



# APPLICATIONS OF MATHEMATICS IN SOCIAL AND BIOLOGICAL SCIENCES

by

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# ABSTRACT

The infinite and elegant universe has inspired wise-men in societies to formulate and develop mathematical concepts throughout history. It has served survival needs, been used in conflicts, art and music.

Mathematics has been the mainstay of physics and the disciplines of engineering, and is beginning to play an important role in the biological and social sciences. Today's talk addresses the application of mathematics in these fields through the techniques of modeling.

# INTRODUCTION

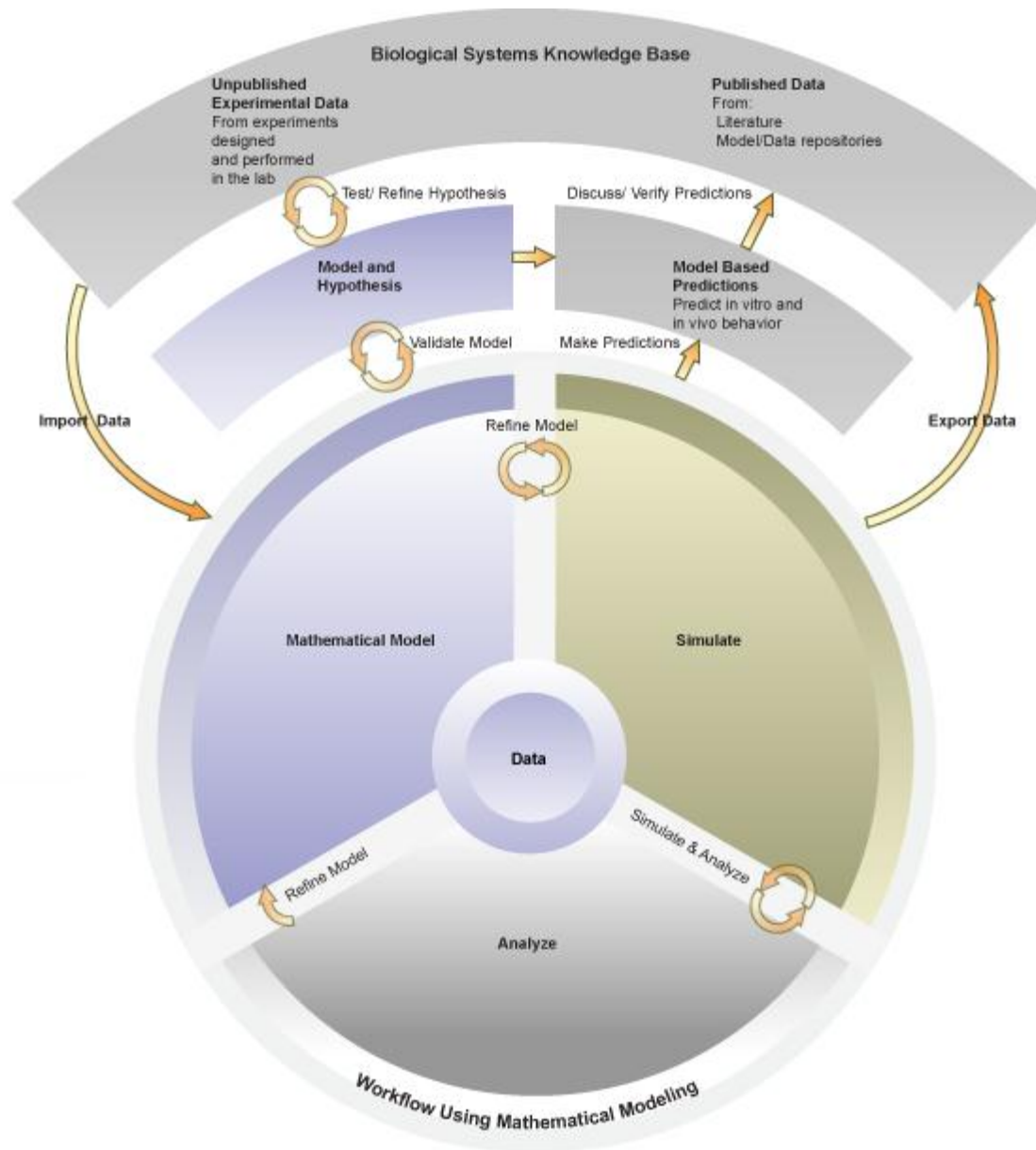
European, Arab, Indian and Chinese philosophers, mathematicians and scientists had been contributing in the variety of disciplines throughout the history. The inter-disciplinary approaches which are considered almost essential nowadays, existed and practiced in the past as well.

