

DETERMINISTIC AND STOCHASTIC MODELLING OF TECHNICAL RESERVES IN SHORT-TERM INSURANCE CONTRACTS

Patrick G. O. Weke

School of Mathematics, University of Nairobi, Kenya.

Email: pweke@uonbi.ac.ke

ABSTARCT

Claims reserving for general insurance business has developed significantly over the recent past. This has been occasioned by the growth of the insurance market, with the risk underwriting process becoming more and more complex. New insurance products have been developed that cater for the more specific needs of the policyholder. Latent claims have also arisen in recent years, putting major strains on company resources. The case of asbestosis related claims testifies to this, having received widespread attention. Furthermore, recent disasters, such as the floods in Europe and the September 11th terrorist attacks on the U.S. have contributed to the need for more complex ways of analyzing claims experience. The suitability of the models used in claims reserving, have had to be reviewed to ensure that they do not give false impressions.

The object of this paper, therefore, is to come up with a comparison of different deterministic and stochastic methods of claims' reserving for a general insurer with a given claims' experience. The suitability of each of the estimates is noted to depend on the purpose of the reserving exercise. The paper discusses some of the methods (for instance, the basic chain ladder method, inflation adjusted chain ladder method, separation technique, Bornhuetter-Fergusson technique and copula) used in claims' reserving, and for a particular claims experience, it gives an analysis of how well each of the methods models claims experience.

1. INTRODUCTION

The settlement of claims is the prime objective of insurance. Policyholders effect insurance so that in return for the payment of a premium, an insurance company accepts the liability to make a monetary payment to the insured on the occurrence of a specified event within a specified period of time.

In theory, the insurer's liability to pay a claim crystallizes at the instant of occurrence of the insured contingency. However, there are many factors which can lead to ver considerable delays between occurrence and payment. Firstly, the insured contingency itself may not occur at a single instant and may not even be recognized as claimable events until many years after commencement. Secondly, the legal liability of the insurer may not always be clear-cut, and there may be considerable delays before the insurer (or the court) decides that liability exists. Thirdly, the quantum of damages may be impossible to determine until some period of time has elapsed since occurrence of the event. Fourthly, there will be processing delays, within the insurer's

office, in the recording of the claims, processing of the claims file, authorization of payment and drawing, despatch and encashment of the claim payment.

The prediction of outstanding claims amounts in non-life insurance with short-term policies is, by its very nature, highly speculative. Specific details of methodologies for making such predictions are contained in a comprehensive and highly detailed survey conducted by Taylor (1986). One feature common to all of these methods is the utilization of current and past records of claims amounts in the form of run-off triangle to calibrate the proposed prediction model before use. Kremer (1982) has shown how the classical chain ladder method for estimating outstanding claims on general insurance business is strongly related to a two-way analysis of variance. This paper follow closely on the work on the statistical treatment of claims reserving in Mack (1991) which noted the connection between the methods of estimating 'Incurred But Not Reported' (IBNR) claims reserves and automobile rating methods. This parametric model is now implemented in GLIM and applied to claims data (see Weke, 2003). Our purpose is not to add to the existing plethora of methodologies but rather to return to the grass roots of the subject by exploring more fully the statistical setting for the basic chain-ladder and related techniques.

2. CLAIMS DATA

2.1 Data Presentation

The methods for estimating claims reserves that are discussed require data to be presented in the form of a run-off triangle. This presentation cross classifies the data according to the period of origin and the period of development. The period of origin may be the year when the claim was incurred, or reported, or when the policy relating to the claim was underwritten, while the development period refers to the length of time since the period of origin in which the claims were incurred, reported or paid. By convention, the development year relating to the year of origin is denoted as development year zero. A claim cohort is defined depending on the definition used for claims from each origin period and development period. For example, we could have each entry in the triangle as being the value of the claim paid in development year j , the claim having occurred in year of origin i . The general form of the run-off triangle is given by:

