UNIVERSITY OF NAIROBI

GENERAL RELATIVITY AND ITS APPLICATIONS

BY

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DECLARATION

This is my own research work and has not been presented for a degree award in any other university.

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DEDICATION

I dedicate this work to my sons Kevin Kenguru Matoke, Justuce Nyarangi Matoke ; daughters F. Kemunto Matoke, Lilian Nyaboke Matoke and granddaughters Nicole Bosibori and Brianna Kemunto. Their care, patience, understanding and tolerance enabled me finish this work successfully. I thank you all and may God bless you in abundance.

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I would like to thank God most sincerely for seeing me through the of this course.

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ABSTRACT

Several Philosophers, Mathematicians and Physicists have in recent years tried to interpret general relativity. General relativity or the general theory of relativity is the geometric theory of gravitation published by Albert Einstein in 1916. Most of General relativity are Einstein's predictions which were subject to interpretation. Mathematicians and Physicists have tried to test and apply the predictions. The aim of this study was to establish the tests , applications and future development of general relativity such as how general relativity can be reconciled with the laws of quantum physics to produce a complete and self-consistent theory of quantum gravity.

The study has looked at the predictions, tests and their applications. The predictions have been tested by international observatory bodies such as, National Aeronautics and Space Administration (NASA). This predictions have found their applications in study of the universe. But some of the predictions have not been fully tested, their research is still on, such as, the production of gravitational waves. However, it is still an open question as to how the concepts of quantum theory can be reconciled with those of general relativity. Despite major efforts, no complete and consistent theory of quantum gravity is currently known, even though a number of promising candidates exist.

`LIST OF ABREVIATIONS

1. LIGO: Laser interferometer Gravitational Observatory
2. ICRF: International Celestial Reference Frame
3. VLBI: Very Long Baseline Interferometer
4. GPS: Global Positioning System
5. LAGEOS: Laser Geodynamics Satellite
6. MGS: Mars Global Surveyor
7. LISA: Laser Interferometer Space Antenna
8. NASA: National Aeronautics and Space Administration
9. ESA: European Space Agency
10. GEO-600, TAMA-300 and VIRGO are Gravitational wave detectors in Germany, Japan and Italy respectively.

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CHAPTER 1: Introduction

1.1 General Relativity

General relativity or the general theory of relativity is the geometric theory of gravitation published by Albert Einstein in 1916. It is the current description of gr

avitation in modern physics. General relativity generalises special relativity and Newton's law of universal gravitation, providing a unified description of gravity as a geometric property of space and time, or spacetime. In particular, the curvature of spacetime (x, y, z, ct) is directly related to the four-momentum (mass-energy and linear momentum) of whatever matter and radiation are present, p = $\left(\frac{E}{c}, p_x, p_y, p_z\right)$, where E is mass-energy, c is velocity of light and p_x, p_y, p_z are space linear momentum. The relation is specified by the Einstein field equations, a system of partial differential equations.

1.2 Predictions of General Relativity

Some predictions of general relativity differ significantly from those of classical physics, especially concerning the passage of time, the geometry of space, the motion of bodies in free fall, and the propagation of light. Examples of such differences include gravitational time dilation, gravitational lensing, the gravitational redshift of light, and the gravitational time delay. General relativity's predictions have been confirmed in all observations and experiments to date.

Einstein's theory proposes the existence of black holes—regions of space in which space and time are distorted in such a way that nothing, not even light, can escape—as an end-state for massive stars. There is ample evidence that such stellar black holes as well as more massive varieties of black hole are responsible for the intense radiation emitted by certain types of astronomical objects such as active galactic nuclei or microquasars.

The bending of light by gravity can lead to the phenomenon of gravitational lensing, where multiple images of the same distant astronomical object are visible in the sky. General relativity also predicts the existence of gravitational waves, which have since been measured indirectly; a direct measurement is the aim of projects such as LIGO and NASA/ESA Laser Interferometer Space Antenna. In addition, general relativity is the basis of current cosmological models of a consistently expanding universe.

1.3 Problem Statement

Although general relativity is not the only relativistic theory of gravity, it is the simplest theory that is consistent with experimental data. However, unanswered questions remain, the most fundamental being how general relativity can be reconciled with the laws of quantum physics to produce a complete and self-consistent theory of quantum gravity.

Soon after publishing the special theory of relativity in 1905, Einstein started thinking about how to incorporate gravity into his new relativistic framework. In 1907, beginning with a simple thought experiment involving an observer in free fall, he embarked on what would be an eight-year search for a relativistic theory of gravity. After numerous detours and false starts, his work culminate in the November, 1915 presentation to the Prussian Academy of Science of what are now known as the Einstein field equations,

$$R_{\mu v} - \frac{1}{2}R g_{\mu v} = \frac{8\pi G}{c^4} T_{\mu v} \qquad (1.1)$$

These equations specify how the geometry of space and time is influenced by whatever matter is present, and form the core of Einstein's general theory of relativity.

The Einstein field equations are nonlinear and very difficult to solve. Einstein used approximation methods in working out initial predictions of the theory. But as early as 1916, the astrophysicist Karl Schwarzschild found the first non-trivial exact solution to the Einstein field equations, the so-called Schwarzschild metric,

$$ds^2 = \left(1 - \frac{2a}{r}\right)c^2dt^2 - \frac{1}{\left(1 - \frac{2a}{r}\right)}dr^2 - r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right) \qquad (1,2)$$

This solution laid the groundwork for the description of the final stages of gravitational collapse, and the objects known today as black holes. In the same year, the first steps towards generalizing Schwarzschild's solution to electrically charged objects were taken, which eventually resulted in the Reissner-Nordström solution, now associated with electrically charged black holes.

$$c^2d\tau^2 = \left(1 - \frac{r_s}{r} + \frac{r_Q^2}{r^2}\right)c^2dt^2 - \frac{dr^2}{1 - \frac{r_s}{r} + \frac{r_Q^2}{r^2}} + r^2\,d\Omega^2 \qquad (1.3)$$

where

τ is the proper time (time measured by a clock moving with the particle) in seconds,

c is the speed of light in meters per second,

t is the time coordinate (measured by a stationary clock at infinity) in seconds,

r is the radial coordinate (circumference of a circle centered on the star divided by 2π) in meters,

Ω is the solid angle,

$$d\Omega^2 = d\theta^2 \sin^2\theta d\phi^2 \qquad (1.4)$$

r_s is the Schwarzschild radius (in meters) of the massive body, which is related to its mass M by

$$r_s = \frac{2GM}{C^2} \qquad (1.5)$$

where G is the gravitational constant, and

r_Q is a length-scale corresponding to the electric charge Q of the mass

$$r_Q^2 = \frac{Q^2 G}{4\pi\epsilon_0 C^4} \qquad (1.6)$$

where $1/4\pi\varepsilon_0$ is Coulomb's force constant. In 1917, Einstein applied his theory to the universe as a whole, initiating the field of relativistic cosmology. In line with contemporary thinking, he assumed a static universe, adding a new parameter to his original field equations—the cosmological constant. By 1929, however, the work of Hubble and others had shown that our universe is expanding. This is readily described by the expanding cosmological solutions found by Friedmann in 1922, which do not require a cosmological constant. Lemaître used these solutions to formulate the earliest version of the big bang models, in which our universe has evolved from an extremely hot and dense earlier state. Einstein later declared the cosmological constant the biggest blunder of his life.

During that period, general relativity remained something of a curiosity among physical theories. It was clearly superior to Newtonian gravity, being consistent with special relativity and accounting for several effects unexplained by the Newtonian theory. Einstein himself had shown in 1915 how his theory explained the anomalous perihelion advance of the planet Mercury without any arbitrary parameters. Similarly, a 1919 expedition led by Eddington confirmed general relativity's prediction for the deflection of starlight by the Sun during the total solar eclipse of May 29, 1919, making Einstein instantly famous. Yet the theory entered the mainstream of theoretical physics and astrophysics only with the developments between approximately 1960 and 1975, now known as the Golden age of general relativity. Physicists began to understand the concept of a black hole, and to identify these objects' astrophysical manifestation as quasars. Ever more precise solar system tests confirmed the theory's predictive power, and relativistic cosmology, too, became amenable to direct observational tests.

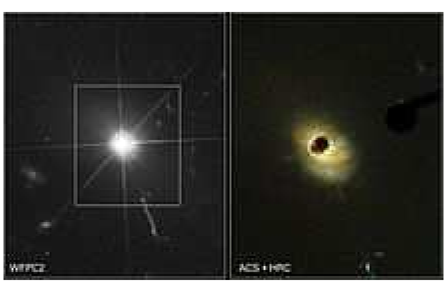


Figure 1

A Hubble picture showing a quasar core

Quasars show a very high redshift, which is an effect of the expansion of the universe between the quasar and the Earth. They are among the most luminous, powerful, and energetic objects known in the universe. They tend to inhabit the very centers of active young galaxies and can emit up to a thousand times the energy output of the Milky Way

Chapter 2: Evolution of General Relativity

2.1. From classical mechanics to general relativity

General relativity is best understood by examining its similarities with and departures from classical physics. The first step is the realization that classical mechanics and Newton's law of gravity admit of a geometric description. The combination of this description with the laws of special relativity results in a heuristic derivation of general relativity.