The primary application of mathematics in the real world has been through models by which the real world situation is represented and interpreted by the use of abstract symbols. Examples have been abundant throughout history, with a leap in applications during and after World War II. The process of mathematical modeling is considered to be an art as well as science.

Models are considered as the representation or abstraction of actual objects, processes, situations whose behavior patterns researchers wish to analyse and are essential for problem description in a holistic manner. The relationship between real world and conceptual world is described as follows:

The real world	The conceptual world
Phenomenon (under study)	Observation Models (analysis) Predictions

# Modeling Process



\*From Mathworks

- 
- The basic classification of models include:

*Iconic or physical models*

*Analogue or diagrammatic models*

*Symbolic or mathematical models*

- The mathematical models are further classified into:

*Standard vs Custom made*

*Qualitative vs Quantitative*

*Stochastic vs Deterministic*

*Optimizing vs Descriptive*

*Static vs Dynamic*

*Simulation vs Non-simulation*

# Essentials of a mathematical model

Before constructing a mathematical model, following questions about the undertaken research problem must be considered

Questions	Description
How shall the solution of the problem be expressed?	what is the objective measure of effectiveness of the problem?
What are the aspects to be controlled?	What are the aspects of the problem that can be controlled and adjusted (represented by controllable variables)?
What aspects are beyond our control?	What are the aspects that are beyond the control of researcher and have to be accepted as constants
What is the relation between aspects and objectives?	What is the relationship between these (two types of) aspects as well as objectives and can this relationship be expressed mathematically?

# Analysis Process - Descriptive

After resolving the above question through satisfactory answer, the model for the research problem is constructed. It is then solved and finally the model is evaluated, before implementation.

Research design deals with the data collection methods, various analysis tools and related activities, which is vital to acquire adequate data for analysis. This assists in developing the empirical framework for research.

The first step in data analysis is to display the data diagrammatically. The next step of data analysis is to move from graphs to well chosen numerical descriptions and includes computation of four fundamental measures:

*Measures of central tendency; Dispersion; Skewness; and Kurtosis*

Researcher then looks into the following aspects also:

*Correlation and regression; Association, and causation; Predication.*



# Analysis Process - Sampling

Most of the Applied and Social Science studies require the sampling techniques to collect data for analysis, such that formulated research questions and research hypotheses are investigated, through parametric and non parametric tests.

Through theory of estimation the analysed results are extended (through statistical inferences) to the population using various levels of significances.

Statistics faces the variability and uncertainty of the real world directly. Statistical reasoning can produce data whose utility is not destroyed by variation and uncertainty. It can analyse data to separate systematic patterns from the already existing variations. Statistical reasoning also allows us to state how uncertain our conclusion (through data analysis) are going to be.

Research is inherently an uncertain enterprise. Statistics cannot make up for the deficiencies in the questions asked, in the precisions, reliability or validity of measurements, but can clarify and quantify the inherent aspects of uncertainty, so that conclusions can be drawn for decision making.



# Analysis Process - Tools

The analysis of data often requires the use of various mathematical and statistical tools that are often regarded by most of the researchers as difficult to handle. However, with computing innovation and increased use of Information Technology tools, the data analysis is conveniently done through various software packages:

- Word processor
- SPSS, static, mat lab
- CAD/CAM software
- The internet
- Artificial intelligence
- Virtual reality
- Simulation and modeling software
- Neural networks, etc

“ However, researchers must be knowledgeable enough to adequately interpret the results obtained through data analysis (using software packages) for the decision making processes.

# Analysis Process - Checks and Balances

In order to appraise the data analysis techniques, researchers must ensure that adequate data (both in quality and quantity) has been collected for the undertaken research problem.