Year of origin	Year of development							
	0	1	2		j	...	$n-1$	n
0	$C_{0,0}$	$C_{0,1}$	$C_{0,2}$				$C_{0,n-1}$	$C_{0,n}$
1	$C_{1,0}$	$C_{1,1}$	$C_{1,2}$				$C_{1,n-1}$	
.	.							
.	.							
i	.							
.	$C_{i,0}$	$C_{i,1}$	$C_{i,2}$				$C_{i,n-i}$	
.	.							
.	.							
.	.							
n	$C_{n,0}$							

2.2 Claims Data

Claims run-off data are generated when delay is incurred in the settlement of insurance claims. Typically the format for such data is that a triangle in which the row i denotes accident years and column j delay or development years. The analysis is based on a set of general insurance data, shown in Table 1. This data is taken from a paper by Renshaw (1989) and consists of claims from a portfolio of general insurance policies with exposure factors.

Table 1: Run-off Claims Data and Exposure

35784	766940	610542	482940	527326	574398	146342	139950	227229	67948
352118	884221	933894	1183289	445745	320996	527804	266172	280405	
290507	1001799	926219	1016654	750816	146923	495992	2480405		
310608	1108250	776189	1562400	272482	352053	206286			
443160	693190	991983	769488	504841	470639				
396132	937085	847498	805037	705960					
440832	847631	1131398	1063269						
359480	1061648	1443370							
376686	986608								
344014									

Exposures

610	721	697	621	600	552	543	503	525	420
-----	-----	-----	-----	-----	-----	-----	-----	-----	-----

The exposures for each year of business are divided into the claims data before the analysis is carried out.

3. THE CHAIN LADDER METHOD

3.1 The Basic Chain Ladder Method

The chain ladder method assumes that all external factors, for example, inflation of claim costs, change in the mix of business, change in the rate of settlement of claims, can effectively be ignored and the model the assumes the form

$$C_{ij} = x_i y_j + \varepsilon_{ij} \quad (3.1)$$

where C_{ij} , $i, j = 1, \dots, n$ denote the total amount of claims in development year j in respect of accident year i ,

x_i is the ultimate total cost of claims in accident year i , and

y_j is the proportion of total payments made by the end of development year j .

Let S_{ij} represents claims amount written in development year j with respect to accident year i then

$$C_{i1} = S_{i1} \quad (3.2)$$

$$C_{i,j+1} = C_{ij} + S_{i,j+1}, \quad \text{for all } i, j; \quad j \leq n - i + 1.$$

In the absence of the external factors the distribution of delays between the incident giving rise to a claim and the payments made in respect of that claim remain relatively stable over time. If it is assumed that the exogeneous influences are small, then we may regard expected value of the ratio of cumulative claims amounts for year j to year $(j+1)$

$$E[C_{ij}/C_{i,j+1}]$$

as an estimate of the progression from $C_{i,j}$ to $C_{i,j+1}$ for incomplete rows, i.e. for values i for which $C_{i,j}$ is known but $C_{i,j+1}$ is not.

The method assumes that the factors y_j are constant for all years of accident. If b_j represents the ratio of the cumulative payments made by the end of year j to the expected value of the cumulative payments made by the end of year $j-1$, then b_j may be estimated by

$$b_j = \frac{\sum_{i=1}^{n-j+1} C_{ij}}{\sum_{i=1}^{n-j+1} C_{i,j-1}}, \quad j = 2, \dots, n. \quad (3.3)$$

The b_j factors are thus calculated by summing each column in the run-off triangle (Table 1) and taking the ratio to the previous column total excluding the last entry. Let us denote the product of $(n-j)$ b_j 's by B_j , that is,

$$B_j = \prod_{z=j+1}^n b_z, \quad j = 1, \dots, n-1 \quad (3.4)$$

then we can estimate the amount of claims still outstanding as at the end of year $(i+j)$ in respect of accident year i by

$$C_{ij}(B_j - 1).$$

If we represent the ratio of outstanding liability at the end of development year n for year of accident 1 to the cumulative claims amount C_{1n} by b_{n+} , then b_{n+} is the estimate of the outstanding liability as at the end of development year n (for year of origin 1). And Equation (3.4) now becomes

$$B_j = \prod_{z=j+1}^n b_z, \quad j = 1, \dots, n-1 \quad (3.5)$$

$$B_n = b_{n+}.$$

These estimates can then be used to complete the run-off of the later years of origin up to the point for which past experience is available.