2.2. Geometry of Newtonian Gravity

At the base of classical mechanics is the notion that a body's motion can be described as a combination of free (or inertial) motion, and deviations from this free motion. Such deviations are caused by external forces acting on a body in accordance with Newton's second law of motion, which states that the net force acting on a body is equal to that body's (inertial) mass multiplied by its acceleration,

$$F = ma \qquad (2.1)$$

The preferred inertial motions are related to the geometry of space and time: in the standard reference frames of classical mechanics, objects in free motion move along straight lines at constant speed. In modern parlance, their paths are geodesics, straight world lines in curved spacetime.

According to general relativity, a ball will fall the same way in an accelerating rocket as it does in a gravitational field, such as on Earth. Conversely, one might expect that inertial motions, once identified by observing the actual motions of bodies and making allowances for the external forces (such as electromagnetism or friction), can be used to define the geometry of space, as well as a time coordinate. However, there is an ambiguity once gravity comes into play. According to Newton's law of gravity, and independently verified by experiments such as that of Eötvös and its successors, there is a universality of free fall (also known as the weak equivalence principle): the trajectory of a test body in free fall depends only on its position and initial speed, but not on any of its material properties. A simplified version of this is embodied in Einstein's elevator experiment, for an observer in a small enclosed room, it is impossible to decide, by mapping the trajectory of bodies such as a dropped ball, whether the room is at rest in a gravitational field, or in free space aboard an accelerating rocket generating a force equal to gravity.

Given the universality of free fall, there is no observable distinction between inertial motion and motion under the influence of the gravitational force. This suggests the definition of a new class of inertial motion, namely that of objects in free fall under the influence of gravity. This new class of preferred motions, too, defines a geometry of space and time—in mathematical terms, it is the geodesic motion associated with a specific connection which depends on the gradient of the gravitational potential. Space, in this construction, still has the ordinary Euclidean geometry. However, space*time* as a whole is more complicated. As can be shown using simple thought experiments following the free-fall trajectories of different test particles, the result of transporting spacetime vectors that can denote a particle's velocity (time-like vectors) will vary with the particle's trajectory; mathematically speaking, the Newtonian connection is not integrable. From this, one can deduce that spacetime is curved. The result is a geometric formulation of Newtonian gravity using only covariant concepts, i.e. a description which is valid in any desired coordinate system. In this geometric description, tidal effects—the relative acceleration of bodies in free fall—are related to the derivative of the connection, showing how the modified geometry is caused by the presence of mass.

2.3. Relativistic Generalization

As intriguing as geometric Newtonian gravity may be, its basis, classical mechanics, is merely a limiting case of (special) relativistic mechanics. In the language of symmetry: where gravity can be neglected, physics is Lorentz invariant as in special relativity rather than Galilei invariant as in classical mechanics. The defining symmetry of special relativity is the Poincaré group which also includes translations and rotations. The differences between the two become significant when we are dealing with speeds approaching the speed of light, and with high-energy phenomena.

Special relativity is defined in the absence of gravity, so for practical applications, it is a suitable model whenever gravity can be neglected. Bringing gravity into play, and assuming the universality of free fall, an analogous reasoning: there are no global inertial frames. Instead there are approximate inertial frames moving alongside freely falling particles. Translated into the language of spacetime: the straight time-like lines that define a gravity-free inertial frame are deformed to lines that are curved relative to each other, suggesting that the inclusion of gravity necessitates a change in spacetime geometry. A priori, it is not clear whether the new local frames in free fall coincide with the reference frames in which the laws of special relativity hold—that theory is based on the propagation of light, and thus on electromagnetism, which could have a different set of preferred frames. But using different assumptions about the special-relativistic frames (such as in free fall), one can derive different predictions for the gravitational redshift, that is, the way in which the frequency of light shifts as the light propagates through a gravitational field. The actual measurements show that free-falling frames are the ones in which light propagates as it does in special relativity. The generalization of this statement, namely that the laws of special relativity hold to good approximation in freely falling (and non-rotating) reference frames, is known as the Einstein equivalence principle, a crucial guiding principle for generalizing special-relativistic physics to include gravity. The same experimental data shows that time as measured by clocks in a gravitational field called proper time, does not follow the rules of special relativity. In the language of spacetime geometry, it is not measured by the Minkowski metric. As in the Newtonian case, this is suggestive of a more general geometry. At small scales, all reference frames that are in free fall are equivalent, and approximately Minkowskian. Consequently, we are now dealing with a curved generalization of Minkowski space. The metric tensor that defines the geometry—in particular, how lengths and angles are measured—is not the Minkowski metric of special relativity, it is a generalization known as a semi- or pseudo-Riemannian metric. Furthermore, each Riemannian metric is naturally associated with one particular kind of connection, the Levi-Civita connection, and this is, in fact, the connection that satisfies the equivalence principle and makes space locally Minkowskian (that is, in suitable locally inertial coordinates, the metric is Minkowskian, and its first partial derivatives and the connection coefficients vanish).

2.4. Einstein's Equations

Having formulated the relativistic, geometric version of the effects of gravity, the question of gravity's source remains. In Newtonian gravity, the source is mass. In special relativity, mass turns out to be part of a more general quantity called the energy-momentum tensor, which includes both energy and momentum densities as well as stress (that is, pressure and shear). Using the equivalence principle, this tensor is readily generalized to curved space-time. Drawing further upon the analogy with geometric Newtonian gravity, it is natural to assume that the field equation for gravity relates this tensor and the Ricci tensor, which describes a particular class of tidal effects: the change in volume for a small cloud of test particles that are initially at rest, and then fall freely. In special relativity, conservation of energy-momentum corresponds to the statement that the energy-momentum tensor is divergence-free

$$T_{\mu v} = \left(\rho + \frac{p}{C^2}\right)u_\mu u_v - g_{\mu v}p \qquad (2.2)$$

This formula, too, is readily generalized to curved spacetime by replacing partial derivatives with their curved-manifold counterparts, covariant derivatives studied in differential geometry. With this additional condition—the covariant divergence of the energy-momentum tensor, and hence of whatever is on the other side of the equation, is zero— the simplest set of equations are what are called Einstein's (field) equations:

$$R_{\mu v} - \tfrac{1}{2}Rg_{\mu v} = \frac{8\pi G}{C^4}T_{\mu v} \qquad (2.3)$$

On the left-hand side is the Einstein tensor, a specific divergence-free combination of the Ricci tensor and the metric. In particular,

$$R_{\mu v} = Rg_{\mu v} \qquad (2.4)$$

is the curvature scalar. The Ricci tensor itself is related to the more general Riemann curvature tensor as

$$R_{\mu v} = R^{\delta}_{\mu\delta v} \qquad (2.5)$$

On the right-hand side, $T_{\mu v}$ is the energy-momentum tensor. All tensors are written in abstract index notation. Matching the theory's prediction to observational results for planetary orbits (or, equivalently, assuring that the weak-gravity, low-speed limit is Newtonian mechanics), the proportionality constant can be fixed as $\kappa = 8\pi G/c^4$, with G the gravitational constant and c the speed of light. When there is no matter present, so that the energy-momentum tensor vanishes, the result are the *vacuum Einstein equations*,

$$R_{\mu v} = 0 \qquad (2.6)$$

There are alternatives to general relativity built upon the same premises, which include additional rules and/or constraints, leading to different field equations. Examples are Brans-Dicke theory, teleparallelism, and Einstein-Cartan theory.

CHAPTER 3: Definition and Tests of General Relativity

3.1. Definition and Basic Properties

General relativity is a metric theory of gravitation. At its core are Einstein's equations, which describe the relation between the geometry of a four-dimensional, pseudo-Riemannian manifold representing spacetime, and the energy-momentum contained in that spacetime. Phenomena that in classical mechanics are ascribed to the action of the force of gravity (such as free-fall, orbital motion, and spacecraft trajectories), correspond to inertial motion within a curved geometry of spacetime in general relativity. There is no gravitational force deflecting objects from their natural, straight paths. Instead, gravity corresponds to changes in the properties of space and time, which in turn changes the straightest-possible paths that objects will naturally follow. The curvature is, in turn, caused by the energy-momentum of matter. Paraphrasing the relativist John Archibald Wheeler, spacetime tells matter how to move; matter tells spacetime how to curve.http://en.wikipedia.org/wiki/General_relativity_resources - cite_note-34

While general relativity replaces the scalar gravitational potential of classical physics by a symmetric rank-two tensor, the latter reduces to the former in certain limiting cases. For weak gravitational fields and slow speed relative to the speed of light, the theory's predictions converge on those of Newton's law of universal gravitation. As it is constructed using tensors, general relativity exhibits general covariance: its laws formulaed within the general relativistic framework—take on the same form in all coordinate systems. Furthermore, the theory does not contain any invariant geometric background structures, i.e. it is background independent. It thus satisfies a more stringent general principle of relativity, namely that the laws of Physics are the same for all observers. Locally, as expressed in the equivalence principle, spacetime is Minkowskian, and the laws of Physics exhibit local Lorentz invariance.