Research is basically a systematized version of activities that strive to create or add to the knowledge set. It is assumed that all the people involved in research are curious, thoughtful as well as problem solving creatures, who deal with the current situation and also predict the future events and their related outcomes.

The basic attributes of Applied and Social Science research are human processes that exist in natural as well as created world of human beings. The main purpose of research is to create new knowledge, that can be accomplished, in several ways through:

*Description*

*Determination*

*Analysis*

*Exploration/experiments*

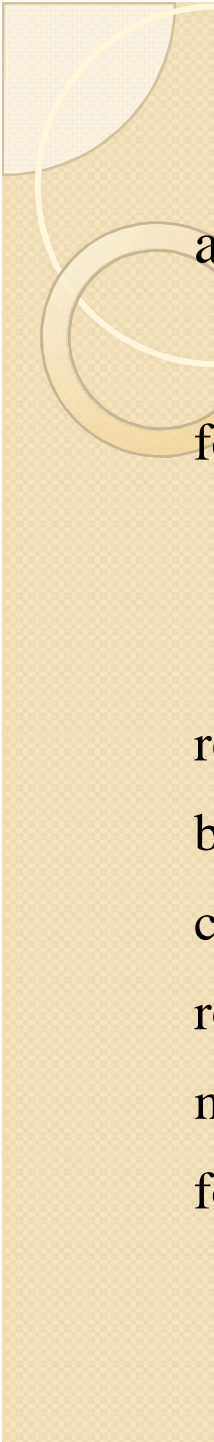
The first step in the research is the identification of the topic. It must be ensured that the selected problem is researchable. Thus a researcher then has to develop conceptual/theoretical framework. This involves in identifying variables and their relationships, which then leads to the formulation of the research problem. The theory of cause and effect plays an important role in understanding the problem

# DETERMINISTIC AND STOCHASTIC MODELS IN BIOLOGICAL PROCESSES

Mathematical modeling is the process by which the real world situation is represented and interpreted by the use of abstract symbols. The process of mathematical modeling is considered to be an *art* as well as *science*. The science aspect deals with the topics needed to execute the relevant steps in the modeling process. It is well known that the problems of reality are never same; therefore features such as creativity, intuition and foresight also contribute quite importantly in the modeling process. These features constitute the art aspect of mathematical modeling and make it very demanding as well as challenging activity. The science aspect can be appreciated in a passive learning mode but the art aspect can be appreciated only by learning in an active mode that is, by building mathematical models and learning from experience. The art of modeling is considered to be very important during iterative phase. The mathematical modeling can be conducted through various approaches. The two approaches presented here are: Systems approach, differential equation (and other mathematics discipline's) approach.

## Systems Approach

In this approach, the real world associated with the problem is viewed as a system. Identifying the features of the system that are relevant for the problem is called system characterization. A system is usually defined as a collection of one or more related objects, which are physical entities with specific attributes or characteristics. The objects can either be non-interacting or interacting in some sense and out of these the interacting ones are of special interest since they closely resemble the real world. The attributes of an object are described in terms of parameters and variables. Parameters are attributes intrinsic to an object whereas variables are attributes needed to describe interaction between objects. The interactions between objects are described through relationships linking the variables of the interacting objects. The *“cause-effect”* interactions are considered to be of particular importance since these produce what are termed as *“causal relationships”*. The causal relationships are popularly indicated through either graph-theoretic or matrix tabular display.



Systems (like traditional models) can also be broadly classified as: static vs. dynamic; continuous vs. discrete; deterministic vs. stochastic.

The representation of systems (like that of models) can also be of the following types:

scaled; pictorial; verbal; figurative or schematic and symbolic.

It is often remembered that, for a symbolic formulation to become representation of a system, a one-to-one correspondence must be established between the symbols of the formulation and the physical features or characteristic of the system. Without this correspondence the symbolic representation of the system becomes abstract (and approaches the format of mathematical model). Based on the mathematical structure of the underlying formulation,



The mathematical models can then be classified into four categories that are suitable for modeling:

*deterministic static systems*

*deterministic dynamic systems*

*probabilistic static systems*

*probabilistic dynamic systems.*

The *operation research models* use alternative (other than calculus) optimization techniques to solve problems. The *reflexion model* technique allows software engineers to quickly and cost effectively gain task specific knowledge about a systems code. The *high level model* is employed by users on the basis of desired software engineering work.



