Assessment

Assessment of credit risk involves modelling the probability of default of a counter party on its obligation. A credit decision involves two things: making the decision to extend credit, which involves taking a risk but there is a probability of getting a reward, or refusing to extend credit and not being able to get a reward. The financial institution will be faced with the credit decision of which applications to accept, what limit to set with regard to the amount extended and whether or not a modification is required with time. This brings about credit risk which is the risk that the counter party will not honor their end of the bargain. Credit risk management is the process of determining which counter party may default. The cost of replacing cash flows in the event that a default occurs is used to measure the credit exposure (Brown and Moles, 2014). Figure 2.2.1 shows the process of making a credit decision.

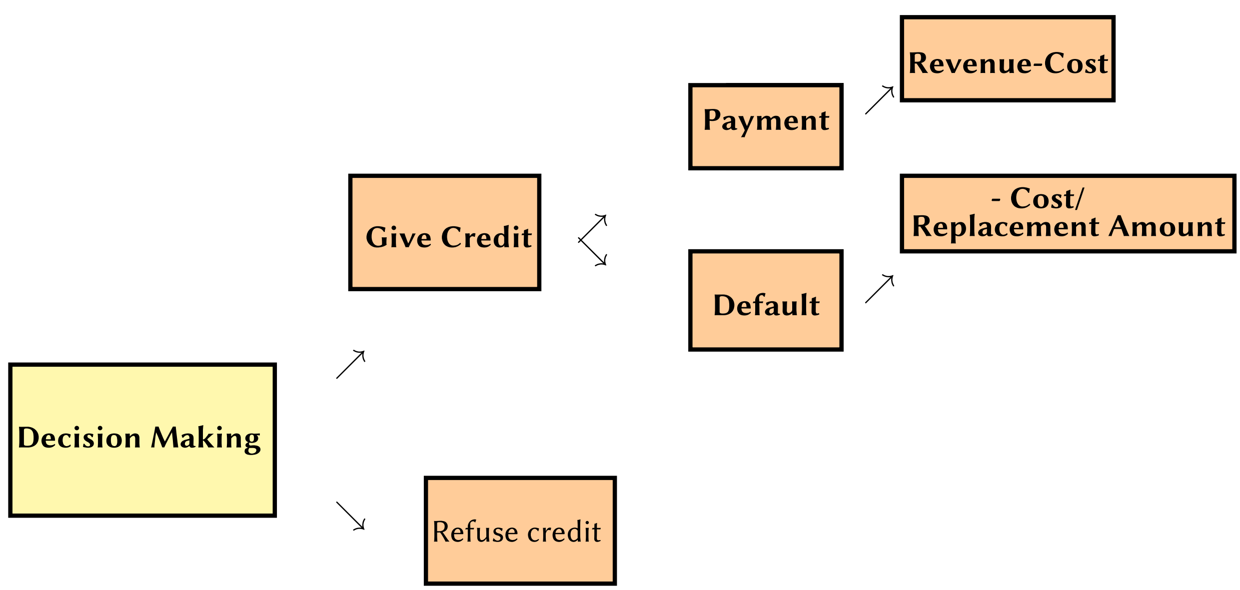

Figure 2.2.1. Visualization of a Credit Decision

There are only two outcomes when a financial institution extends credit, either the credit performs according to expectation or it defaults. The probability of default when a firm extends credit is given by p. A firm will gain if no default happens and face potential loss in the likelihood that credit default takes place. Figure 2.2.1 is the theoretical framework in the assessment of credit risk. Evaluating credit risk involves calculating the payoffs as follows (Brown and Moles, 2014):

$$EC = PV(Revenue - cost) * (1 - p) - PV(Cost) * p$$
$$RC = 0$$

Where;
EC = Extension of Credit
RC = Refusal of credit
PV = Present value
p = Probability of default

Credit Evaluation and Assessment Methods

Credit Evaluation

There are two types of errors that arise when evaluating Credit risk. Type I error occurs when a bad credit is classified as a good credit whereas the type II error occurs when a good credit is classified as a bad credit. In the latter, the opportunity to make profit is forgone whereas in the former the financial institution will incur unexpected losses. Credit quality analysis is with regard to the expected amount of loss brought about by credit risk. The credit quality is portrayed in table 2.2.1 (Brown and Moles, 2014).

A good financial Institution is accepted as good	A Bad Financial institution is accepted as good resulting to unexpected risk of loss (Type I Error)
A good financial institution is rejected as bad resulting to opportunity lose (Type II Error)	A bad financial institution is rejected as bad.

Table 2.2.1. Credit Quality Assessment

Credit analysis experts tend to devote more time in eliminating the type I error, i.e assessing bad credits as good ones, because financial consequences are greater compared to the occurrence of the type II error. This is due to the uncertainties in opportunity cost and the subsequent loss recovery rates. Therefore, any analytic model needs to factor in the probability of default.

Credit Assessment Methods

Credit assessment methods are a crucial part of the process of credit risk management because they determine the creditworthiness of the other counter party. Credit analysts will use a combination of financial and non-financial variables as well as different analytic tools and models to establish the financial status of the counter party. Table 2.2.2 shows the different approaches used and their methodologies.

Approach	Methodology
Judgmental Approach	Uses the Credit Analyst Expert's experience and understanding to make a credit decision
Expert System	Uses a panel approach to make a judgment or make a credit decision using lending systems.
Market Models	Uses Market Information e.g financial market prices as indicators of financial solvency.
Analytic Models	Makes a credit decision using analytic methods.
Statistical Models	Derives appropriate relationships and makes credit decisions by using statistical inference.
Behavioral Models	Makes a credit decision by deriving appropriate relationships through observation of behaviour over time.

Table 2.2.2. Credit Risk Assessment Process

The first step is to define the problem, that is, whether it is a non-default of a default variable. The next step is to analyze the problem as per one of the approaches mentioned in table 2.2.2. These approaches can be mapped into the analytic space which requires data and the business environment information e.g securities, market prices, financial statements e.t.c. The analytic approaches can be categorized into Knowledge models (which involves expert judgment that is subjective), Effect modes (which is a combination of systematic analysis and the expert's judgment) and statistical models (Which are purely systematic). Examples of these models include; the 6 C's of Credit Appraisal, analytic models, Credit Scoring models, market-based models among others. The results from this analysis, depending on which approach is selected, is then used to make a credit decision of whether of not to extend credit. Figure 2.2.2 is a summary of the credit evaluation and assessment process (Brown and Moles, 2014):

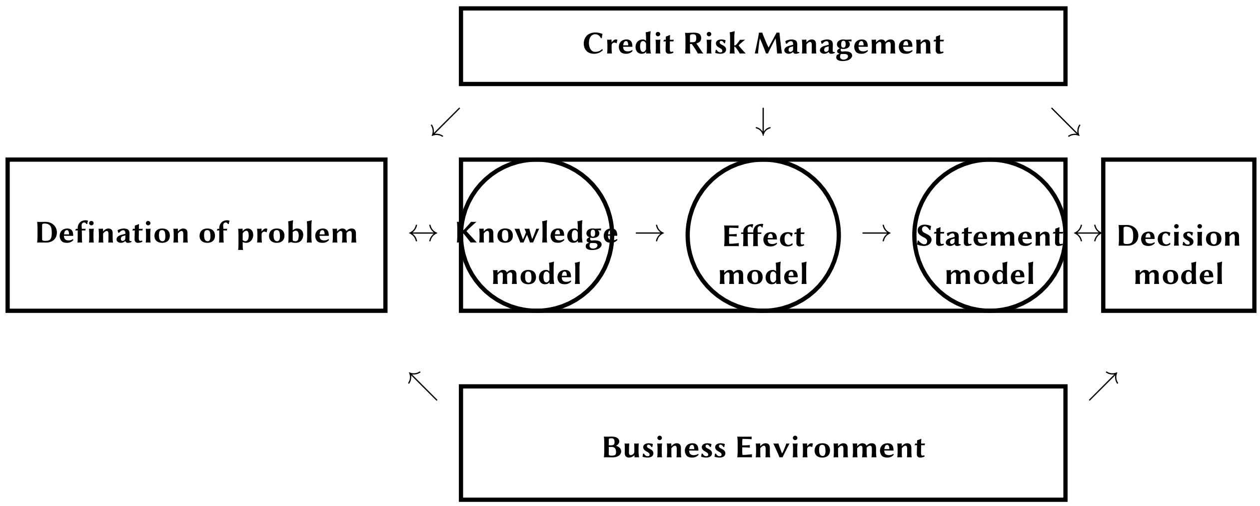

Figure 2.2.2. Credit Risk Management

The 6 C's of credit Appraisal

Credit analysis systems range from simple judgments to formal models. Qualitative models use judgment by credit analysts to determine what is considered as a good or bad credit quality. The most commonly used model of this type is the **6 C's of Credit Appraisal**, which is a model developed with regard to the collective experience of the credit facilitation process. Such Models can be used as a checklist or in complex situations during credit risk assessment (Brown and Moles, 2014). According to Abedi (2000), lending institutions use the 6 C's credit appraisal model to evaluate creditworthiness, whose characterization greatly impacts the financial institution's credit risk policy, in an attempt to decrease the risk of default. The following are the 6 C's of Credit Appraisal:

1. **Character**

   Character is a very essential quality of a borrower. It is the first factor looked into by lenders and it is used to measure the trustworthiness and reliability of a borrower. The lender would want to know if the borrower can be trusted to pay the agreed amount on time within a specified duration of time in the future. The lender will assess the type of borrower one is by looking into the credit history of the borrower. Lenders prefer to lend money to borrowers with high credit scores and are reluctant to extend credit to those with a bad track record of making payments and bad reputation in the credit industry. According to Kwagara (2006), there are a number of ways that a person's character can be determined. They include: Interviews, record of past performances, reference from people who are acquainted with the client, reference from other financial institutions e.t.c. A person considered to have a good character will pay whether it is secured or not and will openly disclose all the relevant facts with regard to the business deal (Karanja, 2010).

2. **Capacity**
   Capacity equates to a borrower's cash flow. It refers to the borrower's ability to fully service his debt. A borrower may not be able to repay his or her loan on time even though he or she is trustworthy and has good intentions. Capacity can be determined by calculating the borrower's debt to income ratio and collection of the borrower's income proof and bank statements. This ratio is inversely proportional to the borrower's eligibility for a loan. Capacity can also be measured by projections from observation and analysis of audited financial statements done by credit analysis experts. A creditworthy borrower is one who has experience in borrowing and paying i.e an individual or organization that has borrowed and repaid money regularly from various financial institutions over long periods of time (Kwagara, 2006).

3. **Condition**
   Condition refers to the overall environment whose factors affect the growth and future of the business. The lender has to determine if the social, economic, political or technological environment is suitable for the facilitation of credit. For example, a borrower, say a small business or firm, may not be granted credit by a lender during an economic recession because the risk of failure is significantly increased due to the economic conditions. In Addition, a lending company may limit granting credit to a potential borrower if the company itself is experiencing slow sales for some of its products (Karanja, 2010). In the case of business loans a lending firm can look at the borrower's relationship with suppliers and clients, industrial factors and also inquire about the intended use of the loan e.t.c

4. **Contribution/Capital**
   Capital helps the borrower analyze the amount of money which has already been invested into the business of corporate entity. This may include the amount of money used for the establishment of the business, the duration in which the firm has been doing business, assets owned by the firm as collateral for other debts e.t.c. Lenders are more confident with borrowers who invest money into their business for future growth because investment in the right channels go hand in hand with the borrower's capital and assets. The lender looks at the borrower's willingness to contribute his/her time and money to the management of his/her assets and projects (Mutonga, 2009).

5. **Collateral**
   These are the securities the borrower can pledge against his or her loan. These securities include accounts receivable, real estate, shares, equipment, insurance policies, inventory and much more. Collateral is given by the borrower to the lender to secure his/her loan, the collateral's value is considered against the amount of the loan (Karanja, 2010) . However, there is some difficulty in disposing off some collateral in the attempt to recover bad loans and therefore some businesses do not consider collateral as an important step towards recovery of unpaid loans (Kwagara, 2006).

6. **Common Sense**
   Common sense refers to the ability to make a good and sensible judgment while making crucial decisions. It is the ability of one to be reasonable in the analysis, presentation and interpretation of financial data or any other business information in general (Kwagara, 2006). The provided financial information should be reasonable enough to support the financing of a project which is an indication that the project has an ability to pay for itself (Abedi, 2000).

2.2.2 Portfolio Theory

The modern portfolio theory also known as the mean-variance portfolio theory was introduced by Markowitz (1968), who came up with the idea of modelling a portfolio as a weighted combination of assets. The Modern portfolio theory reduces the total variance of portfolio return by combining non-identical assets with uncorrelated returns (Ongeri, 2014). It gives maximum return for specified risk or minimum risk for a specified return (Walkling et al., n.d.). The theory has been successfully used to model market risk since the 1980's to date. Interest rates and market risk are managed by value-at-risk (VAR) models in many companies (Gakure et al., 2012), in fact the practice of using modern portfolio theory to measure credit risk has slacked even though credit risk remains to be the largest type of risk companies face (Leeth and Scott, 1989).

The financial performance of a corporate entity can unfavorably be affected by the credit concentration in the industry and therefore firms have resorted to the pursuit of different credit risk measurement approaches. Tools for measuring credit risk in the portfolio context have been developed over the years and many experts have used credit derivatives in an attempt to efficiently transfer risk while preserving client connections. These developments have led to the progression of credit risk management in the portfolio context (Mureithi, 2016). Many financial institutions have used the asset-by-asset approach of managing credit risk which involves identification of the portfolio's expected losses through the systematic evaluation of credit exposures, application of credit risk ratings and the aggregation of the results of this analysis. The asset-by-asset approach whose foundation is a good internal credit risk rating system and a sound credit review, is used in the timely identification portfolio trends and changes in individual credits (Gakure et al., 2012).

The asset-by-asset approach doesn't provide a complete assessment of portfolio credit risk and therefore credit risk experts compliment it with a quantitative portfolio review which involves the use of credit models. Another shortfall of this approach is the difficulty in the identification and measurement of concentration risk, which is an additional risk that comes up due to a rise in exposure to credit extension, and therefore most financial institutions opt to use the portfolio approach instead (Gakure et al., 2012). The portfolio theory involves the maximization of the portfolio expected return with regard to a given level of portfolio risk. The portfolio theory is associated with credit risk appraisal and it is used in decision making when it comes to investments, for instance, buying and selling in the forex market and trading local investment bonds e.g The Akiba bond (Wanjagi, 2018). The portfolio theory therefore brings forth a foundation for the identification and assessment of credit risk and the development of the relationship between risk and return (Lagat et al., 2013).