3.2. Model-Building

The core concept of general-relativistic model-building is that of a solution of Einstein's equations. Given both Einstein's equations and suitable equations for the properties of matter, such a solution consists of a specific semi-Riemannian manifold (usually defined by giving the metric in specific coordinates), and specific matter fields defined on that manifold. Matter and geometry must satisfy Einstein's equations, so in particular, the matter's energy-momentum tensor must be divergence-free. The matter must, of course, also satisfy whatever additional equations were imposed on its properties. In short, such a solution is a model universe that satisfies the laws of general relativity, and possibly additional laws governing whatever matter might be present. Einstein's equations are nonlinear partial differential equations and, as such, difficult to solve exactly. Nevertheless, a number of exact solutions are known, although only a few have direct physical applications. The best-known exact solutions, and also those most interesting from a Physics point of view, are the Schwarzschild solution, the Reissner-Nordström solution and the Kerr metric, each corresponding to a certain type of black hole in an otherwise empty universe, and the Friedmann-Lemaître-Robertson-Walker and de Sitter universes, each describing an expanding cosmos. Exact solutions of great theoretical interest include the Gödel universe (which opens up the intriguing possibility of time travel in curved spacetimes), the Taub-NUT solution (a model universe that is homogeneous, but anisotropic), and Anti-de Sitter space (which has recently come to prominence in the context of what is called the Maldacena conjecture). Given the difficulty of finding exact solutions, Einstein's field equations are also solved frequently by numerical integration on a computer, or by considering small perturbations of exact solutions. In the field of numerical relativity, powerful computers are employed to simulate the geometry of spacetime and to solve Einstein's equations for interesting situations such as two colliding black holes. In principle, such methods may be applied to any system, given sufficient computer resources, and may address fundamental questions such as naked singularities.

3.3. Tests of General Relativity

At its introduction in 1915, the general theory of relativity did not have a solid empirical foundation. It was known that it correctly accounted for the "anomalous" precession of the perihelion of Mercury and on philosophical grounds it was considered satisfying that it was able to unify Newton's law of universal gravitation with special relativity. That light appeared to bend in gravitational fields in line with the predictions of general relativity was found in 1919 but it was not until a program of precision tests was started in 1959 that the various predictions of general relativity were tested to any further degree of accuracy in the weak gravitational field limit, severely limiting possible deviations from the theory.

Beginning in 1974, Hulse, Taylor and others have studied the behaviour of binary pulsars experiencing much stronger gravitational fields than found in our solar system. Both in the weak field limit (as in our solar system) and with the stronger fields present in systems of binary pulsars the predictions of general relativity have been extremely well tested locally.

The very strong gravitational fields that must be present close to black holes, especially those supermassive black holes which are thought to power active galactic nuclei and the more active quasars, belong to a field of intense active research. Observations of these quasars and active galactic nuclei are difficult, and interpretation of the observations is heavily dependent upon astrophysical models other than general relativity or competing fundamental theories of gravitation, but they are qualitatively consistent with the black hole concept as modelled in general relativity.

3.3.1. Perihelion Precession of Mercury

Precession is a change in the orientation of the rotational axis of a rotating body. It can be defined as a change in direction of the rotation axis in which the second Euler angle (nutation) is constant. The orbit of a planet around the Sun is not really an ellipse but a flower-petal shape because the major axis of each planet's elliptical orbit also precesses within its orbital plane, partly in response to perturbations in the form of the changing gravitational forces exerted by other planets. This is called perihelion precession or apsidal precession.

Discrepancies between the observed perihelion precession rate of the planet Mercury and that predicted by classical mechanics were prominent among the forms of experimental evidence leading to the acceptance of Einstein's Theory of Relativity (in particular, his General Theory of Relativity), which accurately predicted the anomalies.

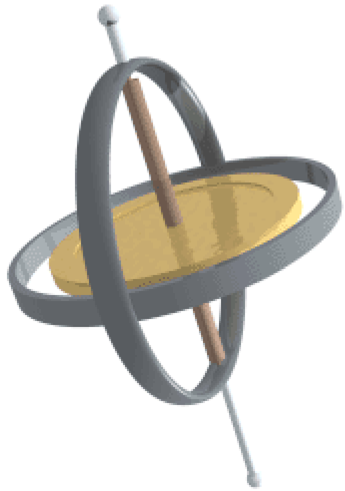


Figure 2

The orientation of Mercury's orbit is found to precess in space over time, as indicated in the adjacent figure (the magnitude of the effect is greatly exaggerated for

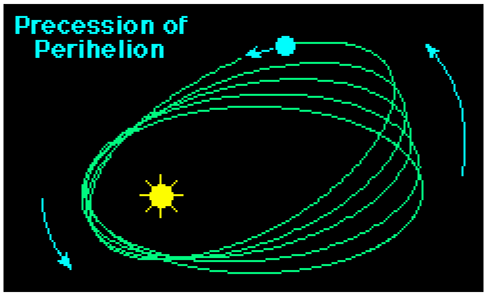


Figure 3

purposes of illustration). This is commonly called the "precession of the perihelion", because it causes the position of the perihelion to move around the center of mass. Only part of this can be accounted for by perturbations in Newton's theory. There is an extra 43 seconds of arc per century in this precession that is predicted by the Theory of General Relativity and observed to occur. This effect is extremely small, but the measurements are very precise and can detect such small effects very well

Under Newtonian physics, a two-body system consisting of a lone object orbiting a spherical mass would trace out an ellipse with the spherical mass at a focus. The point of closest approach, called the periapsis (or, as the central body in our Solar System is the sun, perihelion), is fixed. A number of effects in our solar system cause the perihelions of planets to precess (rotate) around the sun. The principal cause is the presence of other planets which perturb each other's orbit. Another (much more minor) effect is solar oblateness.

Mercury deviates from the precession predicted from the Newtonian effects. This anomalous rate of precession of the perihelion of Mercury's orbit was first recognized in 1859 as a problem in celestial mechanics, by Urbain Le Verrier. His re-analysis of available timed observations of transits of Mercury over the Sun's disk from 1697 to 1848 showed that the actual rate of the precession disagreed from that predicted from Newton's theory by 38" (arc seconds) per tropical century (later re-estimated at 43"). A number of *ad hoc* and ultimately unsuccessful solutions were proposed, but they tended to introduce more problems. In general relativity, this remaining precession, or change of orientation of the orbital ellipse within its orbital plane, is explained by gravitation being mediated by the curvature of spacetime. Einstein showed that general relativity agrees closely with the observed amount of perihelion shift. This was a powerful factor motivating the adoption of general relativity.

Although earlier measurements of planetary orbits were made using conventional telescopes, more accurate measurements are now made with radar. The total observed precession of Mercury is 574.10±0.65 arc-seconds per century relative to the inertial ICFR. This precession can be attributed to the following causes:

Sources of the Precession of Perihelion for Mercury

Amount (arcsec/Julian century)	Cause
531.63 ±0.69	Gravitational tugs of the other planets
0.0254	Oblateness of the Sun (quadrupole moment)
42.98 ±0.04	General relativity
574.64±0.69	Total
574.10±0.65	Observed

The correction by 42.98" is 3/2 multiple of classical prediction.

Thus the effect can be fully explained by general relativity. More recent calculations based on more precise measurements have not materially changed the situation.

The other planets experience perihelion shifts as well, but, since they are farther from the sun and have longer periods, their shifts are lower, and could not be observed accurately until long after Mercury's. For example, the perihelion shift of Earth's orbit due to general relativity is of 3.84 seconds of arc per century, and Venus's is 8.62". Both values are in good agreement with observation. The periapsis shift of binary pulsar systems have been measured, with PSR 1913+16 amounting to 4.2^{o} per year. These observations are consistent with general relativity. It is also possible to measure periapsis shift in binary star systems which do not contain ultra-dense stars, but it is more difficult to model the classical effects precisely - for example, the alignment of the stars' spin to their orbital plane needs to be known and is hard to measure directly - so a few systems such as DI Herculis have been considered as problematic cases for general relativity.