## Differential Equation Approach

Differential equation is a relation, satisfied in a domain, between unknown function and its derivatives. If the unknown function is of one variable only then we have an *ordinary differential equation*, but if it is function of several variables then we have a *partial differential equation*. The derivative terms are used to represent rates of change of physical attributes in an undertaken study. The solution (or integral) of the differential equation satisfies the relationships identically in the considered domain.



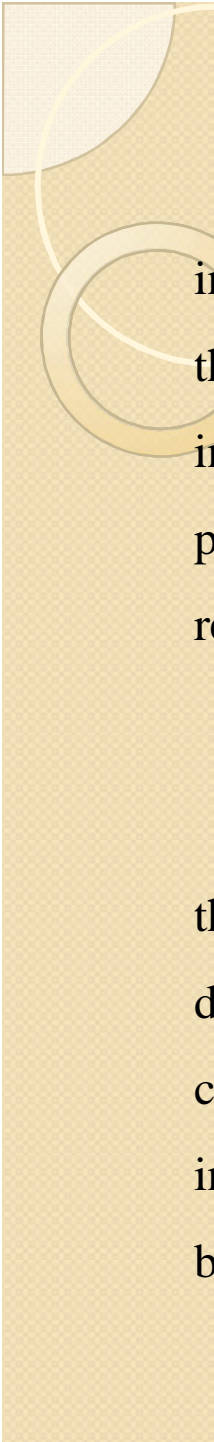
## Biological Processes and Models

The Scientific study of life and living things including their origin, diversity, structures, activities and distribution constitute biology. The facts about organisms and their overall dynamics lead to biological processes. Some of the areas of biology that created interest amongst mathematicians are:

***Epidemiology*** ó deals with spread of various types of infectious diseases and their control mechanism within a chosen population.

***Cellular and molecular biology*** ó deals with DNA structure, genetic mapping, cell and structural biology.

***Organismic biology*** ó deals with physiology, morphology, development and behavior of animals and plants. Neuroscience and immunology.



These disciplines use various branches of mathematics for their studies. The integrated behaviors of complex biological systems is represented and studied through mathematical models. Mathematical models by their very nature have built-in assumptions and approximations that restrict their range of validity, but with proper care in the construction stage the range can be made wide enough to cover relevant aspects of the undertaken problem.

The mathematics of disease is considered a data-driven subject, but the theoretical work has been able to link mathematical model and data values. The disease dynamics requires various mathematical tools required to model construction to solving of differential equation to statistical analysis. The introduction of element of chance into the rules of the program, the model is said to be *stochastic*, which becomes more realistic for representation.

## DETERMINISTIC MODELS

The basic models divide the corresponding populations into susceptible, infectious and recovered (or removed). These are termed as *SIR models* for different diseases. The proportions of individuals in each class of such models are given by differential equations.

A model for growth of infection was constructed (Pokhariyal, 1986) by considering the following factors:

*The structure and resistance (immunity) provided by the tissues.*

*The environment and other external factors (including the other competing bacteria) supporting or opposing the infection.*

*The progeny and their extent of survival.*

The growth of infection at any time (from initial infection to the level-off time) was given as

$$\frac{dp_t(I)}{dt} = \gamma \{1 - p_t(I)\} \{p_t(I) - p_o(I)\} \dots (1)$$

where  $\gamma$  can be termed as a parameter. The explicit solution of (1), does not agree with the boundary conditions (the actual situation), therefore the implicit expression for infection proportion  $p(I)$  was found as:

$$p_t(I) = L - \left\{ L - p_0(I) \right\} \exp \left[ - \int_0^t \gamma \left\{ p_t(I) - p_0(I) \right\} dt \right] \dots\dots(2)$$