2.2.3 Finance Distress Theory

A financial Institution is said to be in a state of financial distress when it can't meet its financial obligations due to its business deteriorating (Baldwin and Mason, 1983). A sign that a firm is under financial distress is when it reduces or fails to pay dividends and when it is dealing with violation of debt payments (Muriithi et al., 2016). According to Whitaker (1999), a company experiences financial distress when its current long-term debt exceeds its current cash flows which will result to the inability of the company to meet its contractual debt obligations.

The reason why financial institutions fall into financial distress is economic instability and decrease in performance together with poor asset and risk management (Wruck, 1990). A combination of poor economic conditions and terrible management leads to costly mistakes and this constitutes of the early stages of financial distress (Muriithi et al., 2016). Corporate entities can use the 6 C's of credit appraisal discussed earlier as a preventive measure with regard to financial distress (Wanjagi, 2018).

The theory of financial distress evidently points out the results of credit management on the monetary ability of any financial institution and it provides an unbiased perspective when it comes to the management of credit risk. Through the financial Distress theory a neutral platform for empirical financial analysis is created within financial institutions (Wanjagi, 2018). Tan (2012), asserts in his research that, financial distress leads to the under-performance of financial institutions. His research uses regression analysis to study the impact of a financial distress with regard to the performance of a company while using financial leverage as a proxy.

During times of economic instability, corporate entities with high leverages loose a significant amount of market shares to other companies that are conservatively financed. Such companies experience a decrease in the market value of equity during these dire times. Companies whose products are specialized are very vulnerable to financial distress. The costs that are indirectly brought about by financial distress are high, positive and essential and therefore companies should ensure that they invest in the setting up of good financial management systems (Opler and Titman, 1994).

2.3 Empirical Literature Review

2.3.1 Credit Risk Default Probabilities

A credit risk default probability is the likelihood that a particular individual or corporate entity will default on a loan. Default probabilities can be calibrated either from ratings or market data where the latter is based on the credit spread of securities prone to credit risk and the former is based on credit ratings obtained from credit rating agencies. Risk-neutral default probabilities are used on securities whose expected return is the risk-free rate of interest. Investors in this scenario are indifferent to credit risk and they require no compensation for risk. Actual default probabilities use historical probabilities in the estimation of capital allocation that is used as a protection against loss in the event that a default occurs. Most credit risk models use the assumption that the financial market is risk neutral during the process of option pricing. (Bluhm et al., 2016).

2.3.2 Default recovery rates and loss given Defaults

Default Recovery Rates

Default Recovery rate is the amount of money a financial institution can be able to recover after the occurrence of a default on a loan. It is normally calculated as a percentage of the face value and it is a vital component in the process of credit risk management. Default recovery rates data sometimes uses prices of bank loans or secondary market bonds. The first research done on default recovery rates was by Altman et al. (1977), who used it in the development of the Zeta Model which is a credit scoring model. It was concluded from this research, that the rate of recovery on non-performing loans was about 30% of the face value plus interest.

Years later, Altman and Eberhart (1994), did a research to determine the effect of seniority on price performance of defaulted bonds. Their conclusion was that in the post default period, the most senior bonds in the capital structure did extremely well whereas the junior bonds did poorly. Similar conclusions were made by Fridson et al. (2000) in their research on recovery rates. Asarnow and Edwards (1995), conclude with regard to ultimate recoveries that, there is a loss given default of around 35% on commercial and industrial loans and 13% on structured loans which is considerably lower.

Research done in the 1990's show that default probabilities and recovery rates are independent. This is evident in the credit pricing models that are used today which are based on the independence assumptions whereby the recovery rates are taken to be constant or taken as a stochastic variable that is not dependent on the probability of default. The Merton model however, shows that default probabilities and recovery rates are inversely related. Recovery risk is considered to be a systematic risk element and therefore should be considered when calculating the risk premium during the process of credit risk management (Jones, 2008).

Loss Given Defaults (LGD)
The loss given default is the monetary loss a bank or a financial institution incurs in the event that a borrower defaults on a loan. It is calculated by subtracting the recovery rate (RR) from 1 i.e (1-RR). LGD estimation is not straightforward as such because it depends on recovery rates which depend on many driving factors, for instance, quality of collateral, economic conditions e.t.c. LGD is a random variable and it is considered as the expected value of the severity of a loss in the event that default occurs. The recovery data these financial institutions use are obtained from rating agencies (Bluhm et al., 2016).

The loss given default comprises of three types of losses which are workout expenses, carrying costs of non-performing loans and the loss of principal. LGD can be measured in the following three ways: Market LGD, where in the event of an occurrence of default, the market prices of defaulted bonds can be used to observe and estimate LGD, Implied market LGD, where theoretical asset pricing models use risky bond prices to derive the LGD, and finally workout LGD, where estimated cash flows brought about by processes related to workout and collections are discounted and used in the estimation of the exposure (Schuermann, 2004).

Loan given default is a very essential element in the financial industry. It is useful in the pricing, trading and investment of bonds, loans and stocks. LGD estimates can also be used to calculate risk capital and provide reserves for credit losses. Calculation and modelling of loss given default is an important part of the management of credit risk as it can be used by financial institutions to know potential credit losses and therefore, prediction errors during the estimation of LGD can be very costly and damaging. Economic and regulatory capital allocation are significantly improved when the accuracy of the estimates of the loss given defaults are increased (Gupton, 2005).

2.3.3 Credit Risk Derivatives

A credit derivative is a financial instrument that financial institutions use to manage their credit-sensitive investments and whose value is derived from the underlying market value. These financial instruments are used as a protection and insurance against unfavorable movements in a borrower's credit quality. For instance, a financial institution can suffer huge losses when a counter party defaults on a loan, however, these losses can be offset by profit made from transacting credit derivatives. The advantage of using credit derivatives is that it provides more liquidity, faster payment and a lower cost of transaction compared to insurance products. Credit derivatives allow free trading of credit risk creating flexibility when it comes to credit risk management. There are many different types of credit derivatives and they can be classified into two main categories:

Default products and Replication products. The following are the most commonly used credit derivatives (Bluhm et al., 2016):

Total Return Swaps

The total return swap provides an investor with total return that is brought about by any given credit asset. The investor also receives capital gains accrued throughout the swap's lifetime. In return, the investor is obligated to pay the asset owner a specified rate, which could be fixed or could vary over time. In the event that there is a depreciation of the price of assets over the duration of the contract of the swap, it is the obligation of the investor to compensate the asset owner with the full capital loss amount. Total return swaps are widely used by banks on loans that lack a liquid market (Jones, 2008).

A total return swap is a means of trading a reference asset without possessing it. This transaction creates a platform where all risks and economic benefit is transferred to another counter party. An essential feature of a total return swap is that it facilitates the acquisition of an almost unlimited amount of leverage. This is among the reasons why it is a popular credit risk derivative which is attractive to various segments of the market. For instance, banks can use total return swaps in the diversification of credit risk while keeping the client's financial records confidential. These financial instruments can also be used by investors to access market assets that were previously not available to them (Bluhm et al., 2016).

Credit default Swaps

This is the most common type of credit risk derivative. It is a contract which involves the periodic payment of a specified amount by a counter party, in return for a contingent payment by a seller following a reference security credit event which is either default or downgrade. Therefore, credit events trigger credit default swaps. These financial instruments reduce credit exposure without removing any asset from a company's balance sheet. Financial institutions can fund risky assets in spite of having a low funding costs, and purchase default protection on those assets. The net spread obtained over cost of funding may be more than the premium used to purchase protection on these securities (Bluhm et al., 2016).

The frequent availability of bid-ask spreads for Credit risk swaps in the financial market is an evidence of the raising liquidity of these derivatives. Credit risk swaps can be used to measure a firm's creditworthiness and credit conditions in the economy at large. In the event of distressed credits, sellers opt to demand an upfront premium instead of a standard spread. In some cases, the buyer and seller may agree on a cash settlement arrangement where the seller is liable for the difference between the recovery values and the face values of the credit asset. However, this type of settlement is not common because of the difficulty in pricing distressed credit assets (Jones, 2008).

Credit Spread Derivatives

A credit spread is the difference between the yield on a loan or bond and that of a corresponding risk-free security. The credit spread is used as a compensation for the risk of default and it reflects the margin relative to a given risk-free rate. The credit risk spread option payoffs depend on performance of a given reference asset with regard to a given credit asset. For the spread forwards, a given amount of money is paid at the maturity date of the contract depending on the actual spread and the agreement (Jones, 2008).

In the case of a credit risk derivative, the return is independent of a given credit event. The return depends on the value of one specific reference credit spread against another. Credit ratings are inversely proportional to default probabilities. These financial instruments can be priced by different credit models. Credit spread call options can be used as a hedging instrument against an increase in the average credit spread. On the other hand, credit spread put options can be used as a hedging instrument against a decrease in the average credit spread (Bluhm et al., 2016).

Credit linked Notes

This is a security that comprises of a credit default spread which enables the issuer to transfer the credit risk of the underlying note to investors. Creation of credit linked notes can be through a special purpose vehicle (SPV) which is collateralized by securities with very high rates. Investors can buy securities from a special purpose vehicle in return of payments of a fixed or floating coupon over the lifetime of the note. In the event of the occurrence of a default event, the investor with only receive the recovery rate of the note at maturity (Jones, 2008).

A credit linked note can be of the form of a synthetic bond with an embedded default swap. An investor's exposure to risk can be defined as the spread between the recovery and face value of a given reference asset. Losses associated with the reference asset can be covered with certainty because proceeds from the Credit linked notes can be kept as a cash collateral or invested in securities that are considered high quality collateral. A merit of the Credit linked notes is that, after the agreement, the protection payment is paid upfront without regarding the investor's credit quality or his correlation with the reference assets (Bluhm et al., 2016).

2.4 Summary of literature and Research Gap

The literature review has asserted the need of credit risk management in financial institutions due to the adverse losses and unexpected costs brought about by credit default. The guidelines constituting of credit risk control strategies are essential for the growth of a company as it reduces the probability of default on loans. A lot of emphasis is put on the whole process of credit risk identification and management which entails Credit risk decision making process, appraisal, assessment, monitoring and evaluation. Each of these stages are crucial in the process of reduction and elimination of bad debts and so every financial institution should ensure that they adapt proper credit risk management systems that amicably address each of the stages mentioned above effectively.

Credit default is brought about by many factors including the economic status of the country, level of political stability, occurrence of a pandemic among many other factors. This brings up the need of setting up credit risk policies and credit risk management systems to curb the effects of credit risk defaults which include but are not limited to bankruptcy, insolvency and worst case scenario economic deterioration because of the effect it has on borrowing and lending, which forms a huge part of economic development. Most credit risk management systems involve the setting up of credit risk models which uses different variables to calculate the probability of default and address the issue of default brought about by credit risk.

It is evident that there are very many methods used since time immemorial to identify and manage credit risk, methods of which have substantially developed over the years. The literature review depicts that different corporate entities use different methods of credit risk management as best suits their individual financial objectives or overall objectives at large, therefore a particular approach may work for one company and fail to work for another. With regard to this fact, it is of great importance that a company should carefully analyze its financial operations in order to know which credit risk management method best suits their financial needs and which one will ultimately increase efficiency.

The whole process of credit risk identification, assessment and management starts with making a credit decision which is basically the contemplation of whether or not to take an opportunity that will result into a return in future. Financial institutions do not have complete information and therefore these decisions are made with a lot of uncertainty which ideally creates a research gap. Researchers and Scholars alike can venture into researching and finding ways to incorporate the modelling of information processing capacity and the borrower's or lender's cognitive capabilities in credit risk models, in an attempt to obtain more information and make sound credit risk decisions which is a significant first step of managing credit risk.

2.5 Conceptual Framework

A conceptual framework can be described as a research tool that acts as a linkage of concepts that when put together, provide an in-depth understanding of a particular research problem under study. Each variable in a conceptual framework plays a major and integral role in finding solutions to the research problem at hand. It provides an interpretative approach of the concepts under scrutiny, and a clear understanding of the framework with regard to the key model variables (Jabareen, 2009). The importance of a conceptual framework is to help a researcher or scholar envision, understand and examine the problem at hand in order to make essential and meaningful findings. Figure 2.5.1 is the conceptual framework of this project:

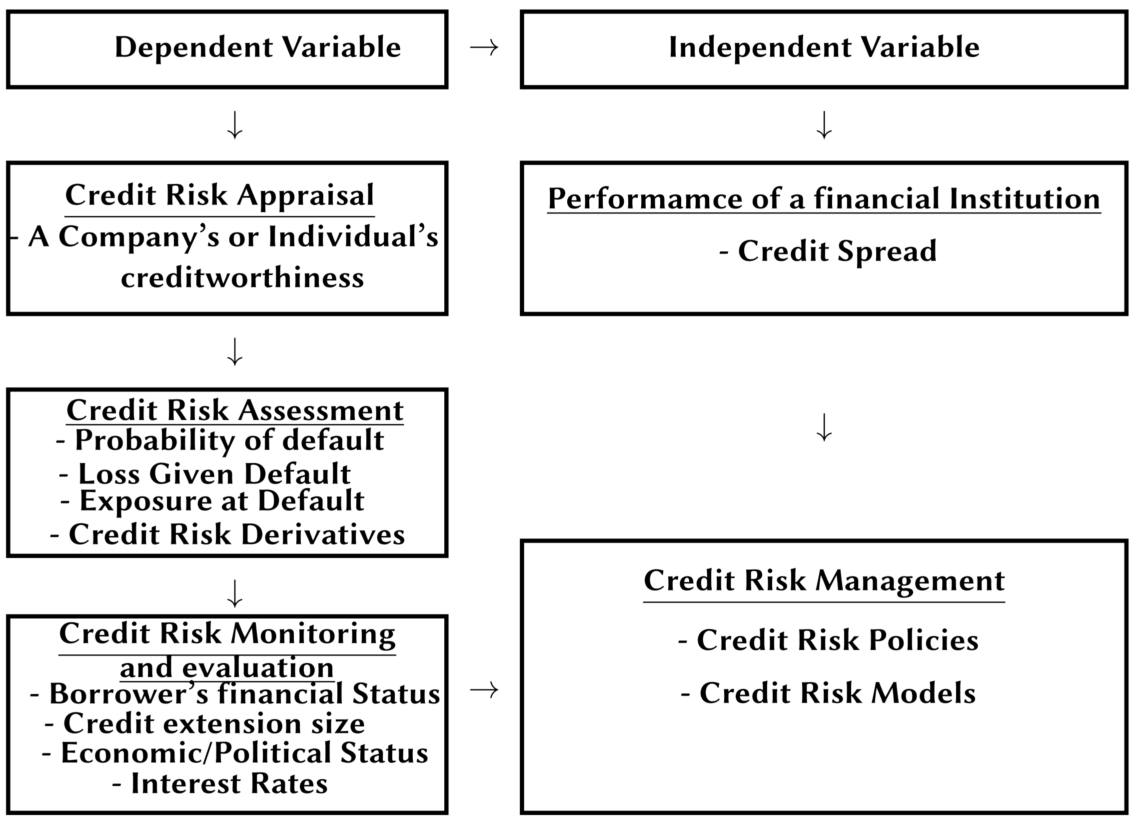

Figure 2.5.1. Conceptual Framework

3 METHODOLOGY

3.1 Introduction

This chapter comprises of some of the most commonly used Models that deal with modelling and pricing of credit risk. The mathematical models used to manage credit risk are a determinant of the loss distribution of a particular loan over a specified period of time and the amount of risk-capital allocation. Credit risk Models can be broadly classified into two i.e Structural Models and Reduced-form Models, however, there exists many other models which will briefly be discussed. The subsequent sections of this Chapter will describe some of the classes of models in detail.