3.3.2. Deflection of Light by the Sun

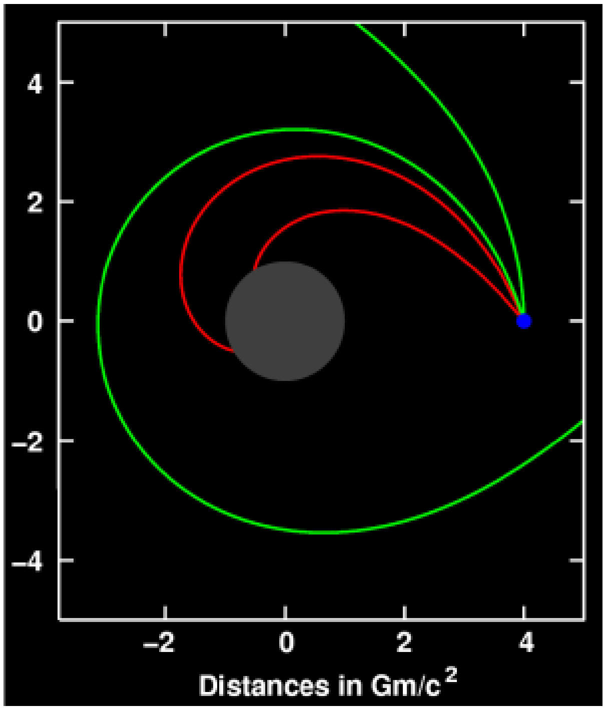


Figure 4

Deflection of light sent out from the location shown in blue, near a compact body shown in grey.

Henry Cavendish in 1784 and Johann Georg von Soldner in 1801 had pointed out that Newtonian gravity predicts that starlight will bend around a massive object. The same value as Soldner's was calculated by Einstein in 1911 based on the equivalence principle alone. However, Einstein noted in 1915 in the process of completing general relativity, that his 1911-result is only half of the correct value. Einstein became the first to calculate the correct value for light bending. The first observation of light deflection was performed by noting the change in position of stars as they passed near the Sun on the celestial sphere. The observations were performed in 1919 by Arthur Eddington and his collaborators during a total solar eclipse, so that the stars near the Sun could be observed. Observations were made simultaneously in the cities of Sobral, Ceará, Brazil and in São Tomé and Príncipe on the west coast of Africa.

The early accuracy, however, was poor. The results were argued by some to have been plagued by systematic error and possibly confirmation bias, although modern reanalysis of the dataset suggests that Eddington's analysis was accurate. The measurement was repeated by a team from the Lick Observatory in the 1922 eclipse, with results that agreed with the 1919 results and has been repeated several times since, most notably in 1973 by a team from the University of Texas. Considerable uncertainty remained in these measurements for almost fifty years, until observations started being made at radio frequencies. It was not until the late 1960s that it was definitively shown that the amount of deflection was the full value predicted by general relativity, and not half that number. The Einstein ring is an example of the deflection of light from distant galaxies by more nearby objects.

3.3.3. Gravitational Redshift of Light

In astrophysics, gravitational redshift or Einstein shift is the process by which electromagnetic radiation originating from a source that is in gravitational field is reduced in frequency, or redshifted, when observed in a region of a weaker gravitational field. This is as a direct result of Gravitational time dilation, frequency of the electromagnetic radiation is reduced in an area of a higher gravitational potential. There is a corresponding reduction in energy when electromagnetic radiation is red shifted, as given by Planck's relation, due to the electromagnetic radiation propagating in opposition to the gravitational gradient. There also exists a corresponding blueshift when electromagnetic radiation propagates from an area of a weaker gravitational field to an area of a stronger gravitational field.

If applied to optical wavelengths this manifests itself as a change in the colour of visible light as the wavelength of the light is increased toward the red part of the light spectrum. Since frequency and wavelength are inversely proportional this is equivalent to saying that the frequency of the light is reduced towards the red part of the light spectrum, giving this phenomena the name redshift.

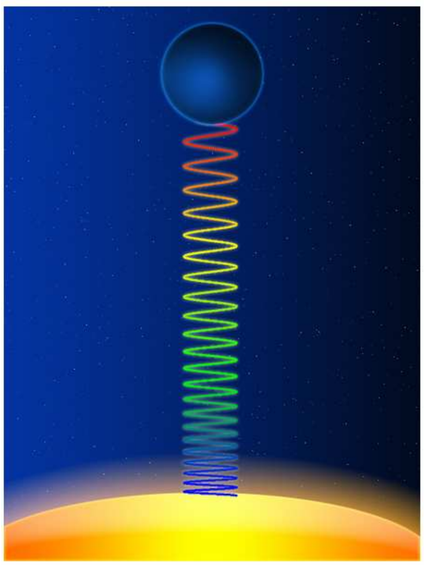


Figure 5

The gravitational redshift of a light wave as it moves upwards against a gravitational field (caused by the yellow star below).

Redshift is often denoted with the dimensionless variable ,

defined as the fractional change of the wavelength.

$$z = \frac{\lambda_e - \lambda_o}{\lambda_o} \qquad (3.1)$$

Where λ_o is the wavelength of the electromagnetic radiation (photon) as measured by the observer. λ_e is the wavelength of the electromagnetic radiation (photon) when measured at the source of emission.

The gravitational redshift of a photon can be calculated in the framework of General Relativity (using the Schwarzschild metric) as

$$1+z = \frac{1}{\sqrt{1-\frac{2GM}{rc^2}}} \qquad (3.2)$$

where denotes G Newton's gravitational constant, M the mass of the gravitating body, c the speed of light, and r the distance between the center of mass of the gravitating body and the point at which the photon is emitted. The redshift is evaluated in at a distance in the limit going to infinity. When the photon is emitted at a distance equal to the Schwarzschild radius, the redshift will be infinitely large. When the photon is emitted at an infinitely large distance, there is no redshift. The redshift is not defined for photons emitted inside the Scharzschild radius. This is because the gravitational force is too large and the photon cannot escape.

Einstein predicted the gravitational redshift of light from the equivalence principle in 1907, but it is very difficult to measure astrophysically (see the discussion under *Equivalence Principle* below). Although it was measured by Walter Sydney Adams in 1925, it was only conclusively tested when the Pound–Rebka experiment in 1959 measured the relative redshift of two sources situated at the top and bottom of Harvard University's Jefferson tower using an extremely sensitive phenomenon called the Mössbauer effect. The result was in excellent agreement with general relativity. This was one of the first precision experiments testing general relativity.

3.4. Modern Tests

The modern era of testing general relativity was ushered in largely at the impetus of Dicke and Schiff who laid out a framework for testing general relativity. They emphasized the importance not only of the classical tests, but of null experiments, testing for effects which in principle could occur in a theory of gravitation, but do not occur in general relativity. Other important theoretical developments included the inception of alternative theories to general relativity, in particular, scalar-tensor theories such as the Brans–Dicke theory; the parameterized post-Newtonian formalism in which deviations from general relativity can be quantified; and the framework of the equivalence principle.

Experimentally, new developments in space exploration, electronics and condensed matter physics have made precise experiments, such as the Pound–Rebka experiment, laser interferometry and lunar rangefinding possible.

3.4.1. Post-Newtonian Tests of Gravity

Early tests of general relativity were hampered by the lack of viable competitors to the theory: it was not clear what sorts of tests would distinguish it from its competitors. General relativity was the only known relativitistic theory of gravity compatible with special relativity and observations. Moreover, it is an extremely simple and elegant theory. This changed with the introduction of Brans–Dicke theory in 1960. This theory is arguably simpler, as it contains no dimensionful constants, and is compatible with a version of Mach's principle and Dirac's large numbers hypothesis, two philosophical ideas which have been influential in the history of relativity. Ultimately, this led to the development of the parameterized post-Newtonian formalism by Nordtvedt and Will, which parameterizes, in terms of ten adjustable parameters, all the possible departures from Newton's law of universal gravitation to first order in the velocity of moving objects. This approximation allows the possible deviations from general relativity, for slowly moving objects in weak gravitational fields, to be systematically analyzed. Much effort has been put into constraining the post-Newtonian parameters, and deviations from general relativity are at present severely limited.

The experiments testing gravitational lensing and light time delay limits the same post-Newtonian parameter, the so-called Eddington parameter γ, which is a straightforward parameterization of the amount of deflection of light by a gravitational source. It is equal to one for general relativity, and takes different values in other theories (such as Brans–Dicke theory). It is the best constrained of the ten post-Newtonian parameters, but there are other experiments designed to constrain the others. Precise observations of the perihelion shift of Mercury constrain other parameters, as do tests of the strong equivalence principle.

One of the goals of the mission BepiColombo is testing the general relativity theory by measuring the parameters gamma and beta of the parameterized post-Newtonian formalism with high accuracy.

3.4.2. Gravitational Lensing

A gravitational lens refers to a distribution of matter (such as a cluster of galaxies) between a distant source (a background galaxy) and an observer, that is capable of bending (lensing) the light from the source, as it travels towards the observer. This effect is known as gravitational lensing and is one of the predictions of Albert Einstein's General Theory of Relativity.

around a massive object (such as a galaxy cluster or a black hole) is curved, and as a result light rays from a background source (such as a galaxy) propagating through spacetime are bent. The lensing effect can magnify and distort the image of the background source.