At the time for the highest infection proportion growth  $t = t_c$  it was found that

$$f_{t_c}(I) = \gamma \left[ \frac{L - p_0(I)}{2} \right]^2 \dots\dots\dots(3)$$

The preventive measures under different conditions were suggested. The mathematical basis for generating and assessing simulated disease profiles in plant pathogen epidemics with emphasis on accuracy was presented (Pokhariyal and Rodrigues, 1993). A computational model was developed and through simulation conclusion about its superiority over other models was established. It was shown that the product of model parameters is a constant as:

$$\gamma t_c \left\{ L - p_0(I) \right\} = 2 \ln 2 = 1.3862 \dots\dots\dots(4)$$

A crop development model considering seed characteristics and climatological parameters was constructed (Pokhariyal, 2002) on similar lines. A relationship between model parameters was shown as:

$$\gamma m t_c^2 = 0.48045 \dots\dots\dots (5)$$

The equation (5) was utilised for suggesting effective crop development strategies. A deterministic model for HIV infection and its application was developed (Pokhariyal and Simwa, 2004) by considering the various stages of infection through corresponding differential equations and the boundary conditions, which are linked with the CD<sub>4</sub> cell count in the patient's body. Using the data from patients' records different scenarios can be simulated for the development of *HIV/AIDS* antiretroviral drugs treatment strategy.

## **STOCHASTIC MODELS**

The models having variables that are involved with chance or probability factors are referred to as *stochastic models*. These models are considered to be representing the real world in a better way. A dynamic model for stage specific HIV incidences with application to sub-Saharan Africa was constructed (Simwa and Pokhariyal, 2003b). In another study "On empirical modelling of HIV/AIDS pandemic with application to East Africa", (Simwa and Pokhariyal, 2003a), the trend patterns of the expected HIV/AIDS epidemic for Kenya and Uganda were determined.



## N-Species Competition Model

The *Chemostat* is an important laboratory apparatus used for the continuous culture of micro-organism. In ecology it is often viewed as a model of a simple lake system, of waste water treatment process or of biological waste decomposition. It is considered to be an excellent experimental venue in which the researchers study the effect of simple microbial interaction, including exploitative competition

The N-Species Competition model in the periodic Chemostat is given by (Wolkowicz and Zhao, 1997):

$$\frac{dS(t)}{dt} = (S^0(t) - S(t))D_0(t) - \sum_{i=1}^n p_i(t, S(t))x_i(t) \dots\dots (6)$$

$$\frac{dx_i(t)}{dt} = x_i(t) \{ p_i(t, S(t) - D_i(t)) \}, \quad 1 \leq i \leq n. \dots\dots(7)$$

where:-

- $S(t)$  denotes the concentration of the nutrient
- $X_i(t)$  denotes the biomass of the  $i$ th species at time  $t$
- $P_i(t, s)$  represents the specific per capita nutrient uptake function of the  $i$ th species



- $S^0(t)$  input nutrient concentration
- $D_0(t)$  dilution rate
- $D_i(t)$  represents the specific removal rate or washout rate of species xi.

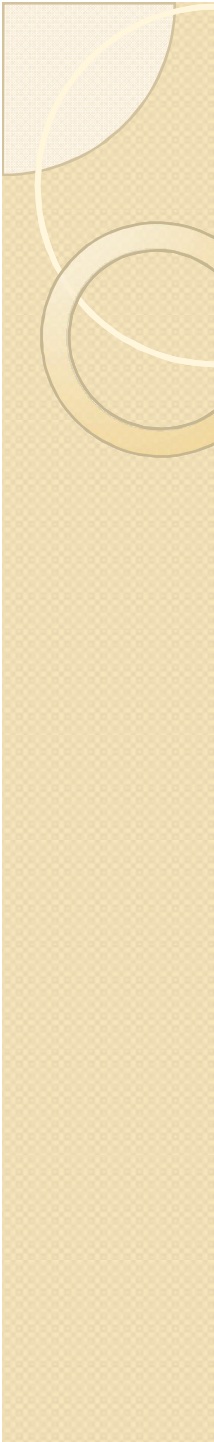
We assume that  $S^0(t)$  and  $D_i(t)$  are all continuous, w-periodic and positive functions, and that each  $P_i(t, s)$  is locally Lipschitz and is strictly increasing for  $s < t$ .