3.2 Structural Models

3.2.1 The Merton Model

This Model was developed by Robert C. Merton in 1974 and has been used to date to value corporate liabilities. It is an extension of Black and Scholes (1973) model which was formulated for the purpose of pricing options. The Merton Model (Merton, 1974), provides a relationship between a financial institution's capital structure and its credit risk. The Model uses the assumption that the corporate entity's assets follow a log normal distribution which is stochastic in nature and whose volatility is constant. The firm's securities that the model uses are its debt and equity, whose sum composes of the firm's asset value, where the latter has no dividend payments and the former is a pure discount bond (Hull et al., 2004).

The financial institution's equity is treated as a call option on the firm's asset therefore allowing the Black and Scholes option pricing methods to be applied in the process of debt valuation (Wang, 2009). Given that the firm's asset value at time t is A_t and the Debt's face value is K with maturity time T, a firm will default at time T if its asset value is less than the debt value (Elizalde, 2005*a*). At maturity time T, the following two scenarios can occur:

- (A_t<K): The firm's value of assets is less than its liabilities. This means that the firm is unable to meet its financial obligations. If this occurs, the shareholders have a right to exercise their limited-liability option and therefore the bondholders will take control of the firm, liquidate it and share the proceeds among themselves. The shareholders are not entitled to anything neither do they have any payoffs, therefore, for the shareholders; $E_T = 0$ and $K_T = V_T$ (McNeil et al., 2005).

- (A_t>K): The firm's value of assets is greater than its liabilities. This means that the firm will not default, hence, the bond holders will receive an amount of K whereas the shareholders are entitled to a sum of A_t-K (McNeil et al., 2005).

The following are the assumptions used by the Merton Model:

Assumptions

1. The firm's underlying assets follow a log-normal stochastic distribution whose volatility is constant.
2. The value of the firm is independent of its capital structure (Modigliani-Miller Theorem).
3. There are no taxes or transaction costs and assets are divisible.
4. Unlimited short-selling is allowed.
5. Assets can be traded continuously over time.
6. There exists a market where investors can buy and sell assets at the same interest rate.
7. The risk-free rate of interest used for borrowing and lending is constant at all times.
8. There exists a sufficient number of investors who are allowed to buy and sell as many assets as desired at the given market price.

The assumptions used by the Merton Model are not realistic in the real world scenario but are only used for expositional convenience. For instance, In the real world markets, the risk-free rate of interest varies over a given period of time and in an unpredictable manner, invalidating the Model's assumption of a constant risk-free rate of interest. However, this assumption is not far from reality when it comes to a derivative that is traded over a short term. Unlimited short selling is not typically allowed with the exception of times when there exists penal interest rates. Furthermore, trading of shares attracts different amounts of transaction costs and the shares can only be traded in integer multiples of one unit invalidating some of the assumptions mentioned above. Despite the invalidity of most of these assumptions, the Merton Model has proven to be a good market approximation, of which most models are only mere approximations of reality. Therefore, the Model is of essence and quite useful as it provides insight with regard to the underlying processes and solutions that are a good approximation of the real world scenario (Merton, 1974).

The following are the variables used in the Merton Model:

- A_t = Value of the firm's assets at time t.
- K = Value of Debt at Maturity.
- r = Continuously compounded risk-free rate of interest.
- σ = Volatility of the firm's assets.
- s = Credit Spread.
- p = Risk-neutral probability of default.
- T = Maturity time.
- E_t = Equity of the firm at time t.

3.2.2 Valuation Using The Merton Model

The asset value of a particular financial institution is assumed to follow the diffusion process given by Equation 3.1.

$$dA_t = rA_t + \sigma A_t dW_t \tag{3.1}$$

Equation 3.2 and 3.3 are important equations during the valuation of credit spread using Merton Model.

$$E_T = max[A_T - K, 0] = (A_t - K)^+ \tag{3.2}$$

$$K_T = min[A_T, K] = K - (K - A_T)^+ \tag{3.3}$$

- Equation 3.2 can be interpreted to mean that the value of equity at maturity time T is equivalent to the payoff of a European Call Option on A_T (McNeil et al., 2005).
- Equation 3.3 can be interpreted to mean that the value of debt at maturity time T is equivalent to the nominal value of the liabilities less the payoffs of a European put option on A_T whose exercise price is K (McNeil et al., 2005).

- The value of the equity is therefore calculated using the Black-Scholes option pricing formula as shown in the equation 3.4:

$$E_t = A_t\Phi(d_1) - Ke^{-r(T-t)}\Phi(d_2) \tag{3.4}$$

where;

$$d_1 = \frac{\ln(\frac{A_t}{k}) + (r + \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{(T-t)}} \tag{3.5}$$

$$d_2 = d_1 - \sigma\sqrt{(T-t)} \tag{3.6}$$

The probability of default at maturity time T is given by:

$$P[A_T < K] = \Phi(-d_2)$$

The credit spread is used as a measurement of the yield to maturity of a non-default zero-coupon bond , which is continuously compounded, and a zero-coupon bond that can default (McNeil et al., 2005). The credit spread of a particular company is calculated using Equation 3.7:

$$s = -\frac{1}{T-t}\ln[\Phi(d_2) + \frac{A_t}{Ke^{-r(T-t)}}\Phi(-d_1)] \tag{3.7}$$

The capital structure of the firm is given by: $A_t = E_t + K_t$, that is, the summation of the value of the firm's equity and the value of its debt. The implementation of the Merton Model requires the estimation of the financial institution's asset value A_t and its volatility σ and the transformation of the debt structure into a zero-coupon bond with a face value of K and a maturity time T (Elizalde, 2005*a*).

3.2.3 The Merton Model with Stochastic Interest rates

The Merton Model has been modified to be used when dealing with default free interest rates that are stochastic in nature. this modification maintains the assumption that the debt is a zero-coupon bond (Lando, 2009) . Treasury bond for instance, have stochastic interest rates that have a correlation with the credit spread of a particular firm (Duffee, 1999). However, interest rates have to be extremely volatile to affect the credit spread significantly. Occasionally, credit spreads are not sensitive to volatility which is mostly used to justify why most models ignore the effects of stochastic interest rates when modelling credit risk (Lando, 2009).

Theorem 3.2.1. *Let Q be a martingale measure whereby the value of assets and short rate follow equations 3.8 and 3.9 respectively (Lando, 2009) .*

$$dV_t = r_t V_t dt + \sigma_v V_t (\rho dW_t^1 + \sqrt{1-\sigma^2} dW_t^2) \tag{3.8}$$

$$dr_t = \kappa(\theta - r)dt + \sigma_r dW_t^1 \tag{3.9}$$

Lemma 3.2.2. *The price of a zero-coupon bond at time t, whose maturity is time T is given by equation 3.10 .*

$$P_t = e^{(x(\bar{T}) - y(\bar{T}) r_t)} \tag{3.10}$$

where $\bar{T} = T - t$ *and;*

$$y(\bar{T}) = \frac{1}{\kappa}(1 - e^{-\kappa \bar{T}}) \tag{3.11}$$

$$x(\bar{T}) = \frac{(y(\bar{T}) - \bar{T})(\kappa^2\theta - \frac{1}{2}\sigma^2)}{\kappa^2} - \frac{\sigma^2 y^2(\bar{T})}{4\kappa} \tag{3.12}$$

- Proof of Theorem 3.2.1 and Lemma 3.2.2 can be found in the Credit Risk Modelling, Theory and Application book by Lando (2009).
- The price of equity at time t can then be derived from equation 3.13.

$$V_e = E_t^Q[e^{(-\int_t^T r_s ds)}(V_T - D)^+] \tag{3.13}$$

- Equation 3.13 is difficult to solve because the discounting stochastic variable and the payoffs are dependent random variables. The difficulty is attributed to the drift in the value of assets being equal to the the stochastic interest rate in Q and also because of the correlation in their Geometric Brownian motions. However the volatility $\sigma(t)$ is deterministic (Lando, 2009).
- Through the application of Ito's formula, the volatility can be expressed as:

$$\sigma(t) = -\sigma_r y(\bar{T}) \tag{3.14}$$

- define:

$$Z_{V,T}(t) = \frac{V(t)}{P_t} \tag{3.15}$$

- Using Ito's formula;

$$\sigma_{V,T}(t) = \sqrt{(\rho\sigma_v + \sigma_r y(\bar{T}))^2 + \sigma_v^2(1-\rho^2)} \tag{3.16}$$

- It follows that;

$$\sum_{V,T}^{2}(T) = \int_0^T ||\sigma_{V,T}(t)||^2 dt \quad (3.17)$$

$$= \int_0^T ((\rho\sigma_v + \sigma_r y(\bar{T}))^2 + \sigma_v^2(1-\rho^2))dt \quad (3.18)$$

$$= \int_0^T (\rho^2\sigma_v^2 + 2\rho\sigma_v\sigma_r y(\bar{T}) + \sigma_r^2 y^2(\bar{T}) + \sigma_v^2 - \rho^2\sigma_v^2)dt \quad (3.19)$$

$$= \int_0^T (2\rho\sigma_v\sigma_r y(\bar{T}) + \sigma_r^2 y^2(\bar{T}) + \sigma_v^2)dt \quad (3.20)$$

- The price of equity is treated as a call option and therefore can be obtained using equation 3.21:

$$S_t = V\Phi(d_1) - DP_t\Phi(d_2) \quad (3.21)$$

Where:

$$d_1 = \frac{ln(\frac{V}{DP_t}) + \frac{1}{2}\sum_{V,T}^{2}(T)}{\sqrt{\sum_{V,T}^{2}(T)}} \quad (3.22)$$

$$d_2 = d_1 + \sqrt{\sum_{V,T}^{2}(T)} \quad (3.23)$$

- After obtaining the value of equity, the value of debt is calculated by subtracting the value of equity from the total value of assets.

- Once the value of equity and debt is known, the credit spread can be obtained using equation 3.7 (Lando, 2009).

3.2.4 Limitations of the Merton Model

Merton's Model restricts the time of default to the maturity of the debt without taking into consideration that the financial institution's value can change before this time. Default does not always occur at a predetermined time in a real world situation. The Merton Model avoids default when the financial Institution's value decreases to minimal levels before the maturity time therefore making it unable to meet the debt's payment at the time of maturity (Elizalde, 2005*a*).

Another limitation of the model is that the capital structure of the firm is assumed to behave like a zero-coupon bond. The capital structure of a financial institution, which comprises of its equity and debt, is a lot more complicated than a bond with no coupon payments. This has brought about a research gap which has been addressed by many researchers. For instance, Geske (1979), extends the Merton Model by considering factors such as debt subordination, sinking funds, payment restrictions e.t.c. He considers the structure of the debt as a coupon bond whereby each coupon payment acts as a compound option which can potentially be a cause of default (Elizalde, 2005*a*).

Another drawback is that the Merton model assumes a constant risk-free rate of interest which is unrealistic and this assumption has brought about a lot of criticism over the years (Elizalde, 2005*a*). Jones et al. (1984), makes a suggestion in their research that the Model should introduce stochastic rates of interests together with taxes to improve the performance of the Model. Interest rates that are stochastic in nature allow the introduction of the correlation between short rates and the financial institution's value of assets (Elizalde, 2005*a*).

3.2.5 The KMV Model

The KMV Model is a model developed by Kealhofer, McQuown, and Vasicek in 1989. This model is based on the Merton (1974) model and so it uses Merton's option pricing approach of assessing credit risk. Moody Corporation, which is a credit rating agency company, adapted the model into its Risk Management services in 2000 bringing forth the Moody KMV Model. The KMV model is a structural credit risk model that uses the asset value of a corporate entity to estimate its probability of default. Unlike the Merton Model, which uses the Probability of default (PD), Moody's KMV Model uses the Expected Default Frequency (EDF), which can be defined as the probability that a firm will default within a given time period, typically one year. The EDF is specific to a particular firm and it is based on the volatility and market value of assets together with the capital structure of the firm (Valášková et al., 2014).

The KMV Model was developed in an attempt to overcome some of the weaknesses associated with the Merton Model. For instance, one of Merton's Model limitations is that the time of default is restricted to the maturity of the debt. The KMV overcomes this limitation by allowing possibility of default before maturity, it takes into account intermediate payments. Merton Model assumes that asset values are lognormally distributed and therefore it uses the lognormal distribution in its calculations which exhibits thin tails. Empirical studies show that asset values often have heavy tails, meaning that there is a higher probability of large deviations than what would be expected. With regard to this, the KMV model adapts an empirical distribution that allows for fatter tails which is better when it comes to extreme events that are more plausible in a real world scenario (Lando, 2009).

The KMV model improves on these limitations by introducing a variable known as the **Distance to Default (DD)**. The Distance to Default can be defined as the distance between the default point and the expected asset value. The proportion of firms that have defaulted within a given time horizon is known for every interval of the values of DD. Moody's KMV model uses a historical data set which is a collection of the proportion of corporate entities defaulting for different values of DD and for different time horizons. These time intervals should be small enough to consider the probability of default as a constant over the interval but huge enough to include enough firms. The default frequency within each time interval would then be an accurate estimate of the probability of a firm defaulting within a given period of time. (Lando, 2009)

3.2.6 Valuation using the KMV Model

Moody's KMV Model's main objective is the prediction of the expected default frequency (EDF). There are a number of steps involved when estimating the expected default frequency. The first step is using the market value to estimate the value of assets and their volatility. The value of equity is obtained from the firm's value of liabilities in its financial statements. The second step is the estimation of the default point (DP) which is obtained by summation of the firm's short term liabilities and half of its long term liabilities. A firm defaults when its market value of assets falls below the DP. The DP is specific to a firm depending on the structure of the firm's liability. The final step is the estimation of distance to default (DD) which is calculated by finding the difference between the expected value of assets at the end of the time interval and at the default point. When a firm's distance to default is great, the firm's assets are safe whereas when a firm has a low distance to default, the firm's assets are said to be risky (Valášková et al., 2014).