Unlike an optical lens, maximum 'bending' occurs closest to, and minimum 'bending' furthest from, the center of a gravitational lens. Consequently, a gravitational lens has no single focal point, but a focal line instead. If the (light) source, the massive lensing object, and the observer lie in a straight line, the original light source will appear as a ring around the massive lensing object. If there is any misalignment the observer will see an arc segment instead. This phenomenon was first mentioned in 1924 by the St. Petersburg physicist Orest Chwolson,[1] and quantified by Albert Einstein in 1936. It is usually referred to in the literature as an Einstein ring, since Chwolson did not concern himself with the flux or radius of the ring image. More commonly, where the lensing mass is complex (such as galaxy groups and clusters) and does not cause a spherical distortion of space–time, the source will resemble partial arcs scattered around the lens. The observer may then see multiple distorted images of the same source; the number and shape of these depending upon the relative positions of the source, lens, and observer, and the shape of the gravitational well of the lensing object.

There are three classes of gravitational lensing:

(i) Strong lensing: where there are easily visible distortions such as the formation of Einstein rings, arcs, and multiple images.

(ii) Weak lensing: where the distortions of background sources are much smaller and can only be detected by analyzing large numbers of sources to find coherent distortions of only a few percent. The lensing shows up statistically as a preferred stretching of the background objects perpendicular to the direction to the center of the lens. By measuring the shapes and orientations of large numbers of distant galaxies, their orientations can be averaged to measure the shear of the lensing field in any region. This, in turn, can be used to reconstruct the mass distribution in the area: in particular, the background distribution of dark matter can be reconstructed. Since galaxies are intrinsically elliptical and the weak gravitational lensing signal is small, a very large number of galaxies must be used in these surveys. These weak lensing surveys must carefully avoid a number of important sources of systematic error: the intrinsic shape of galaxies, the tendency of a camera's point spread function to distort the shape of a galaxy and the tendency of atmospheric seeing to distort images must be understood and carefully accounted for. The results of these surveys are important for cosmological parameter estimation, to better understand and improve upon the Lambda-CDM model, and to provide a consistency check on other cosmological observations. They may also provide an important future constraint on dark energy.

(iii) Microlensing: where no distortion in shape can be seen but the amount of light received from a background object changes in time. The lensing object may be stars in the Milky Way in one typical case, with the background source being stars in a remote galaxy, or, in another case, an even more distant quasar.

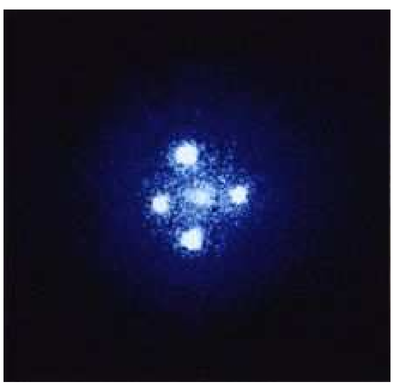
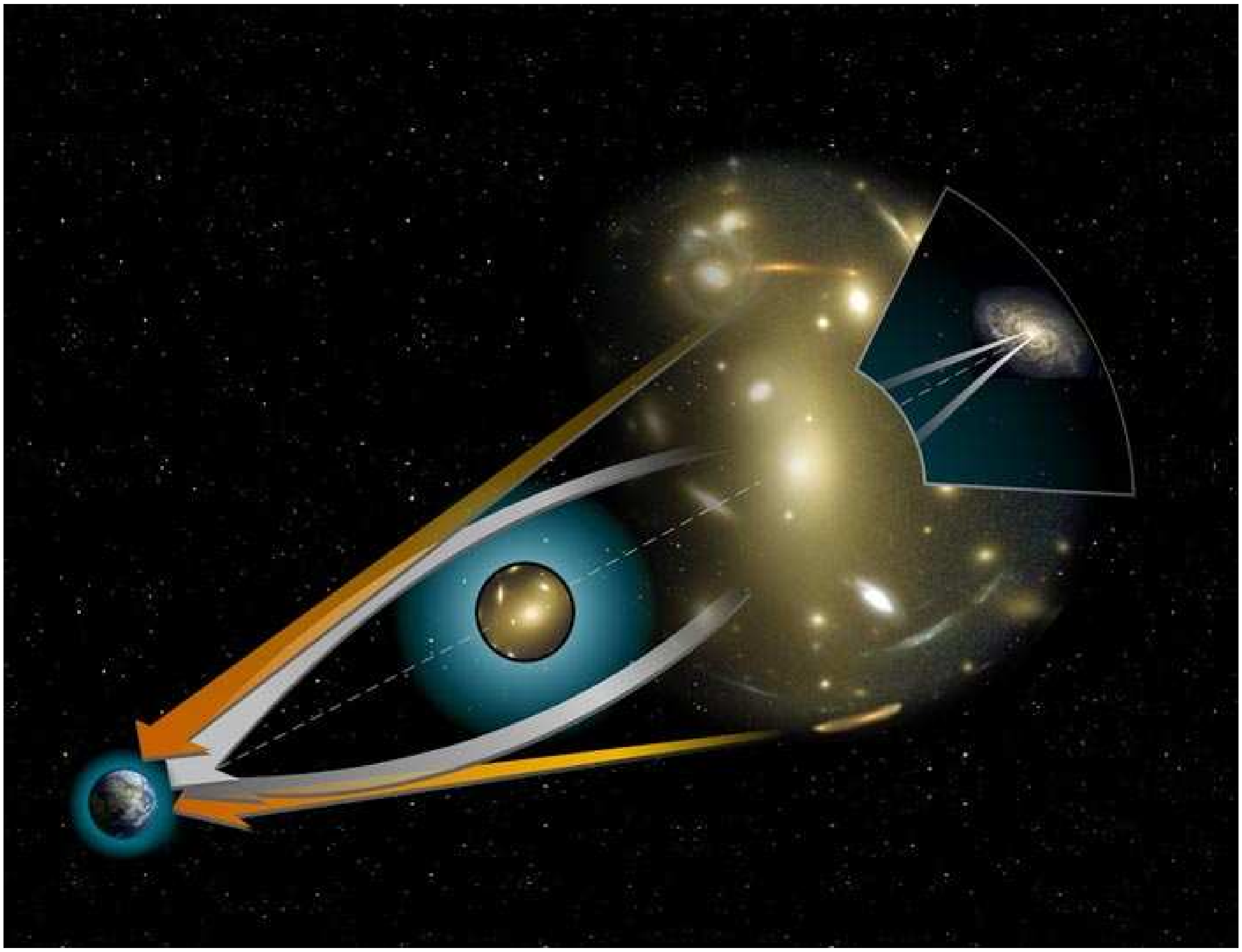


Figure 6

The effect is small, such that (in the case of strong lensing) even a galaxy with a mass more than 100 billion times that of the sun will produce multiple images separated by only a few arcseconds. Galaxy clusters can produce separations of several arcminutes. In both cases the galaxies and sources are quite distant, many hundreds of megaparsecs away from our Galaxy.

Figure 7



Bending light around a massive object from a distant source. The orange arrows show the apparent position of the background source. The white arrows show the path of the light from the true position of the source

One of the most important tests is gravitational lensing. It has been observed in distant astrophysical sources, but these are poorly controlled and it is uncertain how they constrain general relativity. The most precise tests are analogous to Eddington's 1919 experiment: they measure the deflection of radiation from a distant source by the sun. The sources that can be most precisely analyzed are distant radio sources. In particular, some quasars are very strong radio sources. The directional resolution of any telescope is in principle limited by diffraction; for radio telescopes this is also the practical limit. An important improvement in obtaining positional high accuracies (from milli-arcsecond to micro-arcsecond) was obtained by combining radio telescopes across the Earth. The technique is called very long baseline interferometry (VLBI). With this technique radio observations couple the phase information of the radio signal observed in telescopes separated over large distances. Recently, these telescopes have measured the deflection of radio waves by the Sun to extremely high precision, confirming the amount of deflection predicted by general relativity aspect to the 0.03% level. At this level of precision systematic effects have to be carefully taken into account to determine the precise location of the telescopes on Earth. Some important effects are the Earth's nutation, rotation, atmospheric refraction, tectonic displacement and tidal waves. Another important effect is refraction of the radio waves by the solar corona. Fortunately, this effect has a characteristic spectrum, whereas gravitational distortion is independent of wavelength. Thus, careful analysis, using measurements at several frequencies, can subtract this source of error.

The entire sky is slightly distorted due to the gravitational deflection of light caused by the Sun. how general relativity can be reconciled with the laws of quantum physics to produce a complete and self-consistent theory of quantum gravity.

. It measured the positions of about 10^5 stars. During the full mission about 3.5×10^6 relative positions have been determined, each to an accuracy of typically 3 milliarcseconds (the accuracy for an 8–9 magnitude star). Since the gravitation deflection perpendicular to the Earth-Sun direction is already 4.07 mas, corrections are needed for practically all stars. Without systematic effects, the error in an individual observation of 3 milliarcseconds, could be reduced by the square root of the number of positions, leading to a precision of 0.0016 mas. Systematic effects, however, limit the accuracy of the determination to 0.3% (Froeschlé, 1997).