For the linear periodic equation

$$\frac{dV(t)}{dt} = S^0(t)D_0(t) - D(t)V(t) \dots \dots \dots (8)$$

We have a unique positive w-periodic solution  $V(t)$  given by

$$V(t) = e^{-\int_0^t D(s) ds} \left[ \frac{\int_0^w e^{\int_0^s D(u) du} S^0(s) D_0(s) ds}{e^{\int_0^w D(s) ds} - 1} + \int_0^t e^{\int_0^s D(u) du} S^0(s) D_0(s) ds \right] \dots \dots \dots (9)$$



It was shown that sufficient conditions ensure uniform persistence of all the species and guarantee that the full system admits at least one positive, periodic solution. Improved results in the case of 3-species competition were also given.

In a study (Wasike, Muiruri and Pokhariyal, 2006) a competition model between 2-species in an aquatic system that incorporates monotone response functions and periodic nutrient supply is considered. It was shown that outcome of this (2-species) competition depends on the relative sizes of the break-even concentration. As long as these concentrations are distinct, the species with the smallest break-even concentration survives. The nutrient and surviving species approach limiting values. In a recent study Kimathi, Wasike and Pokhariyal 2012 have considered the situation of competing species and their survival nature.



# Simulation and Regression Models in Baboons and Human Brains (Pokhariyal and Hassanali, 2011)

- The variations in morphometric parameter of mammalian brains may be influenced by process of functional complexity, evolution and adaptation. Comparative analysis of linear measurements of cerebrum in the human and baboon has shown morphometric differences.
- In the present study linear measurements from human and baboon cerebrum (n=10 each) were used to predict various values for human and baboon brain and body parameters through multiple regression models.
- The average brain weights were found to be 2.08% and 0.84% of the body weights for humans and baboons respectively. The elasticity of regression models revealed that unit percentage increase in Occipital-Frontal (OF) distance would increase the human brain weight by 66.19%, while the baboon brain weight would increase by 7.63%.
- The unit percentage increase in the Height of Temporal Lobe (HTL) would increase the human brain weight by 16.28%, while the baboon brain weight would increase by only 0.28%. Unit percentage increase in Frontal-Temporal (FT) distance would decrease the human and baboon brain weights by 14.04% and 0.46% respectively.
- Inter-species values were also predicted through simulation techniques by using the ratios of model parameters with application of programming language Python. The OF, FT and HTL values for human were found to be 2.01 times, 1.55 times and 1.91 times respectively to that of baboon