The inflation adjusted chain ladder method which is based on adapting the generalized model by introducing an assumed index of claims cost can also be considered. Other methods, for example, the chi-square method (due to Bailey and Simon, 1960), the method of marginal totals and the method of weighted squares can be used. Details of these methods are available in Weke (1992, 2003).

3.2 Inflation Adjusted Chain Ladder Method

This method adopts the general model in the form:

$$C_{ij} = S_i \cdot R_j \cdot X_{i+j} + e_{ij} \quad (3.6)$$

and the parameters thus become:

$$C_{ij} = s_i \cdot r_j \cdot \lambda_{i+j} + e_{ij} \quad (3.7)$$

where

- C_{ij} are the payments made in development year j of year of origin i , (i.e., non-cumulative)
- s_i is the ultimate total cost in real terms of claims incurred in the period of origin i .
- r_j is the proportion of total payments in real terms made in development year j .
- λ_{i+j} is an assumed index of claims cost

Under the inflation adjusted method, the run-off triangle has to be presented as incremental claims for each year of origin and development. Using a claims inflation index, the past values are brought to current monetary values. Incremental claims along the same diagonal (moving from bottom left to top right) arise from the same year and hence the same inflation index value is applied on them. The adjusted incremental claims are then accumulated and the normal procedures of the basic chain ladder method are applied. These estimated claims reserves are also in current monetary terms. In order to estimate the cash value of future claim payments, an assumption has to be made about the likely level of future claim inflation.

- Assumptions:**
- The claims development pattern is stable
 - Claims inflation will be at the assumed future rate

3.3 The Separation Technique

It has the form of equation (3.6):

$$C_{ij} = S_i \cdot R_j \cdot X_{i+j} + e_{ij}$$

with parameters

$$C_{ij} = n_i \cdot r_j \cdot \lambda_{i+j} + e_{ij} \quad (3.8)$$

where n_i is the number of claims incurred in the year of origin i and λ_{i+j} is related to the year of payment. In this case λ_{i+j} is derived from the data rather than assumed from external sources. The derived factors will be related to increases in claim costs but will also be affected by other external factors and by random fluctuations in the claim size. As a result, they are likely to correspond to any assumed index considered suitable for use with the chain ladder method.

The method for analysing the run-off triangle is as follows. In respect of each year of origin i , the claim payment C_{ij} , made in each development year j are divided by some exposure index S_i , attributable to the period of origin. These exposure measures may be vehicle years or earned premiums. However, they may not accurately reflect the differences in the risks underwritten (a particular problem in using earned premiums). Furthermore, results will be affected by changes in claim frequency. To avoid these problems, normalizing factors are used, the most common being the number of claims n_i (Hossack *et al*, 1999).

The method of assessing the number of claims should be consistent from year to year. As the latest year of origin has developed least, the number of claims assumed for earlier years should be of the same duration. For this reason, the claim numbers are usually related only to the claims reported in the first year. The r_j and λ_{i+j} parameters are then estimated as follows:

Year of origin	Year of development							
	0	1	2		j	...	$n-1$	n
0	$r_0\lambda_0$	$r_1\lambda_0$	$r_2\lambda_0$				$r_{n-1}\lambda_0$	$r_n\lambda_n$
1	$r_0\lambda_1$	$r_1\lambda_1$	$r_2\lambda_1$				$r_{n-1}\lambda_1$	
.	.							
.	.							
i	.							
.	$r_0\lambda_i$	$r_1\lambda_i$	$r_2\lambda_i$		$r_j\lambda_i$			
.	.							
.	.							
.	.							
n	$r_0\lambda_n$							

The (r_j) are the proportion of claims paid in year j so by definition $\sum r_j = 1$.