Components of the EDF
Figure 3.2.1 is a visual representation of the key factors that are used to determine the Expected Default Frequency (EDF) as used in Moody's KMV Model.

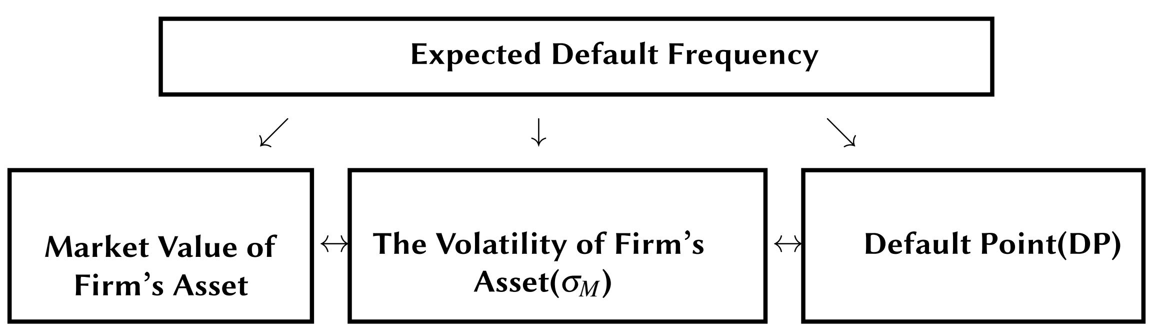

Figure 3.2.1. Components of KMV's EDF

The following are variables used in Moody's KMV Model:

- V_E = The value of the firm's equity.
- V_M = The Market Value of firm's assets.

- σ_M = The market volatility of the firm's asset.
- σ_E = Volatility of the firms's equity.
- D = Book value of the firm's debt.
- SD = Short-term value of firm's debt.
- LD = Long-term value of firm's debt.
- T = Time to Maturity of the firm's debt.
- r = The risk-free rate of interest.
- DP = The firm's Default Point.
- DD = The firm's Distance to default.
- EDF = The expected default frequency.

The estimation of the probability of default requires the calculation of the market value of assets and their volatility and the calculation of the value of equity as well, which changes identically. The volatility of the assets is the vulnerability of the market value of assets to large changes. The book value of a firm's debt, payable at a given time period T, is obtained from it's financial statement. Using the idea borrowed from Merton (1974) Model that the equity of the firm is treated as a call option on the firm's asset, the following equations can be solved simultaneously to get the market value and volatility of the firm's assets (Valášková et al., 2014).

- Equations 3.24 and 3.25 represent the value and the volatility of a firm's equity respectively:

$$V_E = V_M \Phi(d_1) - e^{-rT} D \Phi(d_2) \tag{3.24}$$

$$\sigma_E = \frac{V_M}{V_E} \Phi(d_1) \sigma_M \tag{3.25}$$

- The relationship between the volatility of the firm's equity and the volatility of its assets is given by equation 3.25 (Yang et al., 2013):

  Where;

$$d_1 = \frac{\ln\left(\frac{V_M}{D}\right) + \left(r + \frac{1}{2}\sigma_M^2\right)(T)}{\sigma_M \sqrt{(T)}} \tag{3.26}$$

$$d_2 = d_1 - \sigma_M \sqrt{(T)} \tag{3.27}$$

- The next step is the calculation of the firm's default point (DP) as shown by Equation 3.28 :

$$DP = SD + \frac{1}{2}LD \tag{3.28}$$

- Once the default point of a firm is known, the distance to default (DD) can be calculated using Equation 3.29 :

$$DD = \frac{V_M - DP}{\sigma_M V_M} \tag{3.29}$$

- The Expected Default Probability is then estimated once the firm's distance to default is known. The EDF is easy to calculate if the probability distribution of the firm's assets is known. Most scientific research assume that the assets follow the normal or log-normal distribution. The EDF can be calculated using Equation 3.30 assuming the standard-normal distribution (Yang et al., 2013) :

$$EDF_{KMV} = Pr(E(V_M) < DP) = \Phi(\frac{DP - E(V_M)}{E(V_M)\sigma_M}) = \Phi(-DD) \tag{3.30}$$

3.2.7 Limitations of the KMV Model

Moody's KMV Model assumes that the firm's assets and liabilities follow an empirical distribution. In reality a firm's balance sheet is more complicated than that assumed by the model. Most studies that have been done empirically, more often than not, remove firms that exhibit a complicated capital structure from the sample during modelling to acquire a more reliable data set. It would be difficult to use the model when one is dealing with a lot of versatile and concrete cases (Lando, 2009).

The KMV Model uses the expected default frequency approach to estimate the probability that a firm will default. This approach is suitable when dealing with a short-term loan. Implementation of the model will be strenuous when dealing with long-term maturity periods of time. Furthermore, it is difficult to know exactly what sets off the default, making Modelling of credit risk cumbersome. It is also hard to accurately measure liability because of unexpected variation of cash flow experienced in firms (Lando, 2009).

3.3 Other Credit Risk Models

3.3.1 Reduced-form Models

Reduced-form models were introduced in the 90's by many scholars and have developed drastically over the years. Jarrow and Turnbull (1995), Duffie and Singleton (1999) and Artzner and Delbaen (1995) are the pioneers of these kind of credit risk models among others. Reduced-form models have been created as a result of the various limitations of structural models (Giesecke, 2004). These models are intensity-based. There is limited amount of research on the empirical estimation and validation of models that are based on the intensity of default. Most of the work done on intensity-based models use cross-sections to derive estimates. These cross-sections are commonly based on credit ratings from credit rating agencies or the industry at large. Intensity-based models are suitable for markets experiencing data scarcity (Frühwirth and Sögner, 2006).

The reduced-form models are as a result of standard structural models with incomplete information. Unlike structural models which assume that there is complete information about a firm's assets and default processes, the reduced-form models relaxes the complete information assumption of structural models by making default an unpredictable event. (Elizalde, 2005*b*). The intensity in intensity-based models is measured from market prices and not from the reason why the firm is defaulting, therefore the variation of the model's intensity is based on and specified with regard to the pricing probability (Giesecke, 2004). Default risk in reduced-form models are not dependent on the firm's parameters. The recovery rates and default probabilities of these models are exogenous in nature and the default time is unpredictable, therefore default has a **default process** which is the time taken by the first jump of a stochastic process exhibiting random intensities that determine the probabilities of default that vary over time (Kao, 2000).

Definition 3.3.1. *(**Intensity of default**) Given that τ is the default stopping time, then τ has an intensity process λ_t which is adapted to the filtration $(\mathscr{F}_t)$, if λ_t is a positive measurable process where:*

$$\int_0^t \lambda_u du < \infty \quad \forall t : \{1_{\{\tau \leq t\}} - \int_0^t \lambda_u du : t \geq 0\} \tag{3.31}$$

*Equation 3.31 is a $\{\mathscr{F}_t\}$-Martingale whose intensity λ_t is the rate of default just after time t (Elizalde, 2005*b*).*

Definition 3.3.2. *(**Default Process**) Given that the random time of default is τ, a default process is a point process with one jump whose size is one at default and can be defined by (Giesecke, 2004);*

$$D_t = 1_{\{\tau \leq t\}} = \begin{cases} 1 & if \quad \tau \leq t \\ 0 & \quad otherwise \end{cases} \tag{3.32}$$

- The default process increases over time and it therefore has an upward trend.
- The probability at time t that a firm will default by time s $\geq$ t is as huge as D_t itself. this process is a **submartingale**.
- If no trend exists the process is referred to as a **martingale**.

Definition 3.3.3. *(Martingale) A stochastic process Y is said to be an $\{\mathscr{F}_t\}$-**martingale** on a probability space $\{\Omega, \mathscr{F}, \mathbb{P}\}$ if the following properties hold;*

(a) Y is adapted to the filtration $\{\mathscr{F}_t\}_{t\geq 0}$

(b) $E[|Y(t)|] < \infty$

(c) $\forall$ s & t where $s \leq t$;

$$E[Y(T)|\mathscr{F}_s] = Y(s)$$

Definition 3.3.4. *(Submartingale) A stochastic process Y on a probability space $\{\Omega, \mathscr{F}, \mathbb{P}\}$,which has properties (b) and (c), is known as a **submartingale** if $\forall$ s & t where $s \leq t$;*

$$E[Y(T)|\mathscr{F}_s] \geq Y(s)$$

Remark 1. *Given that λ is a non-negative likelihood of default which is constant (also known as the hazard rate), then D is a homogeneous Poisson process whose intensity is λ with a stopping time of 1 i.e its first jump. In this case τ has an exponential distribution with parameter λ whose default probability is given by equation 3.33 (Giesecke, 2004):*

$$P(T) = 1 - e^{-\lambda T} \tag{3.33}$$

Suppose that the default probability is given, the intensity can be calculated using equation 3.34 :

$$\lambda = \frac{\rho(T)}{1 - P(T)} \tag{3.34}$$

where ρ is the density function of P □

Remark 2. *Given that λ is a deterministic function where $\lambda = \lambda(t)$ at time t, then D is an in-homogeneous Poisson process whose intensity is λ with a stopping time of 1 i.e its first jump and whose default probability is given by (Giesecke, 2004):*

$$P(T) = 1 - e^{\int_0^T \lambda(s)ds} \tag{3.35}$$

An essential parametric intensity-based model can be expressed as:

$$\lambda(t) = r_i \qquad where \quad t \in [T_{i-1}, T_i) \qquad \& \quad i \geq 1 \tag{3.36}$$

r_i and T_i are obtained from market data.

Remark 3. *(**Cox Process**) Given that λ is a stochastic function where $\lambda = \lambda_t$ at time t, where λ is conditional on the intensity realization, then D is an in-homogeneous Poisson process whose intensity is λ with a stopping time of 1 i.e its first jump. The conditional probability of default given λ is (Giesecke, 2004):*

$$P(T) = 1 - e^{-\int_0^T \lambda(s)ds} \tag{3.37}$$

Therefore the default probability is given by;

$$P(T) = 1 - E[e^{-\int_0^T \lambda(s)ds}] \tag{3.38}$$

Proposition 3.3.5. *Suppose that $D_t = 1_{\{r \leq t\}}$ is a cox process that generates a default stopping time with an intensity of λ_t that depends on a set of state variables X_t, then the assumption is that N_t follows a diffusion process, that is (Jarrow and Protter, 2004):*

$$\mathscr{F}_t = \sigma(\tau X_u : u \leq t) \subset M_t \tag{3.39}$$

Reduced form processes are more often than not specified under the martingale measure Q (Jarrow and Protter, 2004).

The proof of proposition 3.3.5 can be found in Karatzas and Shreve (2012)'s book.

Theorem 3.3.6.
- *The default probability before time T in a reduced-form model is given by:*

$$Q(\tau \leq T) = E^Q(E^Q\{D_T = 1 | \sigma(X_u : u \leq \bar{T})\}) = E^Q[e^{-\int_0^T \lambda(u)du}] \tag{3.40}$$

where λ_u is the intensity and T is the default time.

- *The valuation of the firm's debt is given by equation 3.41:*

$$V_t = E^Q[\{1_{\{\tau \leq T\}}\delta_\tau + 1_{\{\tau > T\}}1\}e^{-\int_0^T r(u)du}] \tag{3.41}$$

where δ_τ is the recovery rate at time T, which is stochastic in nature and r(u) is the default free spot rate of interest (Jarrow and Protter, 2004).

More information on proof of Theorem 3.3.6 can be found in work done by Jarrow and Protter (2004).

3.3.2 Liquidation Process Models

A firm liquidates after default occurs i.e when the value of assets of a firm is lower than a given threshold. Liquidation can loosely be described as an event whereby a firm halts all operations to sell its assets in order to pay outstanding debts. In liquidation process models, the occurrence of a default doesn't instantly cause liquidation but leads to a process that may or may not cause liquidation once it is over. This process is different from a default event which signifies the beginning of a firm's financial distress period which will not necessarily result to the liquidation of the firm in question (Elizalde, 2005*a*).

Liquidation process doesn't occur abruptly but rather takes a long period of time (Couderc et al., 2004), implying that past information is an essential exploratory variable when doing an empirical study of the causes of liquidation as it consists of information relating to the whole process of liquidation (Elizalde, 2005*a*). When a default event leads to liquidation of a firm's assets and subsequently lead to a period of financial distress, a firm has a number of options to deal with this financial distress. Some firms sell their assets while others file for bankruptcy (François and Morellec, 2004).

When a firm's value of assets goes below a threshold D, it is liquidated if it remains there consecutively for a given period of time m, usually 2 years. Consecutively as used in liquidation process refers to the number of times that the firm has managed the occurrence of debt and liquidation successfully in the past (Elizalde, 2005*a*). François and Morellec (2004)'s liquidation process model has been built to solve the issue of corporate debt while trying to analyze the model's implications for credit spread and optimal leverage. Their conclusion is that the credit spread is an increasing function of the time period of length m (Elizalde, 2005*a*).

Consequently, Moraux (2002) has come up with a model based on François and Morellec (2004)'s liquidation process. He introduces another cause of liquidation into the model where liquidation happens when the cumulative time a firm's value of assets falls below the default threshold and exceeds m. Moraux (2002) concludes that a firm's value of equity is an increasing function of m and the credit spreads can either decrease or increase with respect to m depending on the seniority of the debt. Many other scholars have ventured into researching the validity of these kind of models over the years. Whilst the liquidation process models have a wonderful theoretical appeal, they have not been tested empirically (Elizalde, 2005*a*).

3.3.3 Factor Models

Factor Models are also known as conditionally independent credit risk models and can reproduce a realistic default correlation. Default correlation is admissible in the process of risk management and pricing of debt obligations. Different obligors have different defaults whose dependence is brought about by a number of different important factors (Schönbucher, 2000). Multi-factor credit risk models use the Monte-Carlo simulation to model credit risk. These models can also be used for capital allocation and hedging of credit risk in different portfolios. The hedging process in overall, is concentrated on each component of the portfolio and its sensitivities (Rosen and Saunders, 2009).