In future, Gaia spacecraft will conduct a census of a thousand million stars in our Galaxy and measure their positions to an accuracy of 24 microarcseconds. Thus it will also provide stringent new tests of gravitational deflection of light caused by the Sun which was predicted by General relativity.

3.4.3. Gravitational Lensing

Irwin I. Shapiro proposed another test, beyond the classical tests, which could be performed within the solar system. It is sometimes called the fourth "classical" test of general relativity. He predicted a relativistic time delay (Shapiro delay) in the round-trip travel time for radar signals reflecting off other planets.

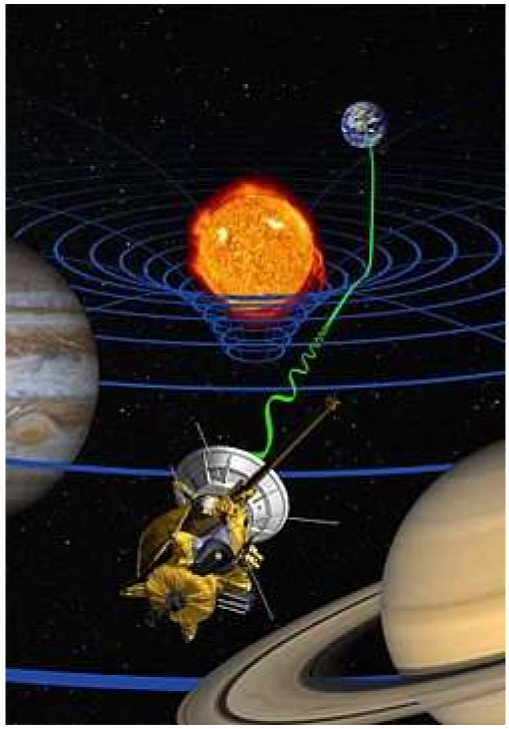


Figure 8: High-precision test of general relativity by the Cassini space probe (artist's impression): radio signals sent between the Earth and the probe (green wave) are delayed by the warping of space and time (blue lines) due to the Sun's mass.

The mere curvature of the path of a photon passing near the Sun is too small to have an observable delaying effect (when the round-trip time is compared to the time taken if the photon had followed a straight path), but general relativity predicts a time delay which becomes progressively larger when the photon passes nearer to the Sun due to the time dilation in the gravitational potential of the sun. Observing radar reflections from Mercury and Venus just before and after it will be eclipsed by the Sun gives agreement with general relativity theory at the 5% level. More recently, the Cassini probe has undertaken a similar experiment which gave agreement with general relativity at the 0.002% level. Very Long Baseline Interferometry has measured velocity-dependent (gravitomagnetic) corrections to the Shapiro time delay in the field of moving Jupiter.

3.4.4. The Equivalence Principle

The equivalence principle, in its simplest form, asserts that the trajectories of falling bodies in a gravitational field should be independent of their mass and internal structure, provided they are small enough not to disturb the environment or be affected by tidal forces. This idea has been tested to incredible precision by Eötvös torsion balance experiments, which look for a differential acceleration between two test masses

A version of the equivalence principle, called the strong equivalence principle, asserts that self-gravitation falling bodies, such as stars, planets or black holes (which are all held together by their gravitational attraction) should follow the same trajectories in a gravitational field, provided the same conditions are satisfied. This is called the Nordtvedt effect and is most precisely tested by the Lunar Laser Ranging Experiment. Since 1969, it has continuously measured the distance from several rangefinding stations on Earth to reflectors on the Moon to approximately centimeter accuracy. These have provided a strong constraint on several of the other post-Newtonian parameters.

Another part of the strong equivalence principle is the requirement that Newton's gravitational constant be constant in time, and have the same value everywhere in the universe. There are many independent observations limiting the possible variation of Newton's gravitational constant, but one of the best comes from lunar rangefinding which suggests that the gravitational constant does not change by more than one part in 10^{11} per year.

3.4.5. Gravitational Redshift

The first of the classical tests discussed above, the gravitational redshift, is a simple consequence of the Einstein equivalence principle and was predicted by Einstein in 1907. As such, it is not a test of general relativity in the same way as the post-Newtonian tests, because any theory of gravity obeying the equivalence principle should also incorporate the gravitational redshift. Nonetheless, confirming the existence of the effect was an important substantiation of relativistic gravity, since the absence of gravitational redshift would have strongly contradicted relativity. The first observation of the gravitational redshift was the measurement of the shift in the spectral lines from the white dwarf star Sirius B by Adams in 1925. Although this measurement, as well as later measurements of the spectral shift on other white dwarf stars, agreed with the prediction of relativity, it could be argued that the shift could possibly stem from some other cause, and hence experimental verification using a known terrestrial source was preferable.

Experimental verification of gravitational redshift using terrestrial sources took several decades, because it is difficult to find clocks (to measure time dilation) or sources of electromagnetic radiation (to measure redshift) with a frequency that is known well enough that the effect can be accurately measured. It was confirmed experimentally for the first time in 1960 using measurements of the change in wavelength of gamma-ray photons generated with the Mössbauer effect, which generates radiation with a very narrow line width. The experiment, performed by Pound and Rebka and later improved by Pound and Snyder, is called the Pound–Rebka experiment. The accuracy of the gamma-ray measurements was typically 1%. The blueshift of a falling photon can be found by assuming it has an equivalent mass based on its frequency (where *h* is Planck's constant) along with , a result of special relativity. Such simple derivations ignore the fact that in general relativity the experiment compares clock rates, rather than energies. In other words, the "higher energy" of the photon after it falls can be equivalently ascribed to the slower running of clocks deeper in the gravitational potential well. To fully validate general relativity, it is important to also show that the rate of arrival of the photons is greater than the rate at which they are emitted. A very accurate gravitational redshift experiment, which deals with this issue, was performed in 1976, where a hydrogen maser clock on a rocket was launched to a height of 10,000 km, and its rate compared with an identical clock on the ground. It tested the gravitational redshift to 0.007%.

Although the Global Positioning System (GPS) is not designed as a test of fundamental physics, it must account for the gravitational redshift in its timing system, and physicists have analyzed timing data from the GPS to confirm other tests. When the first satellite was launched, some engineers resisted the prediction that a noticeable gravitational time dilation would occur, so the first satellite was launched without the clock adjustment that was later built into subsequent satellites. It showed the predicted shift of 38 microseconds per day. This rate of discrepancy is sufficient to substantially impair function of GPS within hours if not accounted for.

Other precision tests of general relativity not discussed here, are the Gravity Probe A satellite, launched in 1976, which showed gravity and velocity affect the ability to synchronize the rates of clocks orbiting a central mass; the Hafele–Keating experiment, which used atomic clocks in circumnavigating aircraft to test general relativity and special relativity together; and the forthcoming Satellite Test of the Equivalence Principle.

3.4.6. Frame-Dragging Tests

Tests of the Lense–Thirring precession, consisting of small secular precessions of the orbit of a test particle in motion around a central rotating mass like, e.g., a planet or a star, have been performed with the LAGEOS satellites, but many aspects of them remain controversial. The same effect may have been detected in the data of the Mars Global Surveyor (MGS) spacecraft, a former probe in orbit around Mars; also such a test raised a debate. First attempts to detect the Sun's Lense–Thirring effect on the perihelia of the inner planets have been recently reported as well. Frame dragging would cause the orbital plane of stars orbiting near a supermassive black hole to precess about the black hole spin axis. This effect should be detectable within the next few years via astrometric monitoring of stars at the center of the Milky Way galaxy. By comparing the rate of orbital precession of two stars on different orbits, it is possible in principle to test the no-hair theorems of general relativity.

The Gravity Probe B satellite, launched in 2004 and operated until 2005 detected frame-dragging and the geodetic effect. The experiment used four quartz spheres the size of ping pong balls coated with a superconductor. Data analysis continued through 2011 due to high noise levels and difficulties in modelling the noise accurately so that a useful signal can be found. Principal investigators at Stanford University reported on May 4, 2011, that they had accurately measured the framing effect relative to the distant star IM Pegasi, and the calculations proved to be in line with the prediction of Einstein's theory. The results, published in *Physical Review Letters* measured the geodetic effect with an error of about 0.2 percent. The results reported the frame dragging effect (caused by the Earth's rotation) added up to 37 milliarcseconds with an error of about 19 percent.

3.4.7. Strong Field Tests

Pulsars are rapidly rotating neutron stars which emit regular radio pulses as they rotate. As such they act as clocks which allow very precise monitoring of their orbital motions.