Then by summing the diagonal involving λ of the run-of triangle, we have:

$$d_n = \lambda_n (r_0 + r_1 + \dots + r_n) = \lambda_n \quad (3.9)$$

where d_j is the total of the j^{th} diagonal.

Thus $\lambda_n = d_n$.

If v_j is the sum of the column of the triangle containing r_j , then

$$v_j = r_j(\lambda_n + \lambda_{n-1} + \dots + \lambda_j) \quad (3.10)$$

hence $v_n = r_n \lambda_n$ or $r_n = v_n / \lambda_n$

Summing the next diagonal gives

$$d_{n-1} = \lambda_{n-1}(r_0 + r_1 + \dots + r_{n-1}) = \lambda_{n-1}(1 - r_n) \quad (3.11)$$

and so

$$\lambda_{n-1} = d_{n-1} / (1 - r_n) \quad (3.12)$$

and therefore, recursively:

$$r_j = v_j / (\lambda_j + \lambda_{j+1} + \dots + \lambda_n) \quad (3.13)$$

$$\lambda_j = d_j / (1 - r_{j+1} - r_{j+2} - \dots - r_n) \quad (3.14)$$

To estimate the claims reserves, it is necessary to make an assumption for future inflation factors. This is done by examining the ratios of past λ_{i+j} factors in the light of the known inflation rates as at the time, and taking into account the expected levels of inflation throughout the future run-off period. This enables the future factors to be calculated which, together with the relevant r_j and n_i factors will produce an estimate for each future development period for the years of origin not fully run-off.

Assumptions: The claims development pattern is stable.

3.4 The Bornhuetter–Fergusson Technique (The B – F method)

The B – F method differs from the basic chain ladder method in that the ultimate claim, S_i , is replaced by an alternative estimate, S_i^{BF} , which is based on external information and expert judgment. The model is thus of the form:

$$C_{ij} = S_i^{BF} \cdot R_j + e_{ij} \quad (3.15)$$

with parameters:

$$C_{ij} = s_i^{BF} \cdot r_j + e_{ij} \quad (3.16)$$

where r_j is the proportion of total payments made by the end of development period j .

S_i^{BF} could be an estimate found by using a simple loss ratio on written premiums (or some other suitable measure of exposure).

Assumptions: - the given loss ratio is correct
 - the claims development pattern is stable
 - the past claims development does not provide any additional information on the future development of claims.

If b_j is the ratio of the expected amount of claims paid by the end of period $(j-1)$, then b_j can be estimated by:

$$b_j = \frac{\sum_{i=0}^{n-j} C_{ij}}{\sum_{i=0}^{n-j} C_{i,j-1}}$$

which is the same parameter as that of the chain ladder method and defining

$$B_j = \prod_{z=j+1}^n b_z$$

The estimated claims still outstanding at the end of year $i+j$ with respect to the origin year i is given by:

$$S_i^{BF} (1 - B_j) \quad (3.17)$$

The Bornhuetter-Ferguson technique assumes that there is prior knowledge about the parameters of the model, making it analogous to a Bayesian approach. The B-F method may also be applied on inflation adjusted claims data (as is the case with the inflation adjusted chain ladder method), and then future claims reserves estimated on an assumption of the future rate of claims inflation.

In the next section, the gamma distribution model with multiplicative errors is described and applied to claims data. However, if we are dealing with a situation in which past inflation rates change with time (and are known) then a slight modification is necessary.

4. A STOCHASTIC APPROACH: A CASE OF THE GAMMA MODEL

Let us suppose that a run-off triangle we have total claims amount variable S_{ij} (with a realization s_{ij}) of $n \times n$ cells labeled (i, j) each with a known measure of exposure, n_i (independent of j in this particular case of claims reserving). Let us also suppose that the total claims amount R_{ijk} of each unit $k = 1, \dots, n_i$ of cell (i, j) has a gamma

distribution with a constant shape parameter, α and a mean value m_{ij} . Then the probability density function (pdf) of the total claims amount variable is

$$f(s_{ij}) = \frac{1}{\Gamma(\alpha)} \left(\frac{\alpha s_{ij}}{m_{ij}} \right)^{\alpha} \frac{1}{s_{ij}} \exp(-\alpha s_{ij}/m_{ij}) \quad (4.1)$$