- Suppose a portfolio with N obligors whose default events at the end of a given time period are described by a multi-factor Merton Model.
- Obligor i will default when a random variable V_i, which is continuous in nature, falls below a particular threshold. V_i is a description of the obligor's creditworthiness.
- Let the probability of default be P_i and let the creditworthiness of an obligor be standard-normal then we can express the default threshhold as shown in equation 3.42 (Rosen and Saunders, 2009):

$$T_i = \Phi^{-1}(P_i) \tag{3.42}$$

Theorem 3.3.7. *The creditworthiness of an obligor i is determined by a number of factors K and can be obtained using the linear model represented by equation 3.43 (Rosen and Saunders, 2009):*

$$V_i = \sum_{j=1}^{K} \beta_{ij} X_j + \sigma_i \varepsilon_i \tag{3.43}$$

where;

$$\sigma_i^2 = 1 - \sum_{j=1}^{K} \beta_{ij}^2 \tag{3.44}$$

- X_j = *Standard normal random variables representing the systematic factors driving credit events.*
- β_{ij} = *Factor loadings for the* i^{th} *instrument where* $\sum_{j=1}^{K} \beta_{ij} < 1$.
- ε_i = *standard-normal independent variables which are a representation of idiosyncratic movement of an obligor's creditworthiness.*

Proof of Theorem 3.3.7 can be found in Rosen and Saunders (2009)'s work. Some risk factors have an instantaneous effect on asset returns while others affect the return with a time lag. However, these lag effects on the value of assets are not a contradiction of the considerations about market efficiency. Factor models are relevant in cases where the proxies for asset returns can be observed (Hamerle et al., 2003).

3.3.4 The Credit Risk Mixture Model Approach

Credit risk can be valuated using Mixture models whereby the risk of default is presumably dependent on a set of some prevalent economic factors which are stochastically modelled. Suppose that these factors are known then it is assumed that the defaults of individual firms are independent. However, default dependency comes from dependence of the individual probabilities of default that are on a set of some prevalent factors (McNeil et al., 2005). When working with mixture models, the assumption is that a suitable probability space $\{\Omega, \mathscr{F}, \mathbb{P}\}$ has been chosen, which is a reflection of the probabilistic domain. The most commonly used mixture models are the Bernoulli and the Poison mixture Models (Bluhm et al., 2016).

Theorem 3.3.8. *(**The Bernoulli Mixture Model**) Suppose that the loss of a portfolio follows a loss statistic* $X = X_1, \cdots, X_n$ *which has a Bernoulli distribution i.e* $X_i \sim B(1; P_i)$. *Let* F *be a distribution function in* $[0,1]^n$ *and* P *be the loss default probabilities, which are random variables, whereby* $P = (P_1, \cdots, P_n) \sim F$.

- *The **conditional independence** of the losses is given by equation 3.45.*

$$X_i|_{P_i=p_i} \sim B(1; p_i), \quad (X_i|_{P=p})_{i=1,\cdots,n} \tag{3.45}$$

- *The joint distribution of loss variables X_i is determined by equation 3.46 .*

$$\mathbb{P}[X_1 = x_1, X_2 = x_2, \cdots, X_n = x_n] = \int_{[0,1]^n} \prod_{i=1}^{n} p_i^{x_i}(1-p_i)^{1-x_i} dF(p_1, \cdots, p_n) \quad \textbf{(3.46)}$$

where $\quad x_i \in \{0,1\}$

- *Equations 3.47 and 3.48 represent the first and second moments of the loss variable X_i, respectively.*

$$E[X_i] = E[P_i] \quad \textbf{(3.47)}$$

$$V[X_i] = E[P_i](1 - E[P_i]) \quad \textbf{(3.48)}$$

- *The covariance of the losses is thereby represented by equation 3.49.*

$$Cov[X_i, X_j] = Cov[P_i, P_j] = E[X_i X_j] - E[X_i]E[X_j] \quad \textbf{(3.49)}$$

- *Therefore, a Bernoulli Mixture Model has a **default correlation** represented by equation 3.50.*

$$Corr[X_i, X_j] = \frac{Cov[X_i, X_j]}{\sqrt{E[P_i](1-E[P_i])}\sqrt{E[P_j](1-E[P_j])}} \quad \textbf{(3.50)}$$

Equations 3.49 and 3.50 are an indication that dependence between losses in a given portfolio is as a result of the covariance structure of the multivariate distribution function F of P (Bluhm et al., 2016).

Theorem 3.3.9. *(**The Poisson Mixture Model**) Suppose that the loss of a portfolio follows a loss statistic $Y = (Y_1, \cdots, Y_n)$ which has a Poisson distribution i.e $Y_i \sim Pois(\Lambda_i)$. Let F be a distribution function in $[0,\infty)^n$ and λ_i be the loss default probabilities, which are random variables, whereby; $\lambda = (\lambda_1, \lambda_2, \cdots, \lambda_n)$ of Λ which is the random intensity vector. Let the random variables $(Y_1, \cdots, Y_n)$ be independent. It follows that:*

- *The **conditional independence** of the losses is given by equation 3.51.*

$$Y_i|_{\Lambda_i = \lambda_i} \sim Pois(\lambda_i), \qquad (Y_i|_{\Lambda=\lambda})_{i=1,\cdots,n} \quad \textbf{(3.51)}$$

- *The joint distribution of loss variables Y_i is determined by equation 3.52 .*

$$\mathbb{P}[Y_1 = y_1, Y_2 = y_2, \cdots, Y_n = y_n] = \int_{[0,\infty)^n} e^{-(\lambda_1 + \cdots + \lambda_n)} \prod_{i=1}^{n} \frac{\lambda_i^{y_i}}{y_i!} dF(\lambda_1, \cdots, \lambda_n) \quad \textbf{(3.52)}$$

where $y_i \in \{0,1,2,3,\cdots\}$

- *Equations 3.53 and 3.54 represent the first and second moments of the loss variable y_i, respectively.*

$$E[Y_i] = E[\Lambda_i] \qquad (i = 1, \cdots, n) \tag{3.53}$$

$$V[Y_i] = V[\Lambda_i] + E[\Lambda_i] = V(E[Y_i|\Lambda]) + E(V[Y_i|\Lambda]) \tag{3.54}$$

- *The covariance of the losses is thereby represented by equation 3.55.*

$$Cov[Y_i, Y_j] = Cov[\Lambda_i, \Lambda_j] = E[Y_iY_j] - E[Y_i]E[Y_j] \tag{3.55}$$

- *Therefore, a Poisson Mixture Model has a* ***default correlation*** *represented by equation 3.56.*

$$Corr[Y_i, Y_j] = \frac{Cov[\Lambda_i, \Lambda_j]}{\sqrt{V[\Lambda_i] + E[\Lambda_i]}\sqrt{V[\Lambda_j] + E[\Lambda_j]}} \tag{3.56}$$

Proof of Theorem 3.3.8 and Theorem 3.3.9 can be found in the book by McNeil et al. (2005). Equations 3.55 and 3.56 are an indication that dependence between losses in a given portfolio is as a result of the covariance structure of the multivariate distribution function F of Λ (Bluhm et al., 2016). A good example of a Poisson Mixture model is the CreditRisk+ Model. Mixture Models are more convenient when dealing with statistical fitting. Generalized mixed models can be useful in the process of fitting mixture models to empirical default data collected over several periods of time (McNeil et al., 2005).

3.4 Credit Premium

The complete diversification of aggregated credit risk can not be achieved. This risk therefore requires a premium to act as a compensation for investors who are risk averse. Credit risk models reflect the historical default experience and therefore can be a useful tool in pricing of credit sensitive derivatives by fitting market prices that have been observed over a given period of time. The credit risk premium's role is to map the actual probability of default into the market implied probability of default (Giesecke, 2004).

3.4.1 Actuarial Approach to Credit risk Pricing

Theorem 3.4.1. *(**The Actuarial Principle**). Suppose that a zero-coupon bond matures at time T and has a face value of 1. The bonds payoff will be $1_{\{\tau > T\}}$ at maturity time. The value of the bond incorporating the actual probabilities of default is given by equation 3.57 .*

$$e^{-rT}E^P[1_{\{\tau > T\}}] = e^{-rT}P[\tau > T] = e^{-rT}(1 - p(T)) \tag{3.57}$$

- The deficiency with the actuarial principle is that is does not cover a credit risk premium for the investor's risk of default.
- However, the actuarial principle allows for the difference between the risk-free bond and the risky bond which only covers the expected default losses. i.e $p(T)e^{-rT}$.
- therefore, the actuarial principle has to undergo modification to account for risk aversion. This modification is done in order to generate lower prices of the bond or higher compensation for the risk averse investor.
- Modification involves the substitution of the actual probability of default P with a pricing measure Q.
- The value of the bond calculated using probability Q accounts for both the expected default loss and and the risk premium. Q is also known as the *Market-Implied Volatility* (Giesecke, 2004).

Lemma 3.4.2. *The price of a bond whose pricing measure is Q, is the expected discounted payoff and can be computed using equation 3.58 .*

$$D_0^T = e^{-rT}E^Q[1_{\{\tau>T\}}] = e^{-rT}Q[\tau > T] = e^{-rT}(1-q(T)) \tag{3.58}$$

- The Market-Implied volatility Q has the following two properties:

1. ***Martingale Property***: A default contingent security whose price is S and whose discounting price process is $S_t e^{-rt}$, must be a martingale with respect to the pricing measure. Therefore Equation 3.59 holds. Equation 3.59 is the expected discounted future cash flow with respect to the pricing probability Q (Giesecke, 2004) .
$$S_0 = E^Q[S_T e^{-rT}] \tag{3.59}$$
2. ***Equivalence***: The actual probability of default and the market-implied probability of default agree on which probabilistic event has zero probability i.e An event has zero probability under P iff it has zero probability under Q (Giesecke, 2004).

Proof of Theorem 3.4.1 and Lemma 3.4.2 can be found in Giesecke (2004)'s work.

Theorem 3.4.3. *(**The Radon-Nikodym Density**): It is a likelihood ratio that represents the relationship between the actual probability of default P and the market-implied probability of default Q. The relationship is given by Equation .*

$$Z_t = \frac{dQ}{dP} \tag{3.60}$$

Example 3.4.4. *Suppose that the actual probability of default of a given bond is 0.5 whereas its market-implied default probability is 0.75. For a default-free path, the Radon-Nikodym density will be given by equation 3.61.*

$$Z_1(\rho_1) = \frac{Q(\rho_1)}{P(\rho_1)} = \frac{1-q}{1-p} = \frac{1}{2} \tag{3.61}$$

For a default path, the Radon-Nikodym density will be given by equation 3.62.

$$Z_1(\rho_2) = \frac{Q(\rho_2)}{P(\rho_2)} = \frac{q}{p} = \frac{3}{2} \tag{3.62}$$

Suppose that the payoff of the bond is given by $Y(\rho_1) = 20$ and $Y(\rho_2) = 0$, It follows that:

$$E^Q[Y] = Y(\rho_1)Q[\rho_1] = Y(\rho_1)Z_1(\rho_1)P[\rho_1] = E^P[YZ_1] = 5 \tag{3.63}$$

$$E^P[Y] = E^Q[Y\frac{1}{Z_1}] = 10 \tag{3.64}$$

Under the Radon-Nikodym denisty, It follow that $E^P[Z_T] = 1$ (Giesecke, 2004) .

Theorem 3.4.5. *Using the option pricing analogy, the Radon-Nikodym Density has an explicit characterization in structural models represented by equation 3.65.*

$$Z_T = e^{(-\beta W_T - \frac{1}{2}\beta^2 T)} \tag{3.65}$$

where:

- *W_T is a geometric Brownian motion which drives the firm's credit risk.*
- *β is the credit risk premium*

The proof of Theorem 3.4.3 and Theorem 3.4.5 can be found in the book by Calin (2015).

Theorem 3.4.6. *(**Credit Risk Premium**): It can be defined as the excess return on firm assets over the riskless return per unit of firm risk. The Credit risk premium can be evaluated using equation 3.66.*

$$\beta = \frac{\mu - r}{\sigma} \tag{3.66}$$

The proof of Theorem 3.4.6 can be found in work done by Giesecke (2004). The credit risk premium is positive when the market is risk averse. When the assets of a particular firm are risky, investors will need a return that is higher than the risk-free return (Giesecke, 2004). The ***total spread*** of a loan is the difference between interest rate charged on the loan and the interest rate of a bond with no default but identical characteristics. It can be calculated using the following equation (McNeil et al., 2005) .

Total Spread = Administrative costs + Expected loss + credit risk premium

The expected loss can be calculated by multiplying the default probability with the expected percentage loss given default. The total spread formula above is of a general form and can vary depending on a variety of firm-based factors (McNeil et al., 2005).

4 DATA ANALYSIS AND RESULTS

4.1 Introduction

This Chapter comprises of the use of Merton Model described in Chapter 3 to valuate and analyze credit risk. The subsequent sections of this chapter include the description of the data that was used, the analysis of data, and the discussion of the results obtained. The data has been analyzed using the R software.

4.2 Data Description

Data used in this research is secondary data collected from secondary sources. This project has chosen three corporate entities to act as a sample of all the financial institutions that facilitate credit transactions. The three companies chosen are: Britam Holdings Ltd., Absa Bank Kenya PLC and Jubilee Holdings Ltd. The Credit spread of these three companies will be calculated and added as a risk premium to the risk-free interest rate and the subsequent rate of interest will be used as a guideline for both borrowers and lenders in the Industry at large. The financial Institutions have been chosen at random and data has been obtained from each of the firm's audited financial statements which have been retrieved from each of the firm's website.

4.3 Data Analysis

4.3.1 Data Variables

Merton Model assumes that the value of Assets and its volatility are known at all times. However, it is difficult to know the exact value of these variables at all times because of constant variation.

Observable Variables

- **Asset and Debt Values**
  Asset and Debt values used in this analysis are obtained directly from each of the company's financial statements.

- **Asset Volatility**
  Merton Model assumes that the volatility of assets is constant until the time to maturity.

  Theorem 4.3.1. *The Asset Volatility for each Company in the Model is calculated from the assumption that the Assets follow a log-normal distribution. The volatility is calculated using Equation 4.1 :*

$$\sigma = \left(ln[\frac{Var(x)}{[E(x)]^2} + 1] \right)^{\frac{1}{2}} \tag{4.1}$$

  Proof. The following is the proof for Theorem 4.3.1:
  $x \sim \log(\mu, \sigma^2)$
  $E(x) = e^{\mu + \frac{1}{2}\sigma^2}$
  $Var(x) = e^{2\mu + \sigma^2}(e^{\sigma^2} - 1)$
  $Var(x) = [E(x)]^2(e^{\sigma^2} - 1)$
  $e^{\sigma^2} = \frac{Var(x)}{[E(x)]^2} + 1$
  $\sigma^2 = ln[\frac{Var(x)}{[E(x)]^2} + 1]$
  $\sigma = \left(ln[\frac{Var(x)}{[E(x)]^2} + 1] \right)^{\frac{1}{2}}$ □

- **Risk-free rate of interest**
  The risk-free rate of interest used in the data analysis has been obtained from the central bank of Kenya (CBK). The model assumes a constant risk-free rate of interest throughout the specified period of time to maturity.