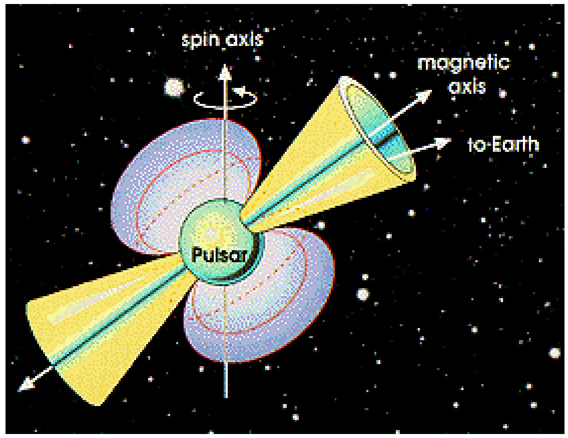


Figure 9: Model of a pulsar rotating around its spin axis and emitting radio waves along its magnetic axis

Observations of pulsars in orbit around other stars have all demonstrated substantial periapsis precessions that cannot be accounted for classically but can be accounted for by using general relativity. For example, the Hulse–Taylor binary pulsar PSR B1913+16 (a pair of neutron stars in which one is detected as a pulsar) has an observed precession of over 4^o of arc per year (periastron shift per orbit only about 10^{-6}). This precession has been used to compute the masses of the components.

Similarly to the way in which atoms and molecules emit electromagnetic radiation, a gravitating mass that is in quadrupole type or higher order vibration, or is asymmetric and in rotation, can emit gravitational waves.   These gravitational waves are predicted to travel at the speed of light. For example, planets orbiting the Sun constantly lose energy via gravitational radiation, but this effect is so small that it is unlikely it will be observed in the near future. Gravitational waves have been indirectly detected from the Hulse–Taylor binary. Precise timing of the pulses shows that the stars orbit only approximately according to Kepler's Laws, – over time they gradually spiral towards each other, demonstrating an energy loss in close agreement with the predicted energy radiated by gravitational waves. Thus, although the waves have not been directly measured, their effect seems necessary to explain the orbits.

A "double pulsar" discovered in 2003, PSR J0737-3039, has a perihelion precession of 16.90^{o} per year; unlike the Hulse–Taylor binary, both neutron stars are detected as pulsars, allowing precision timing of both members of the system. Due to this, the tight orbit, the fact that the system is almost edge-on, and the very low transverse velocity of the system as seen from Earth, J0737−3039 provides by far the best system for strong-field tests of general relativity known so far. Several distinct relativistic effects are observed, including orbital decay as in the Hulse–Taylor system. After observing the system for two and a half years, four independent tests of general relativity were possible, the most precise (the Shapiro delay) confirming the general relativity prediction within 0.05% .

3.4.8. Gravitational Waves

A number of gravitational wave detectors have been built, with the intent of directly detecting the gravitational waves emanating from such astronomical events as the merger of two neutron stars. Currently, the most sensitive of these is the Laser Interferometer Gravitational-wave Observatory (LIGO), which has been in operation since 2002. So far, there has not been a single detection event by any of the existing detectors. Future detectors are being developed or planned, which will greatly improve the sensitivity of these experiments, such as the Advanced LIGO detector being built for the LIGO facilities, and the proposed Laser Interferometer Space Antenna (LISA). It is anticipated, for example, that Advanced LIGO will detect events possibly as often as daily.

If gravitational waves exist as predicted, they should be detected by these gravitational wave detectors. Finding the existence of gravitational waves as predicted by general relativity is a critical test of the validity of the theory.

3.4.9. Cosmological Tests

Tests of general relativity on the largest scales are not nearly so stringent as solar system tests. The earliest such test was prediction and discovery of the expansion of the universe. In 1922 Alexander Friedmann found that Einstein equations have non-stationary solutions. In 1927 Georges Lemaître showed that static solutions of the Einstein equations, which are possible in the presence of the cosmological constant, are unstable, and therefore the static universe envisioned by Einstein could not exist (it must either expand or contract). Lemaître made an explicit prediction that the universe should expand. He also derived a redshift-distance relationship, which is now known as the Hubble Law. Later, in 1931, Einstein himself agreed with the results of Friedmann and Lemaître. The expansion of the universe discovered by Edwin Hubble in 1929 was then considered by many (and continues to be considered by some now) as a direct confirmation of the general relativity. In the 1930s, largely due to the work of E. A. Milne, it was realised that the linear relationship between redshift and distance derives from the general assumption of uniformity and isotropy rather than specifically from general relativity. However the prediction of a non-static universe was non-trivial, indeed dramatic, and primarily motivated by general relativity.

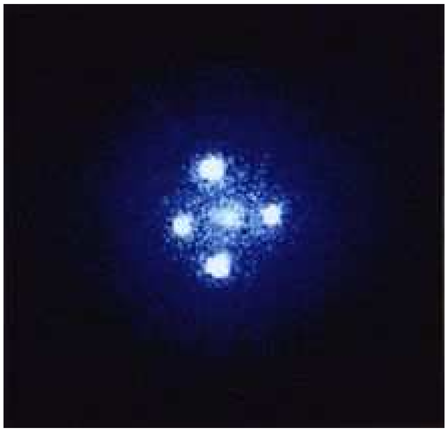
Some other cosmological tests include searches for primordial gravity waves generated during cosmic inflation, which may be detected in the cosmic microwave background polarization or by a proposed space-based gravity wave interferometer called Big Bang Observer. Other tests at high redshift are constraints on other theories of gravity, and the variation of the gravitational constant since big bang nucleosynthesis (it varied by no more than 40% since then).

CHAPTER 4: Astrophysical Applications

4.1. Gravitational Lensing

The deflection of light by gravity is responsible for a new class of astronomical phenomena. If a massive object is situated between the astronomer and a distant target object with appropriate mass and relative distances, the astronomer will see multiple distorted images of the target.

Figure 10



Such effects are known as gravitational lensing. Depending on the configuration, scale, and mass distribution, there can be two or more images, a bright ring known as an Einstein ring, or partial rings called arcs. The earliest example was discovered in 1979; since then, more than a hundred gravitational lenses have been observed. Even if the multiple images are too close to each other to be resolved, the effect can still be measured, e.g., as an overall brightening of the target object; a number of such "microlensing events" have been observed. Gravitational lensing has developed into a tool of observational astronomy. It is used to detect the presence and distribution of dark matter, provide a "natural telescope" for observing distant galaxies, and to obtain an independent estimate of the Hubble constant. Statistical evaluations of lensing data provide valuable insight into the structural evolution of galaxies.

4.2. Gravitational Wave Astronomy

Observations of binary pulsars provide strong indirect evidence for the existence of gravitational waves. However, gravitational waves reaching us from the depths of the cosmos have not been detected directly, which is a major goal of current relativity-related research. Several land-based gravitational wave detectors are currently in operation, most notably the interferometric detectors GEO 600, LIGO (three detectors), TAMA 300 and VIRGO. A joint US-European space-based detector, LISA, is currently under development, with a precursor mission (LISA Pathfinder) due for launch in 2012.

Observations of gravitational waves promise to complement observations in the electromagnetic spectrum.[102] They are expected to yield information about black holes and other dense objects such as neutron stars and white dwarfs, about certain kinds of supernova implosions, and about processes in the very early universe, including the signature of certain types of hypothetical cosmic string.

4.3. Black Holes and other Compact Objects

Whenever the ratio of an object's mass to its radius becomes sufficiently large, general relativity predicts the formation of a black hole, a region of space from which nothing, not even light, can escape. In the currently accepted models of stellar evolution, neutron stars of around 1.4 solar masses, and stellar black holes with a few to a few dozen solar masses, are thought to be the final state for the evolution of massive stars. Usually a galaxy has one supermassive black hole with a few million to a few billion solar masses in its center, and its presence is thought to have played an important role in the formation of the galaxy and larger cosmic structures.

Astronomically, the most important property of compact objects is that they provide a supremely efficient mechanism for converting gravitational energy into electromagnetic radiation. Accretion, the falling of dust or gaseous matter onto stellar or supermassive black holes, is thought to be responsible for some spectacularly luminous astronomical objects, notably diverse kinds of active galactic nuclei on galactic scales and stellar-size objects such as microquasars. In particular, accretion can lead to relativistic jets, focused beams of highly energetic particles that are being flung into space at almost light speed. General relativity plays a central role in modelling all these phenomena, and observations provide strong evidence for the existence of black holes with the properties predicted by the theory.

Black holes are also sought-after targets in the search for gravitational waves. Merging black hole binaries should lead to some of the strongest gravitational wave signals reaching detectors here on Earth, and the phase directly before the merger ("chirp") could be used as a "standard candle" to deduce the distance to the merger events–and hence serve as a probe of cosmic expansion at large distances. The gravitational waves produced as a stellar black hole plunges into a supermassive one should provide direct information about supermassive black hole's geometry.

Simulated view of a black hole (center) in front of the Large Magellanic Cloud. Note the gravitational lensing effect, which produces two enlarged but highly distorted views of the Cloud. Across the top, the Milky Way disk appears distorted into an arc.