And the amount generating function (m.g.f.) is

$$M_{ij}(t) = \left(1 - \frac{m_{ij}t}{\alpha} \right)^{-\alpha} \quad (4.2)$$

Assuming that all n_i units cell (i, j) are independent then $s_{ij} = R_{ij1} + R_{ij2} + \dots$ has m.g.f.

$$M_s(t) = \left(1 - \frac{m_{ij}t}{\alpha} \right)^{-n_i\alpha}$$

and hence $S_{ij} \sim \text{Gamma}(n_i\alpha; \alpha/m_{ij})$. Since the realization s_{ij} of S_{ij} are usually known from the run-off triangle and not those of R_{ijk} we therefore work with distribution of S_{ij} .

Now consider the multiplicative approach displayed in the form $m_{ij} = x_i y_j$, where x_i , y_j are unknown parameters for accident year and development year respectively. These unknown parameters can be estimated by the maximum likelihood method. Assuming that all S_{ij} are independent, the loglikelihood function is given by

$$l = \log L = \sum_{i,j} \left\{ -\alpha s_{ij}/(x_i y_j) + n_i \alpha \log(\alpha s_{ij}) - n_i \alpha \log(x_i y_j) - \log s_{ij} \Gamma(n_i \alpha) \right\} \quad (4.3)$$

The m.l.e.s are those values $\hat{x}_i$, $\hat{y}_j$ of x_i , y_j , respectively which maximize Equation (4.3). These likelihood estimators are given by

$$\begin{aligned} \hat{x}_i &= \frac{1}{n_{\bullet}} \sum s_{ij}/y_j, & i &= 1, \dots, n \\ \hat{y}_j &= \frac{1}{n_{\bullet}} \sum s_{ij}/x_i, & j &= 1, \dots, n \end{aligned} \quad (4.4)$$

where $n_{\bullet} = \sum n_i$ over i . The estimators in Equation (4.4) are similar to approximating $\hat{x}_i$ by the n_i -weighted mean of $S_{ij}/n_i y_i$ and $\hat{y}_j$ by the n_i -weighted mean of $S_{ij}/n_i x_i$; $i, j = 1, \dots, n$. The gamma distribution model with shape

parameter $n_i\alpha$ (α is a constant) and mean value n_im_{ij} can be fitted into a generalized linear model (GLM) by specifying the random component, systematic component and the link function (McCullagh and Nelder, 1989)

5. STATISTICAL ANALYSIS OF THE MODEL

In this computer based statistical model, goodness-of-fit statistic derived from the likelihood ratio (called the *deviance*) is used. The fitted values are set equal to the observed values ($\hat{\mu}_{ij} = S_{ij}$) in Equation (4.3) to give the maximum achievable loglikelihood and the deviance therefore becomes

$$\begin{aligned} Dev &= 2\{\log L(s_{ij}; s_{ij}) - \log L(\hat{\mu}_{ij}; s_{ij})\} \\ &= 2\sum_{i,j} n_i\alpha \{-\log(s_{ij}/\hat{\mu}_{ij}) + (s_{ij} - \hat{\mu}_{ij})/\hat{\mu}_{ij}\}. \end{aligned} \quad (5.1)$$

This test statistic depends on a scale parameter estimate, $n_i\alpha$, and when no scale parameter has been explicitly set by the user, GLIM calculates the deviance using the above expression with $\alpha = 1$ and uses the mean deviance as an estimate of $1/\alpha$ for the purpose of calculating standard errors of the parameters. The standardized Pearson residuals, r_i , are computed in GLIM and used to explore the adequacy of the fit of the model.

$$r_i = (s_{ij} - \hat{\mu}_{ij}) / \{V(\hat{\mu}_{ij})\}^{1/2}, \quad i, j = 1, \dots, n, \quad (5.2)$$

where $V(\hat{\mu}_{ij})$ denotes the variance function of the distribution evaluated at the fitted value $\hat{\mu}_{ij}$, and also produce residual plots for analysis.