- **Time to Maturity (T-t)**
  The model has used a 7-year time period in its data analysis. The Model uses the annual values of both the assets and liabilities and therefore there will be 7 different data points in the various plots, tables and graphs obtained.

Variables to be calculated:

The following are the variables to be obtained from the data analysis:

(a) Probability of Default

(b) Credit Spread

Results

The following variables will be obtained from the R codes during the credit risk analysis;

- V_t - This is the expected firm value at time t<T. It is calculated using the formula represented by Equation 4.2;

$$V_t = V_0(e^{rt}) \tag{4.2}$$

- S_t - This is the expected equity value of the firm at time t<T. It is calculated using Equation 3.4.

- D_t - This is the expected firm value of debt at time t<T.

4.3.2 Absa Bank Kenya

Absa Bank Kenya PLC, previously known as Barclays Bank, is a corporate entity that offers financial services. It is one of the largest companies in the banking industry and serves a variety of countries in Africa. The Merton Model has used an assumed risk-free rate of interest of 14.52% which has been obtained from the Central Bank of Kenya Database. The risk-free rate of interest is assumed to be constant throughout the 7 years. Table 4.3.1 consists of the Asset and Debt Values which have been obtained from Absa Bank Kenya PLC financial statements over the 7-year time period.

ABSA GROUP LTD KE.		
Year	**Total Annual Assets**	**Total Annual Liabilities**
2014	225,845,434	187,659,344
2015	240,877,020	201,160,649
2016	259,692,012	217,303,770
2017	271,177,377	227,078,241
2018	324,839,666	280,632,722
2019	373,981,791	328,792,375
2020	379,440,676	332,936,737

Table 4.3.1. Absa's Total Annual Assets and Liabilities

Figure 4.3.1 represents the time plots for Absa's assets and liabilities respectively over the 7-year time period.

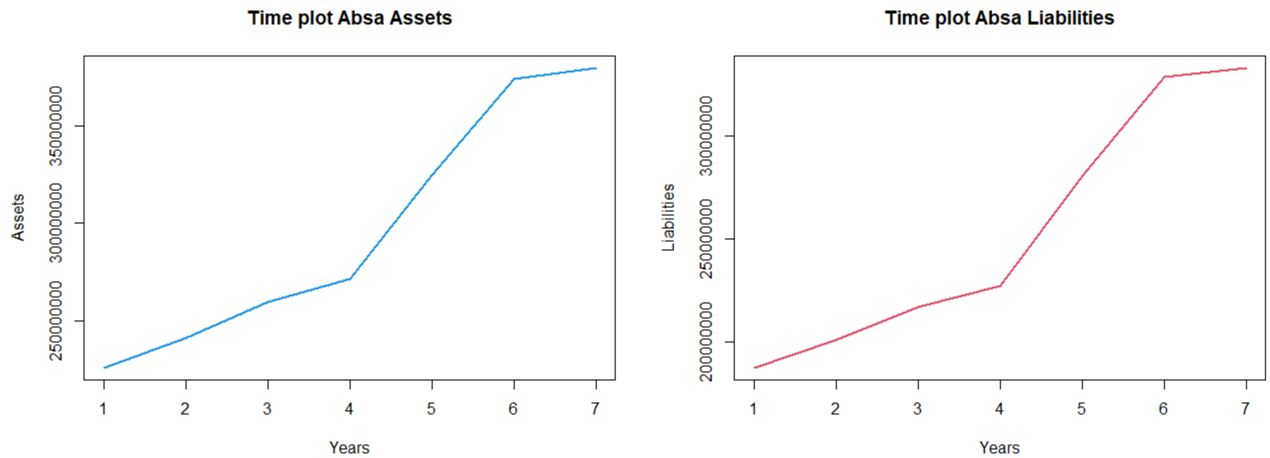

Figure 4.3.1. Time plots for Absa's Assets and Liabilities

Output

As mentioned earlier, analysis in this research has been done using the R software. R has a package known as CreditRisk that is used to evaluate credit risk with structural credit risk models as well as reduced-form credit risk models. The default probabilities have been calculated using the Merton model available in the R package. Calculations of Credit spread requires knowledge of the volatility of assets. The volatility of Absa's assets has been calculated as per Equation 4.1. Table 4.3.2 shows the output obtained during the calculation of the Asset Volatility.

Year	Absa's total Assets		
2014	225,845,434	Mean (μ)	296,550,568
2015	240,877,020	Variance	3,961,291,542,995,820
2016	259,692,012	$e(\sigma^2)$	104.5044%
2017	271,177,377	σ^2	1.9135%
2018	324,839,666	σ	13.83%
2019	373,981,791		
2020	379,440,676		

Table 4.3.2. Absa's Asset Volatility

The asset volatility obtained is 13.83% and is assumed to be constant until the time to Maturity. Therefore, the credit spread can be calculated using Equation 3.7. Table 4.3.3 consists of results obtained with regard to probabilities of default and Credit Spreads of Absa Bank Kenya PLC throughout the 7-year Period.

Maturity	Vt	St	Dt	Survival Prob.	Prob. Of Default	Z (Credit Spread)
1	261138481	182614912	78523569	0.989830	0.010170	0.0000127
2	301946801	211150208	90796592	0.944263	0.055737	0.0000133
3	349132269	244146208	104986061	0.896096	0.103904	0.0000136
4	403691449	282299046	121392403	0.854581	0.145419	0.0000137
5	466776635	326414141	140362494	0.819601	0.180399	0.0000185
6	539720193	377423140	162297053	0.789877	0.210123	0.0000219
7	624062701	436403357	187659344	0.764259	0.235741	0.0000215

Table 4.3.3. Absa Bank Kenya (Probability of Default/Credit Spread)

Figure 4.3.2 is a graphic representation of Absa's Survival and Default Probabilities.

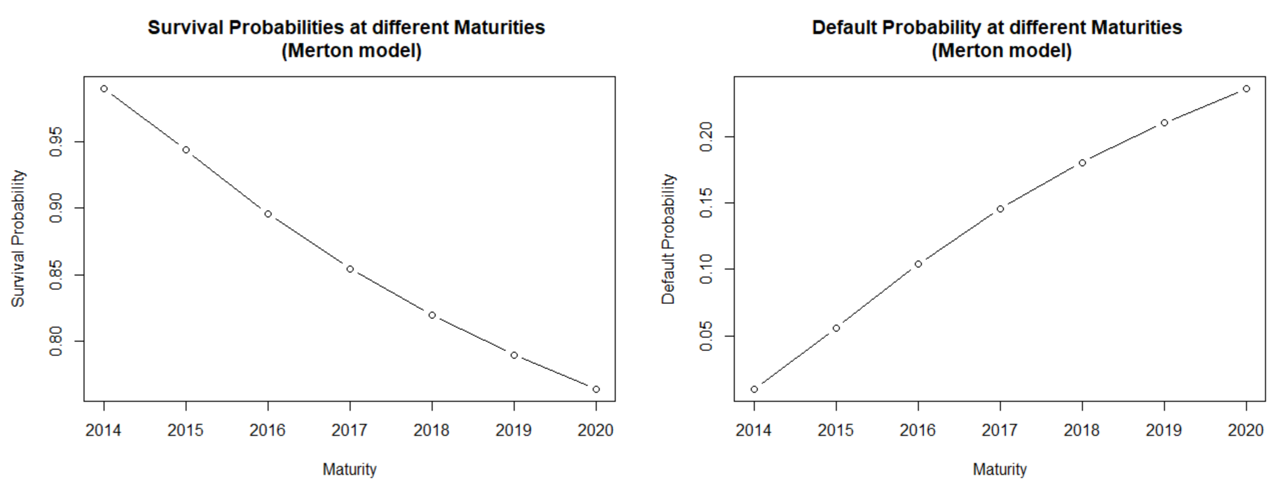

Figure 4.3.2. Absa's Survival and Default Probabilities at different maturity periods

4.3.3 Britam Holdings Ltd.

Britam Holdings Ltd. is a financial provider that offers services and financial products in Insurance, Banking, Asset and Property Management. The assumed risk-free rate of interest used during Britam's data analysis is 14.52% obtained from CBK, and is assumed to be constant throughout the 7 years. Table 4.3.4 consists of the Asset and Debt Values which have been obtained from Britam Holdings financial statements over the 7-year period.

Britam Holdings LTD		
Year	**Total Annual Assets**	**Total Annual Liabilities**
2014	72,450,354	51,010,682
2015	77,632,352	59,957,904
2016	83,642,609	65,765,013
2017	99,024,857	76,354,847
2018	103,656,332	79,700,162
2019	125,243,565	95,866,739
2020	136,962,471	119,895,639

Table 4.3.4. Britam's Total Annual Assets and Liabilities

Figure 4.3.3 represents the time plots of Britam's Assets and Liabilities.

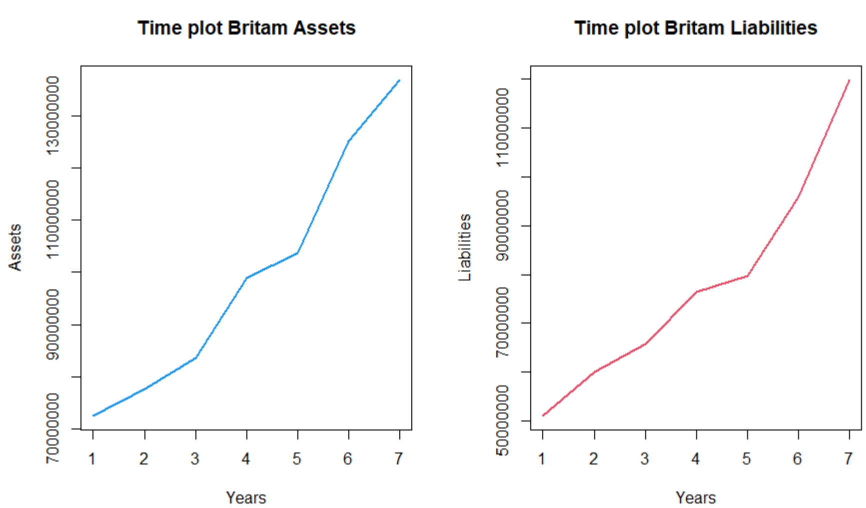

Figure 4.3.3. Time plots for Britam's Assets and Liabilities

Output

Britam's Data analysis has been carried out using the R software. Calculations of its default probabilities have been done using the Merton Model available in the Credit risk Package. Credit spread calculations requires Britam's volatility of assets which is calculated using Equation 3.7. Table 4.3.5 shows the output obtained during the calculation of Britam's Asset Volatility.

Britam Holdings LTD		
72,450,354	Mean	99,801,791
77,632,352	Variance	590,727,568,256,555
83,642,609	$e(\sigma^2)$	105.9308%
99,024,857	σ^2	2.5022%
103,656,332	σ	15.82%
125,243,565		
136,962,471		

Table 4.3.5. Britam's Asset Volatility

Britam's Asset Volatility has been calculated and has been found to be 15.82%. This volatility is assumed to be constant until the time to Maturity. The credit spread can be calculated using Equation 3.7. Table 4.3.6 shows the results obtained with regard to probabilities of default and the Credit Spreads of Britam Holdings Ltd. throughout the 7-year period.

Maturity	Vt	St	Dt	Survival Prob.	Prob. Of Default	Z(Credit Spread)
1	83772229	62427695	21344534	0.998881	0.001119	0.0000168
2	96863382	72182568	24680814	0.982372	0.017628	0.0000361
3	112000301	83462371	28537929	0.952874	0.047126	0.0000416
4	129502677	96505071	32997607	0.920677	0.079322	0.0000356
5	149740164	111585994	38154170	0.889797	0.110203	0.0000348
6	173140180	129023627	44116553	0.861346	0.138654	0.0000336
7	200196935	149186253	51010682	0.835443	0.164557	0.0000950

Table 4.3.6. Britam Holdings (Probability of Default/Credit Spread)

Figure 4.3.4 is a graphic representation of Britam's survival and default probabilities respectively over the 7-year period.

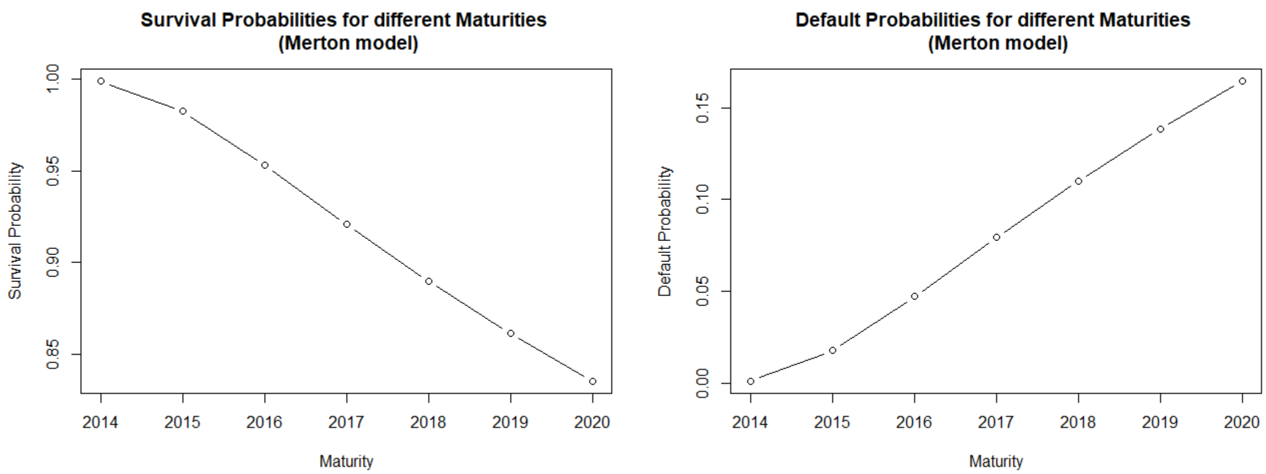

Figure 4.3.4. Britam's Survival and Default Probabilities at different Maturity periods

4.3.4 Jubilee Holdings Ltd.

Jubilee Holdings Ltd. is an insurance company that provides insurance services with a vast range of policies such as motor insurance, health insurance, pension plans e.t.c. Jubilee's Data Analysis has used an assumed risk-free rate of interest of 14.52% obtained from CBK, which is assumed to be constant throughout the seven years. Table 4.3.7 consists of the Asset and Debt Values used during the Data Analysis. These values have been obtained from Jubilee Holdings Ltd. financial statements over the seven year period.