# Baboon/Human brain measurements

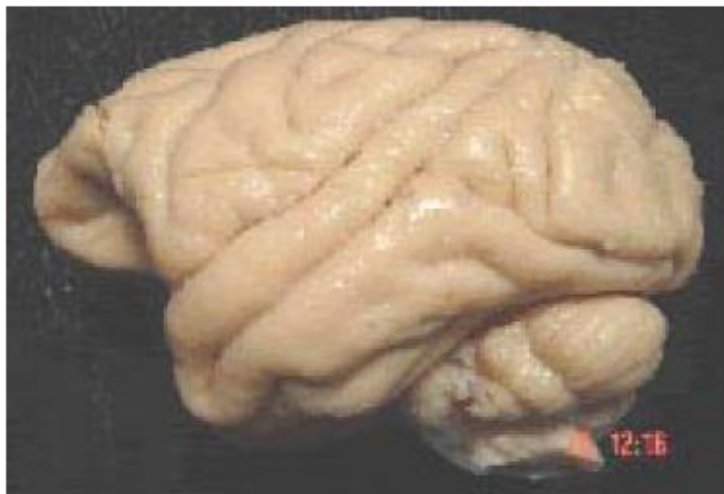
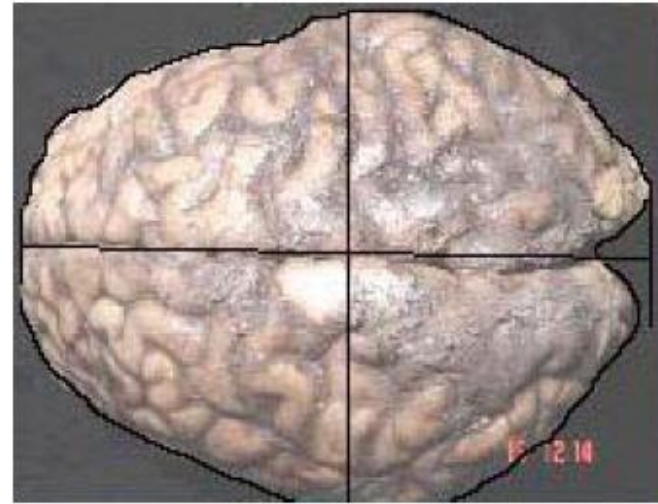
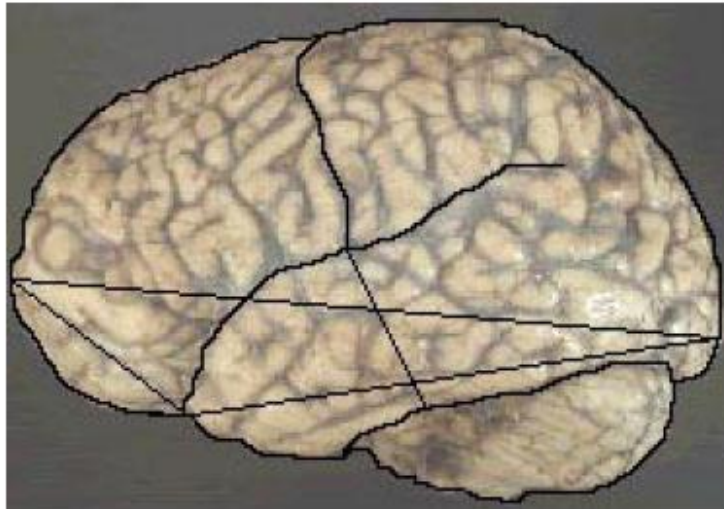


Fig. 1. A. Lateral aspect human brain; B. Lateral aspect baboon brain.

Fig. 1. A. Superior aspect human brain; B. Superior aspect baboon brain.



# SOCIAL SCIENCE MODELS

The most useful model in Social Sciences are the Econometric Models and the Finance Models. The econometric model make use of Economics theory, Mathematical tools and Statistical testing.

Statistical tests involve in:

- \*Computing coefficient of determination, to check the goodness of fit of the model.

- \*Statistical significance of the coefficient of the model parameter (through Z-test or t-test).

- \*Elasticity of the model parameters (that usually conform to laws of economics).

The use of linear (single or multiple regression) as well as non-linear models is quite popular.

Among the non-linear models the Exponential and Geometric models are frequently used, as they can be transformed to the linear models, by use of the logarithms.

# Binary Variable Models

The use of binary (or dummy) variables indicating the presence or absence of an attribute features in qualitative models. In some situations the Likert's scale is used to record the responses of consumers and through computations the qualitative aspects can be transformed to quantitative values.

## Linear/Non-Linear Probability Models

In such models, with given information (say, the income of an individual), the probability of acquiring an asset can be computed. However, LPM suffers from many drawbacks:

- \**Reality is non-linear*
- \**Sum of probabilities can not be guaranteed to add to one.*
- \**Coefficient of determination becomes questionable.*
- \**Heterodasticity.*

Thus the answer lies in the **Non-Linear Models** and the two popularly used models through cumulation are:

- \**Normit (or Probit), cumulative normal.*
- \**Logit (cumulative logistic, log of odd in favour).*

# Finance Models

In the **Financial Models** the Time Series concepts are often used, with elementary addition and product models to:

\* *Auto- Regressive (AR)*

\* *Moving Average (MA),*

\* *ARMA,*

\* *ARIMA*

\* *ARCH.*

\* *GARCH.*

The multiple regression analysis, factor analysis and principal component analysis are frequently used in social science studies





## Finance Models example: Cross Listing and Dividend Policy: Evidence From Cross Listings within East Africa

by Kennedy Munyua Waweru; Ganesh P. Pokhariyal; and Muroki F. Mwaura 2012.