6. IMPLEMENTATION AND APPLICATION

Our main aim is to implement the model in a GLIM statistical package and apply it to the run-off claims data with exposures in Table 1 to produce predicted claims amounts for the empty (i, j) -th cell in the southeast triangular region. We achieve this objective by user defined macros within GLIM and specify four primary macros (see Weke, 1992 for further details). The output of estimated claims, row totals of the estimated future claims and total estimated future claims are significantly important to the practitioners for forecasting purposes. For any general insurance company these values show the expected claims amounts arising from events which have occurred but of which no notification has yet been received (IBNR), the predicted claims for each accident year and the overall total expected claims amount from the year of origin to the present time. The goodness-of-fit statistic of the model can also be performed by using the package since both the deviance and degrees of freedom are given in the output.

7. RESULTS AND DISCUSSION

The results of the application of GLIM package to the non-cumulative run-off data with exposures (Table 1), for which I am grateful to the anonymous supplier, are now be displayed and discussed.

The parameter estimates for the grand mean, the accident year parameters $R(i)$ and the development year parameters $C(j)$; $i, j = 1, \dots, 10$, with their respective standard errors were computed and used to construct Table 2. The GLIM system automatically sets $R(1) = C(1) = 0$. The residuals and the fitted values of the model for various values of accident year and development year are computed and their components used to draw up the residual plots. Table 2 gives the observed claims amounts, S_{ij} , and estimated future claims S_{ij} . The system calculates the estimated future claims which appear down the 'stairs' from the observed claims amounts. The S_{ij} values are in turn summed up columnwise to produce the row totals of the estimated future claims. Since the claims amounts for accident year 1 is full we have that $RT(1) = 0$. And Table 3 shows the row totals of the estimated future claims and total estimated future claims. Practitioners have keen interest in the values in Table 3 since these values are estimates of the outstanding claims provision at the present time (i.e. at the end of accident year 10) with respect to year of origin and the total overall outstanding claims provision for the entire period. The estimates are of significant use in forecasting the IBNR claims provision and in general organization of business.

It is usually necessary to investigate the extent of predictor instability for this model. Generally, it will be observed that the predictor stability weakens as data points further into the spines of the run-off triangle are varied. An improvement of the model in the case of known claims numbers may also be appropriate to consider.

Table 2: Observed claims amounts and estimated future claims

35784	766940	610542	482940	527326	574398	146342	139950	227229	67948
352118	884221	933894	1183289	445745	320996	527804	266172	425046	93278
290507	1001799	926219	1016654	750816	146923	495992	2840405	356653	90296
310608	1108250	776189	1562400	272482	352053	206286	214175	316555	80144
443160	693190	991983	769488	504841	470639	339427	228571	337833	85531
396132	937085	847498	805037	705960	418038	354081	238439	352418	89223
440832	847631	1131398	1063269	614416	452746	383479	258236	381678	96631
359480	1061648	1443370	1225467	685536	505153	427867	288127	425858	107817
376686	986608	982781	1049747	587237	432719	366515	246813	364794	92357
344014	853339	873144	932640	521726	384446	325628	219279	324098	82054

Table 3: Row totals of the estimated future claims

Row	1	2	3	4	5	6	7	8	9	10
RT	0	93278	446949	610874	991363	1452200	2187186	3665823	4122963	4516353

The total estimated future claims is: 18086988

8. CONCLUSIONS

The main focus of this paper was to determine the 'best estimate' of claims reserves for a particular set of circumstances by comparing the reserve estimates produced by the different methods. The study revealed that for the particular data, the separation technique generally tended to give the best fit to the observed claims experience. It gave the lowest mean, median, range and inter-quartile range for the percentage residual errors. It gave the lowest total claims reserves. The method would thus be suitable in the case where it is important to have a fair picture of the reserves without being pessimistic or optimistic. This would generally be so when the reserving exercise is being carried out for management review purposes and when determining premium rates.