Jubilee Holdings Ltd.		
Year	**Total Annual Assets**	**Total Annual Liabilities**
2014	74,505,374	58,026,343
2015	82,378,010	61,996,803
2016	90,567,743	69,146,074
2017	104,967,530	80,392,402
2018	114,189,212	86,834,434
2019	130,076,938	99,555,308
2020	145,863,583	110,328,202

Table 4.3.7. Jubilee's Total Annual Assets and Liabilities

Figure 4.3.5 represents the time plots for Jubilee's Assets and Liabilities over the 7-year time period.

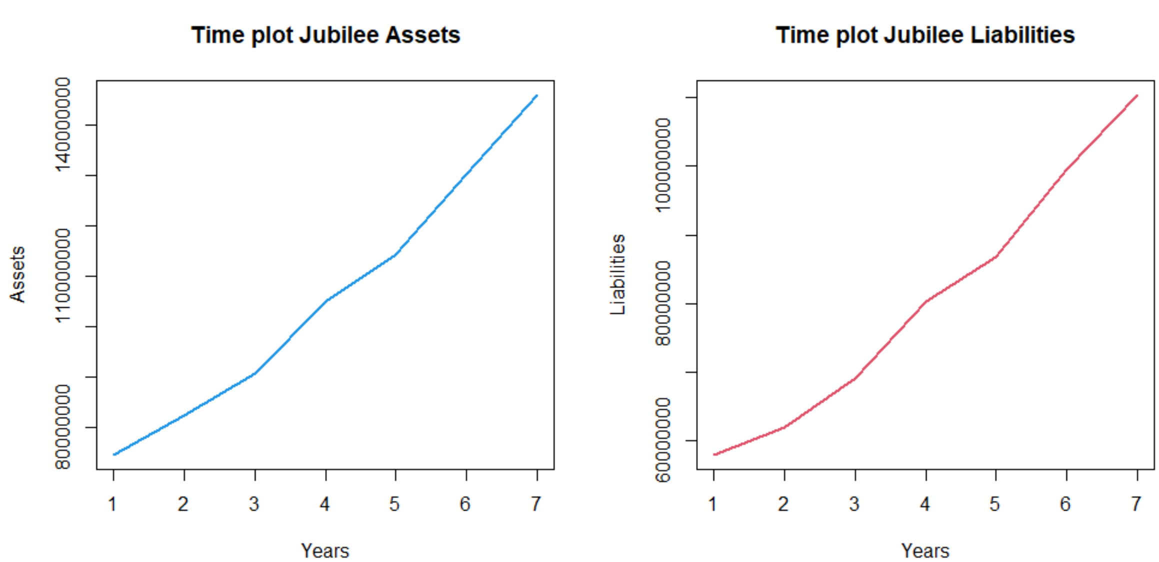

Figure 4.3.5. Time plots for Jubilee's Assets and Liabilities

Output

Jubilee's Data Analysis has been carried out using the R software, whereby the default probabilities have been calculated using the Merton Model available in the CreditRisk package. The Credit Spread calculations require Jubilee's assets volatility which has been calculated using equation 3.7. Table 4.3.8 shows the output obtained during the calculation of Jubilee's Asset Volatility.

Jubilee Holdings		
74,505,374	Mean	106,078,341
82,378,010	Variance	670,825,808,905,597
90,567,743	$e(\sigma^2)$	105.9615%
104,967,530	σ^2	2.5148%
114,189,212	σ	15.86%
130,076,938		
145,863,583		

Table 4.3.8. Jubilee's Asset Volatility

Jubilee Holdings' asset volatility is 15.86% as per the calculations. The volatility is assumed to be constant until the time to maturity. After obtaining the asset volatility, the credit spread is subsequently calculated using equation 3.7. Table 4.3.9 shows the results obtained with regard to the default probabilities and the Credit Spreads of Jubilee Holdings Ltd. throughout the 7-year period.

Maturity	Vt	St	Dt	Survival Prob.	Prob. Of Default	Z(Credit Spread)
1	86148388	61869904	24278484	0.992075	0.007925	0.0000397
2	99610866	71536185	28074681	0.950499	0.049501	0.0000302
3	115177136	82714403	32462733	0.903405	0.096595	0.0000339
4	133175959	95640094	37535866	0.861530	0.138470	0.0000347
5	153987473	110585839	43401634	0.825614	0.174386	0.0000328
6	178051219	127867177	50184042	0.794736	0.205264	0.0000346
7	205875426	147849083	58026343	0.767898	0.232102	0.0000314

Table 4.3.9. Jubilee Holdings (Probability of Default/credit Spread)

Figure 4.3.6 is a graphic representation of Jubilee's survival and default probabilities respectively over the 7-year period.

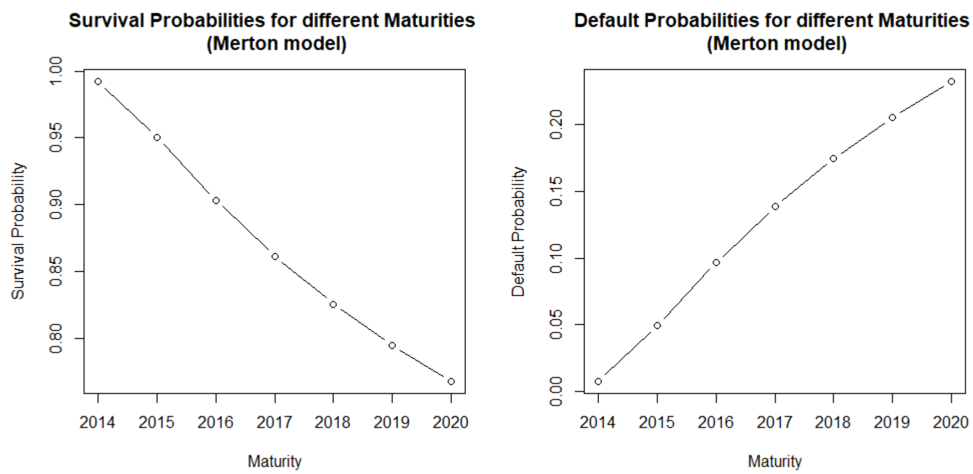

Figure 4.3.6. Jubilee's Survival and Default Probabilities at different maturity periods

4.4 Discussion and Interpretation

4.4.1 Absa Bank Kenya

The standard deviation is normally used to represent volatility. The standard deviation is directly proportional to the variability in market returns i.e The higher the σ, the higher the market return variability and vice versa. Absa Bank Kenya PLC has an asset volatility of 13.83%. An average volatility ranges from 10% - 20% and therefore, Absa's Assets are relatively secure, which means that its assets face very minimal risk when it comes to negative changes in its value .

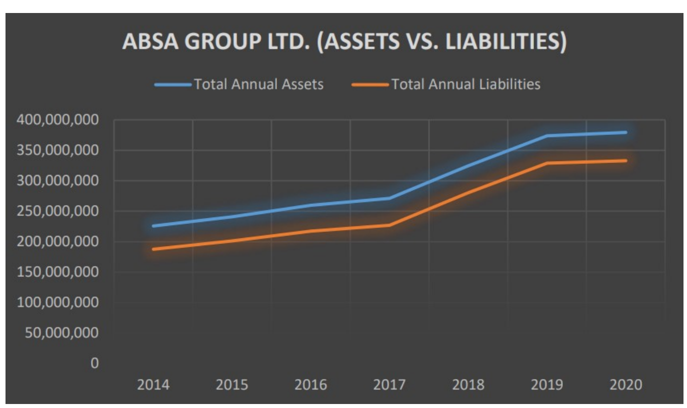

Figure 4.4.1. Absa's time plot of its Assets against its Liabilities

Throughout the 7-year period, it can be noted from Figure 4.4.1 that Absa's Assets are greater than its liabilities (i.e $A_t > K$), which implies that there will be no default during this period because the company will be able to meet its liabilities. The probability of default in the first year is 1.02% and increases to 5.57% in the second year and so on until the final year of maturity of which the probability of default is 23.57%.

The first few years have very low probabilities of default because the asset values are significantly higher than the firm's expected liabilities. As the company's liabilities increase, so does the probabilities of default. Absa's default probabilities exhibit a slow increase over the years but the percentage of the default probabilities remain low until the time to maturity. Generally, the results imply that Absa Kenya PLC has a low likelihood of defaulting on a loan before the end of the seven-year time period.

The credit spread of a corporate entity is directly proportional to the risk of default and subsequently, inversely proportional to the credit quality of that particular firm. A high credit spread implies that the credit quality of the firm will be low. Absa Bank Kenya PLC has very low credit spreads over the 7-year period with 0.00127% in the first year and 0.00215% in year 7, implying that its credit quality is significantly high.

4.4.2 Britam Holdings Ltd

Britam Holdings Ltd. has an asset volatility of 15.82% as calculated under the assumption that the assets are log-normally distributed. This implies that Britam's assets are relatively secure because the average volatility ranges between 10% and 20%. A volatility of 15.82% is an indication that there is minimal risk with the regard to asset values changing negatively over the given period of time.

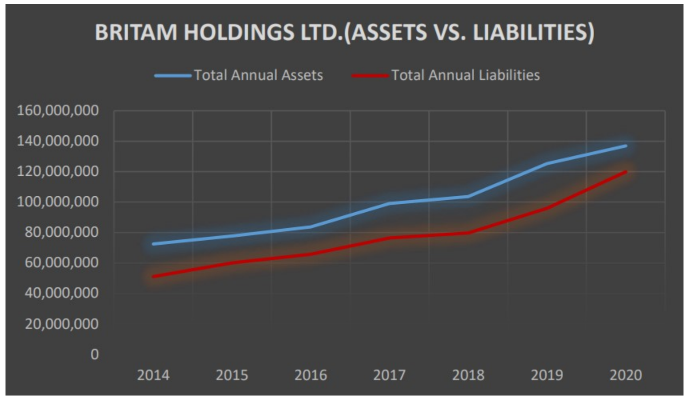

Figure 4.4.2. Britam's time plot of its Assets against its Liabilities

When analyzing the asset values against the liabilities from the onset as seen in Figure 4.4.2 , the value of assets exceed those of the liabilities throughout the 7-year period (i.e $A_t > K$) . Therefore, it is expected that Britam Holdings will not default until maturity. However, it is paramount to know the probability of default in each time period i.e t<T. Results from the analysis indicate a default probability of 0.1119% in the first year followed by a probability of 1.76%, 4.71% and so on with the maturity time having a default probability of 16.46%.

It is evident to see from the analysis that the probability of default is slowly increasing over the time horizon with the probability of default of the first year being significantly low. The low values from the onset is attributed by the fact that the asset value at that time is significantly higher than the expected liability but as the liability increases, the probability of default also slightly increases. Since the probabilities of default are significantly low until the time to maturity, there is a very low likelihood that Britam Holdings will not default on a loan before the time to Maturity.

As mentioned in the previous section, the credit spread is inversely proportional to the credit quality of a particular company. Britam's credit spreads are significantly low from the first year until the time to maturity. The first year has a credit spread of 0.00168% followed by 0.00361% in the second year and so on until the 7^{th} year which has a credit spread of 0.00950%. The very low credit spread can be interpreted to mean that Britam Holdings has a significantly high credit quality.

4.4.3 Jubilee Holdings Ltd.

The numerical computation of Jubilee Holdings' Asset volatility yielded a 15.86% volatility. This figure is still between the average 10% to 20% indicating that Jubilee's assets are relatively secure with minimal risk of negative changes in its asset values until time to maturity. In reality, the volatility changes throughout the period of borrowing and lending, however, the Merton Model assumes that it will remain at 15.86% throughout the given time period.

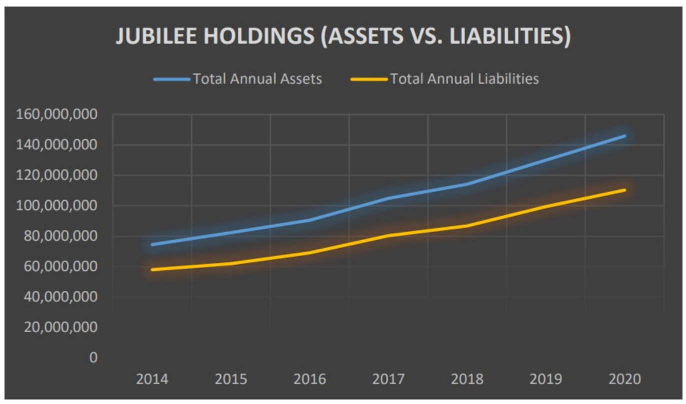

Figure 4.4.3. Jubilee's time plot of its Assets against its Liabilities

It is notable from figure 4.4.3 that Jubilee's value of assets exceed those of its liabilities (i.e $A_t > K$) over the period t<T and therefore the expectation is that Jubilee Holdings Ltd. will not default until the time to maturity as discussed in section 3.2.1. From the data analysis there is a 0.79% probability of default in the first year, followed by that of 4.95% in the second year, 9.67% in the third year and so on until finally a 23.21% default probability in the 7^{th} year.

The probability of default is significantly low in the first year and slowly increases as the time to maturity increases. However, the percentage value remains relatively low throughout the 7-year period . The default probability is low in the first year because Jubilee's Asset values significantly exceed the expected liabilities. The Probabilities slightly increase over the years because its debt values are increasing as the time to maturity nears, but the debt values never exceed those of the assets. There is an overall low default probability over the 7-year time span which can be interpreted to mean that there is a low likelihood of Jubilee Holdings defaulting on a loan before the time to maturity.

Jubilee Holdings' credit spreads are significantly low as well, with the first year having a credit spread of 0.00397%, followed by 0.00302% in the second year and so on. The final year exhibits a 0.00314% credit spread. Since the credit spread is inversely proportional to the credit quality, the low credit spreads can be interpreted to mean that Jubilee Holdings' credit quality is adequately high from the beginning of the financial period until the time to maturity.

5 CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusions

Structural credit risk Models have been used since time immemorial to valuate credit risk by using the company's structural variables, e.g Asset and Debt values, to determine the default time. The main aim of this research is to use the Merton Model to analyze the probability of default of corporate entities on their bonds and subsequently evaluate the credit spread of each of the companies. The Merton (1974) Model is a classic in the literature of structural credit risk model. It provides a relationship between a firm's capital structure and its credit risk. Under the Merton Model, Default occurs when the value of the firm's assets is less than its liabilities ($A_t < K$).

The process of credit risk management consists of credit risk appraisal and credit risk assessment whereby, the former involves the assessment of the creditworthiness of a borrower whereas the latter involves analyzing and modelling the probability of default. However, default probabilities are not the only measure of credit risk. For instance, Moody's KMV model is a structural credit risk model that uses the expected default frequency (EDF) and introduces the distance to default variable to model default before maturity.