- *The purpose of this study was to examine the impact of cross listing on dividend policy for cross listed firms within East Africa and to test the substitute hypothesis.*
- *The study first conducts univariate analysis for the before-after effects of cross listing using paired tests, then includes non cross listed firms in multivariate analysis using pooled Time Series Cross Section, Panel Corrected Standard Errors regressions for a period of 13 years (1998 to 2010).*
- *The study's findings provide empirical evidence to indicate that the dividend payouts for cross listed firms were relatively higher than the dividend payouts for then non cross listed firms. This is how the big firms cross list in many stock markets in the world.*
- *The findings lend credence to the substitute hypothesis, the payment of higher dividends by cross listed firms may well result from a voluntary commitment on the part of these firm to protect their investors and maintain credible reputation for fair treatment. These findings are contrary to the prediction investor recognition/visibility hypothesis*



## Finance Models example: Real Exchange Rate Volatility in Kenya

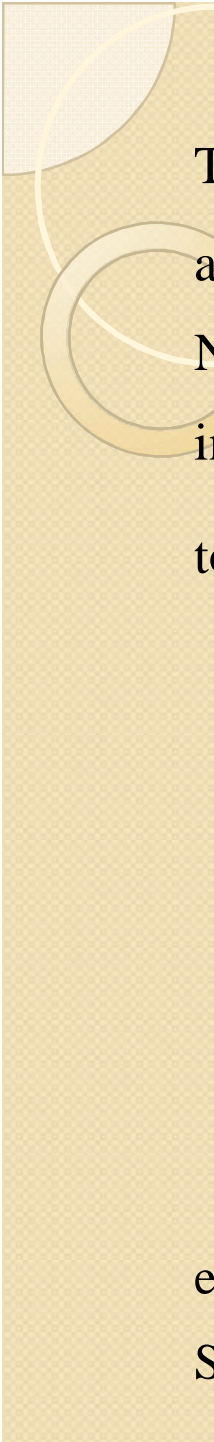
by Danson Musyoki, Ganesh P. Pokhariyal and Moses Pundo - 2012.

- This paper examines Real Exchange Rate (RER) volatility in Kenya by using Generalized Autoregressive Condition of Heteroscedasticity (GARCH) and computation of the unconditional standard deviation of the changes for the period of January 1993 to December 2009.
- Data for the study was collected from Kenya National Bureau of Statistics, Central Bank of Kenya and International Monetary Fund Data Base by taking monthly frequency.
- Thus, 204 data values were analysed, which assisted in evaluating the extent of the trade Kenya had with 182 different countries and used in the construction of the Real Exchange Rate (RER).
- The study found that RER was very volatility for the entire study period. Kenya's RER generally exhibited a appreciating and volatility trend, implying that in general, the country's international competitiveness deteriorated over the study period.
- The RER Volatility reflect negative effect on economic growth of Kenya

## **FUTURE SCOPE AND TRENDS IN MODELLING**

Apart from the traditional branches of physical sciences and engineering, the mathematical models are gaining popularity in various branches of biological sciences. In the social sciences, particularly in economics and finance the use of mathematical and statistical tools is increasing for analysis and decision making strategies.

Walter(Nov, 2010) in his paper "Earthquakes and Weather=quakes : Mathematics and climate change", proposed open problem, "can weather-quakes and global warming/ climate change be deduced from accepted basic principles of mathematics, physics and chemistry assuming every thing we know about or assuming a simplified model of, the atmosphere?"



The future trend in the modeling is to use of Neural Networks, where artificial neurons simulate the behavior of the natural (biological) neurons. Neural Network deal with uncertainties of the phenomena under study. The inaccuracies are dealt with incorporation of Fuzzy logic system (Algebra) to have Fuzzy Neural Network. Neural Networks are used as:

*Pre-processor*

*Post-processor*

*Mathematical Models*

*Baseline control.*

What mathematics has done for Physics and Engineering in the past, it is expected that mathematics would be very important for the development of Social Sciences and Biological Sciences.

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**Thanks**