The inflation adjusted chain ladder also gave a reasonably good fit on the observed claims experience. However a trend for it to overestimate the claims in later years of origin was observed. This may explain why the estimated claims reserves of this method tend to be higher than those given by the separation technique. The basic chain ladder method gave similar results but did give higher total claims reserves. The overestimation was however not large in both cases. The methods would thus seem appropriate where a conservative approach is taken in the claims reserving exercise. Determining claims reserves for the published accounts of the company and also for supervision of solvency may be done using either of these two methods. Furthermore, in the case that the insurance company is being valued for a purchase, a conservative value of the reserves is appropriate and either of the two methods could be used.

The B-F methods gave poor fits to the observed claims data. The inflation-adjusted method was observed to consistently overestimate the claims at all years of origin. It thus would not be considered an appropriate model to estimate claims reserves for this class of business. The B-F method without inflation adjustment would also not be appropriate for estimating the claims reserves.

The implementation of this model is GLIM provided enough statistics in the output for testing the fit of the model to the run-off claims data and also produced the estimated claims amounts for the empty (i, j) -th cell in the south east triangular region. The estimated claims amounts are in turn used to produce the row totals of the estimated future claims and the total estimated future claims which are of interest to practitioners.

The estimated claims amounts, the row totals of the estimated future claims and total estimated future claims reported in Table 2 and Table 3 are consistently lower in value than the corresponding estimates in Renshaw (1989). This slight variation in the results is attributed to the fact that Renshaw (1989) used a log-normal distribution model in which case the expected value of the estimated claims amount is a function of an additional term (i.e. half the variance term). Thus the computation of predicted values is based on both the means and variances of the parameter estimates in the log-response function.

The row totals of the estimated claims reported in Table 3 lie between the values calculated using the dynamic estimation and empirical Bayes methods in Verrall (1989). We find from the above comparisons that the results obtained by the parametric model are satisfactorily acceptable and can be used for practical purposes. Therefore this model provides a simple method whose application in claims reserving is nearly as simple to execute as the chain-ladder method but has the advantage of providing the goodness-of-fit test statistic and the estimation error.

REFERENCES

- Bailey, R. A. and Simon, L. J. (1960). Two Studies in Automobile Rate Making. *ASTIN Bulletin* **1**, pp. 192-217.
- Kremer, E. (1982). IBNR-claims and the two-way model of ANOVA, *Scand, Act, J.*, Vol. 1, 47-55.
- Mack, T. (1991). A Simple Parametric Model for Rating Automobile Insurance or Estimating IBNR Claims Reserves. *ASTIN Bulletin*, Vol. 21, No. 1, 93-109
- McCullagh, P. and Nelder, J. R. (1989). Generalized Linear Models (2nd Edition), Chapman and Hall.
- Renshaw, A. E. (1989). Chain Ladder and Interactive Modelling (Claims Reserving and GLIM). *Journal of the Institute of Actuaries*, Vol. 116, Part 3, 559-587.
- Taylor, G. C. (1986). Claims Reserving in Non-Life Insurance, North Holland.
- Verrall, R. J. (1989). A State Space Representation of the Chain Ladder Linear Model. *Journal of the Institute of Actuaries*, Vol. 116, Part 3, 589-609.
- Weke, P. G. O. (1992). *Statistical Models for Estimating IBNR Claims Reserves*, M.Sc. Dissertation, The City University, London.
- Weke, P. G. O. (2003). Estimating IBNR Claims Reserves using Gamma model and GLIM, *The Nigerian Journal of Risk and Insurance*, 2003, Vol. 4, No. 1: 1 – 11.
- Weke, P. G. O. (2006). Deterministic Claims Reserving in Short-Term Insurance Contracts, *E.A.J. of Statistics*, Vol. 1, No. 2: 198 – 213. (With A. Mureithi).
- Weke, P. G. O. (2008). Estimation of IBNR Claims Reserves using Linear Models, *KJST – Series A*, 2008, Vol. 13, No. 1. (accepted).