Introduction of structural models such as the Merton (1974) model, which was based on the Black and Scholes (1973) model, and Moody's KMV Model have been a major stepping stone in the evolution of modelling, valuation and management of credit risk. For instance, scholars such as Jarrow and Turnbull (1995), Duffie and Singleton (1999) and Artzner and Delbaen (1995) have introduced credit risk models that are intensity-based and do not necessarily depend on the firm's capital structure. Other credit risk models such as the factor models use the monte-carlo simulation to model credit risk. Furthermore, Models such as the Liquidation process model introduce thresholds whereby default occurs when a firm's asset value is lower than a particular threshold leading to the liquidation of that particular firm.

Three financial companies have been used to validate the Merton Model in this research . The model requires observable asset and debt values and the estimation of the asset volatility under the assumption that the assets are log-normally distributed. The Merton

Model has successfully been used to obtain the probability of default even though the model does not provide concise confidence intervals or distribution functions for the estimation of its variables, especially the asset values and the asset volatility. A company's liabilities are directly proportional to its default probabilities. The credit spread is observed to be directly proportional to the risk of default and inversely proportional to the credit quality of a counter party. The higher the credit spread, the riskier the investment and vice versa. These results are paramount in the making of sound and informed financial decisions in the hope to minimize and manage credit risk.

This research provides significant insights in the valuation of credit risk by looking at the extensive credit risk literature and the models used to measure and prevent credit risk. This work can be used as a comprehensive reference guide in the area of assessment and management of credit risk in different and diverse sectors of the financial industries, in the hopes of maintaining a competitive and thriving financial economy.

5.2 Recommendations

Merton Model assumes that a firm's underlying assets follow a log-normal distribution. Assets in the real world market can exhibit a variety of other distributions throughout a given time period depending on the variations in the market over time. With this in mind, further studies can be carried out with regard to cases where the assets are non-Gaussian in nature. This will help in providing significant solutions with respect to returns on assets which display fatter tails and skews as well, during the process of credit risk analysis, management and prevention.

A research done by Finger et al. (2002) has brought about the likelihood of jumps in the dynamics of the asset-value process which is not continuous in nature. Further research can be done on how this approach of credit risk analysis can be used to determine what effects the stochastic jumps have on the company's value and its credit spreads. Work done by Lando (2009) , with regard to the Merton Model with jumps in asset value, can be a good guide in implementing this research.

The Merton Model assumes that the risk-free rate of interest is constant throughout the given time period. This assumption is unrealistic because the interest rates vary a lot in the real financial world. Some researchers have suggested the introduction of stochastic interest rates in the model, it will be interesting to see an implementation of the same in the valuation model.

References

Abedi, S. (2000), 'Highway to success', *Credit Management Journal* **41**, 56–65.

Altman, E. I. and Eberhart, A. C. (1994), 'Do seniority provisions protect bondholders' investments?', *Journal of Portfolio Management* **20**(4), 67.

Altman, E. I., Haldeman, R. G. and Narayanan, P. (1977), 'Zetatm analysis a new model to identify bankruptcy risk of corporations', *Journal of banking & finance* **1**(1), 29–54.

Altman, E. I. and Saunders, A. (1997), 'Credit risk measurement: Developments over the last 20 years', *Journal of banking & finance* **21**(11-12), 1721–1742.

Artzner, P. and Delbaen, F. (1995), 'Default risk insurance and incomplete markets 1', *Mathematical Finance* **5**(3), 187–195.

Asarnow, E. and Edwards, D. (1995), 'Measuring loss on defaulted bank loans: A 24-year study', *Journal of Commercial Lending* **77**(7), 11–23.

Baldwin, C. Y. and Mason, S. P. (1983), 'The resolution of claims in financial distress the case of massey ferguson', *The Journal of Finance* **38**(2), 505–516.

Beaver, W. H. (1966), 'Financial ratios as predictors of failure', *Journal of accounting research* pp. 71–111.

Black, F. and Cox, J. C. (1976), 'Valuing corporate securities: Some effects of bond indenture provisions', *The Journal of Finance* **31**(2), 351–367.

Black, F. and Scholes, M. (1973), 'The valuation of options and corporate liabilities', *Journal of political economy* **81**(3), 637–654.

Bluhm, C., Overbeck, L. and Wagner, C. (2016), *Introduction to credit risk modeling*, Crc Press.

Brown, K. and Moles, P. (2014), 'Credit risk management', *K. Brown & P. Moles, Credit Risk Management* **16**.

Butterworth's, B. (1990), 'Risk management: A modern perspective', *McGraw Hill. Central Bank of Kenya Act, CAP* **491**, 56–60.

Calin, O. (2015), *An informal introduction to stochastic calculus with applications*, World Scientific.

Chen, K. and Pan, C. (2012), 'An empirical study of credit risk efficiency of banking industry in taiwan', *Web Journal of Chinese Management Review* **15**(1), 1–16.

Couderc, F., Renault, O. et al. (2004), 'Times-to-default: Life cycle, global and industry cycle impacts', *Unpublished manuscript, University of Geneva and Warwick Business School* .

Creswell, J. W. and Creswell, J. D. (2017), *Research design: Qualitative, quantitative, and mixed methods approaches*, Sage publications.

Duffee, G. R. (1999), 'Estimating the price of default risk', *The Review of financial studies* **12**(1), 197–226.

Duffie, D. and Singleton, K. J. (1999), 'Modeling term structures of defaultable bonds', *The review of financial studies* **12**(4), 687–720.

Elizalde, A. (2005*a*), 'Credit risk models ii: Structural models'.

Elizalde, A. (2005*b*), 'Credit risk models iii: reconciliation reduced-structural models', *CEMFI and Universidad de Navarra Working Paper* .

Finger, C., Finkelstein, V., Lardy, J.-P., Pan, G., Ta, T. and Tierney, J. (2002), 'Creditgrades technical document', *RiskMetrics Group* pp. 1–51.

François, P. and Morellec, E. (2004), 'Capital structure and asset prices: Some effects of bankruptcy procedures', *The Journal of Business* **77**(2), 387–411.

Fridson, M. S., Garman, C. M. and Okashima, K. (2000), 'Recovery rates: the search for meaning', *Merrill Lynch High Yield, March* .

Frühwirth, M. and Sögner, L. (2006), 'The jarrow/turnbull default risk model—evidence from the german market', *The European Journal of Finance* **12**(2), 107–135.

Gakure, R., Ngugi, J. K., Ndwiga, P. M. and Waithaka, S. M. (2012), 'Effect of credit risk management techniques on the performance of unsecured bank loans employed commercial banks in kenya', *International Journal of Business and Social Research (IJBSR)* **2**(4), 221–236.

Geske, R. (1977), 'The valuation of corporate liabilities as compound options', *Journal of Financial and quantitative Analysis* pp. 541–552.

Geske, R. (1979), 'The valuation of compound options', *Journal of financial economics* **7**(1), 63–81.

Giesecke, K. (2004), 'Credit risk modeling and valuation: An introduction', *Available at SSRN 479323* .

Gordy, M. B. (2000), 'A comparative anatomy of credit risk models', *Journal of Banking & Finance* **24**(1-2), 119–149.

Gupton, G. M. (2005), 'Advancing loss given default prediction models: how the quiet have quickened', *Economic Notes* **34**(2), 185–230.

Hamerle, A., Liebig, T. and Rösch, D. (2003), 'Credit risk factor modeling and the basel ii irb approach'.

Hull, J., Nelken, I. and White, A. (2004), 'Merton's model, credit risk, and volatility skews', *Journal of Credit Risk Volume* **1**(1), 05.

Jabareen, Y. (2009), 'Building a conceptual framework: philosophy, definitions, and procedure', *International journal of qualitative methods* **8**(4), 49–62.

Jarrow, R. A., Lando, D. and Turnbull, S. M. (1997), 'A markov model for the term structure of credit risk spreads', *The review of financial studies* **10**(2), 481–523.

Jarrow, R. A. and Turnbull, S. M. (1995), 'Pricing derivatives on financial securities subject to credit risk', *The journal of finance* **50**(1), 53–85.

Jarrow, R. and Protter, P. (2004), 'Structural versus reduced form models: a new information based perspective', *Journal of Investment management* **2**(2), 1–10.

Jones, E. P., Mason, S. P. and Rosenfeld, E. (1984), 'Contingent claims analysis of corporate capital structures: An empirical investigation', *The journal of finance* **39**(3), 611–625.

Jones, S. (2008), 'Advances in credit risk modelling and corporate bankruptcy prediction'.

Kao, D.-L. (2000), 'Estimating and pricing credit risk: An overview', *Financial Analysts Journal* **56**(4), 50–66.

Karanja, J. M. (2010), A survey of the credit policies in the insurance companies in Kenya, PhD thesis, University of Nairobi, Kenya.

Karatzas, I. and Shreve, S. (2012), *Brownian motion and stochastic calculus*, Vol. 113, Springer Science & Business Media.

Kerlinger, F. N. (1966), 'Foundations of behavioral research.'.

Kithinji, A. M. (2010), 'Credit risk management and profitability of commercial banks in kenya'.

Kwagara, M. P. (2006), An Assessment of credit risk management techniques adopted by micro-Finance institutions in Kenya, PhD thesis, University of Nairobi.

Lagat, F. K., Mugo, R. and Otuya, R. (2013), 'Effect of credit risk management practices on lending portfolio among savings and credit cooperatives in kenya'.

Lando, D. (2009), *Credit risk modeling: theory and applications*, Princeton University Press.

Leeth, J. D. and Scott, J. A. (1989), 'The incidence of secured debt: evidence from the small business community', *Journal of Financial and Quantitative Analysis* pp. 379–394.

Markowitz, H. M. (1968), *Portfolio selection*, Yale university press.

Mbua, S. N. (2017), Effect of interest rates capping by the Central Bank of Kenya on the banks listed on the Nairobi Securities Exchange, PhD thesis, United States International University-Africa.

McMenamin, J. (2002), *Financial management: an introduction*, Routledge.

McNeil, A. J., Frey, R. and Embrechts, P. (2005), *Quantitative risk management*, Princeton univ. press.

Merton, R. C. (1974), 'On the pricing of corporate debt: The risk structure of interest rates', *The Journal of finance* **29**(2), 449–470.

Moraux, F. (2002), 'Valuing corporate liabilities when the default threshold is not an absorbing barrier', *Available at SSRN 314404* .

Mureithi, E. (2016), The effect of credit management techniques on the financial performance of commercial banks in Kenya, PhD thesis, University of Nairobi.

Muriithi, J. G., Waweru, K. M. and Muturi, W. M. (2016), 'Effect of credit risk on financial performance of commercial banks kenya'.

Mutonga, S. (2009), Credit risk management models by commercial banks in Nairobi Kenya, PhD thesis, University of Nairobi.

Ongeri, G. M. (2014), The effect of macroeconomic variables on the financial performance of non-bank financial institutions in Kenya, PhD thesis, University Of Nairobi.

Opler, T. C. and Titman, S. (1994), 'Financial distress and corporate performance', *The Journal of finance* **49**(3), 1015–1040.

Rosen, D. and Saunders, D. (2009), 'Analytical methods for hedging systematic credit risk with linear factor portfolios', *Journal of Economic Dynamics and Control* **33**(1), 37–52.

Schönbucher, P. J. (2000), Factor models for portofolio credit risk, Technical report, Bonn Econ Discussion Papers.

Schuermann, T. (2004), 'What do we know about loss given default?'.

Sukamolson, S. (2007), 'Fundamentals of quantitative research', *Language Institute Chulalongkorn University* **1**, 2–3.

Tan, T. K. (2012), 'Financial distress and firm performance: Evidence from the asian financial crisis', *Journal of Finance and Accountancy* **11**, 1.

Thygerson, K. J. (1995), *Management of financial institutions*, Scott, Foresman.

Valášková, K., Gavlakova, P. and Dengov, V. (2014), Assessing credit risk by moody's kmv model, *in* '2nd International Conference on Economics and Social Science (ICESS 2014), Information Engineering Research Institute, Advances in Education Research', Vol. 61, pp. 40–44.

Walkling, R., Crockford, W., Finn, J. and Hudson, R. (n.d.), 'Institute and faculty of actuaries'.

Wang, Y. (2009), 'Structural credit risk modeling: Merton and beyond', *Risk Management* **16**(2), 30–33.

Wanjagi, A. J. (2018), Effect of credit risk management practices on performance of commercial banks in Kitengela, Kenya, PhD thesis, KCA University.

Whitaker, R. B. (1999), 'The early stages of financial distress', *Journal of economics and finance* **23**(2), 123–132.

Wruck, K. H. (1990), 'Financial distress, reorganization, and organizational efficiency', *Journal of financial economics* **27**(2), 419–444.

Yang, Y., Li, L., Zhou, Z. and Fei, W. (2013), 'The research on applicability of amended kmv model with different industries', *Journal of Risk Analysis and Crisis Response* **3**(1), 27–33.

Appendix A: R Codes

```
#Time Plots

Data<-read.csv(file.choose(),header=TRUE);Data
par(mfrow=c(1,2))
Asset<-Data[,x];Asset
plot.ts(Asset,lwd=2,col=c(4),main="Time plot (Company) 
   Assets",ylab="Assets",xlab="Years")
Debt<-Data[,y];Debt
plot.ts(Debt,lwd=2,col=c(2),main="Time plot (Company) 
   Liabilities",ylab="Liabilities",xlab="Years")

# Survival and Default Probabilities using Merton's Model

Mert<-Merton(L=Value, V0=Value, sigma=Value, r= 0.1452, t=c
   (1, 2, 3, 4, 5, 6, 7));Mert
plot(c(2014, 2015, 2016, 2017, 2018, 2019, 2020), Mert$Surv,
     main = 'Survival Probabilities for different Maturities
        \n (Merton model)',
     xlab = 'Maturity', ylab = 'Survival Probability', type
        = 'b')
p<-(1-Mert$Surv);p
plot(c(2014, 2015, 2016, 2017, 2018, 2019, 2020), p,
     main = 'Default Probabilities for different Maturities 
        \n (Merton model)',
     xlab = 'Maturity', ylab = 'Default Probability', type =
         'b')

#Credit Spread Computation

Sigma<-value;Sigma
k<-Debt;k
r<-0.1452;r
T<-7;T
At<-Asset;At

....
```

```
d1<-(1/(Sigma*sqrt(T)))*(log(At/k)+(r+0.5*Sigma^2)*T);d1
d2<-d1-Sigma*sqrt(T);d2
z<-(-1/T)*log(pnorm(d2)+At/(k*exp(-r*T))*pnorm(-d1));z
```