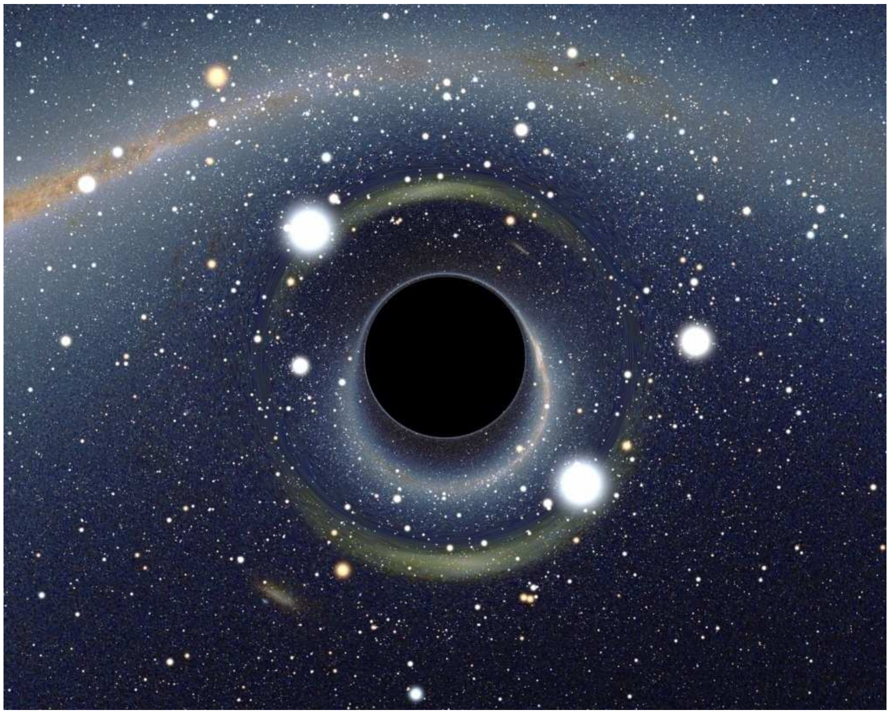


Figure 11

4.4. Cosmology

The current models of cosmology are based on Einstein's field equations, which include the cosmological constant Λ since it has important influence on the large-scale dynamics of the cosmos,where g_{ab} is the spacetime metric. Isotropic and homogeneous solutions of these enhanced equations, the Friedmann-Lemaître-Robertson-Walker solutions, allow physicists to model a universe that has evolved over the past 14 billion years from a hot, early Big Bang phase.

Once a small number of parameters (for example the universe's mean matter density) have been fixed by astronomical observation, further observational data can be used to put the models to the test. Predictions, all successful, include the initial abundance of chemical elements formed in a period of primordial nucleosynthesis, the large-scale structure of the universe, and the existence and properties of a "thermal echo" from the early cosmos, the cosmic background radiation.

Astronomical observations of the cosmological expansion rate allow the total amount of matter in the universe to be estimated, although the nature of that matter remains mysterious in part. About 90% of all matter appears to be so-called dark matter, which has mass but does not interact electromagnetically and, hence, cannot be observed directly. There is no generally accepted description of this new kind of matter, within the framework of known particle physics or otherwise. Observational evidence from redshift surveys of distant supernovae and measurements of the cosmic background radiation also show that the evolution of our universe is significantly influenced by a cosmological constant resulting in an acceleration of cosmic expansion or, equivalently, by a form of energy with an unusual equation of state, known as dark energy, the nature of which remains unclear. A so-called inflationary phase, an additional phase of strongly accelerated expansion at cosmic times of around seconds, was hypothesized in 1980 to account for several puzzling observations that were unexplained by classical cosmological models, such as the nearly perfect homogeneity of the cosmic background radiation. Recent measurements of the cosmic background radiation have resulted in the first evidence for this scenario. However, there is a bewildering variety of possible inflationary scenarios, which cannot be restricted by current observations. An even larger question is the physics of the earliest universe, prior to the inflationary phase and close to where the classical models predict the big bang singularity. An authoritative answer would require a complete theory of quantum gravity, which has not yet been developed.

CHAPTER 5: Advanced Concepts

5.1. Causal Structure and Global Geometry

In general relativity, no material body can catch up with or overtake a light pulse. No influence from an event A can reach any other location X before light sent out at A to X. In consequence, an exploration of all light worldlines (null geodesics) yields key information about the spacetime's causal structure. This structure can be displayed using Penrose-Carter diagrams in which infinitely large regions of space and infinite time intervals are shrunk ("compactified") so as to fit onto a finite map, while light still travels along diagonals as in standard spacetime diagrams.

Aware of the importance of causal structure, Roger Penrose and others developed what is known as global geometry. In global geometry, the object of study is not one particular solution (or family of solutions) to Einstein's equations. Rather, relations that hold true for all geodesics, such as the Raychaudhuri equation, and additional non-specific assumptions about the nature of matter (usually in the form of so-called energy conditions) are used to derive general results.

5.2. Horizons

Using global geometry, some spacetimes can be shown to contain boundaries called horizons, which demarcate one region from the rest of spacetime. The best-known examples are black holes: if mass is compressed into a sufficiently compact region of space (as specified in the hoop conjecture, the relevant length scale is the Schwarzschild radius), no light from inside can escape to the outside. Since no object can overtake a light pulse, all interior matter is imprisoned as well. Passage from the exterior to the interior is still possible, showing that the boundary, the black hole's *horizon*, is not a physical barrier.

Early studies of black holes relied on explicit solutions of Einstein's equations, notably the spherically symmetric Schwarzschild solution (used to describe a static black hole) and the axisymmetric Kerr solution (used to describe a rotating, stationary black hole, and introducing interesting features such as the ergosphere). Using global geometry, later studies have revealed more general properties of black holes. In the long run, they are rather simple objects characterized by eleven parameters specifying energy, linear momentum, angular momentum, location at a specified time and electric charge. This is stated by the black hole uniqueness theorems: "black holes have no hair", that is, no distinguishing marks like the hairstyles of humans. Irrespective of the complexity of a gravitating object collapsing to form a black hole, the object that results (having emitted gravitational waves) is very simple.

Even more remarkably, there is a general set of laws known as black hole mechanics, which is analogous to the laws of thermodynamics. For instance, by the second law of black hole mechanics, the area of the event horizon of a general black hole will never decrease with time, analogous to the entropy of a thermodynamic system. This limits the energy that can be extracted by classical means from a rotating black hole (e.g. by the Penrose process). There is strong evidence that the laws of black hole mechanics are, in fact, a subset of the laws of thermodynamics, and that the black hole area is proportional to its entropy. This leads to a modification of the original laws of black hole mechanics: for instance, as the second law of black hole mechanics becomes part of the second law of thermodynamics, it is possible for black hole area to decrease—as long as other processes ensure that, overall, entropy increases. As thermodynamical objects with non-zero temperature, black holes should emit thermal radiation. Semi-classical calculations indicate that indeed they do, with the surface gravity playing the role of temperature in Planck's law. This radiation is known as Hawking radiation.

There are other types of horizons. In an expanding universe, an observer may find that some regions of the past cannot be observed ("particle horizon"), and some regions of the future cannot be influenced (event horizon). Even in flat Minkowski space, when described by an accelerated observer (Rindler space), there will be horizons associated with a semi-classical radiation known as Unruh radiation.

5.3. Singularities

Another general—and quite disturbing—feature of general relativity is the appearance of **spaceti**me boundaries known as singularities. Spacetime can be explored by following up on timelike and lightlike geodesics—all possible ways that light and particles in free fall can travel. But some solutions of Einstein's equations have regions known as spacetime singularities, where the paths of light and falling particles come to an abrupt end, and geometry becomes ill-defined. In the more interesting cases, these are "curvature singularities", where geometrical quantities characterizing spacetime curvature, such as the Ricci scalar, take on infinite values. Well-known examples of spacetimes with future singularities—where worldlines end—are the Schwarzschild solution, which describes a singularity inside an eternal static black hole, or the Kerr solution with its ring-shaped singularity inside an eternal rotating black hole. The Friedmann-Lemaître-Robertson-Walker solutions and other spacetimes describing universes have past singularities on which worldlines begin, namely big bang singularities, and some have future singularities (big crunch) as well.

Given that these examples are all highly symmetric—and thus simplified—it is tempting to conclude that the occurrence of singularities is an artefact of idealization. The famous singularity theorems, proved using the methods of global geometry, say otherwise: singularities are a generic feature of general relativity, and unavoidable once the collapse of an object with realistic matter properties has proceeded beyond a certain stage and also at the beginning of a wide class of expanding universes. However, the theorems say little about the properties of singularities, and much of current research is devoted to characterizing these entities' generic structure. The cosmic censorship hypothesis states that all realistic future singularities are safely hidden away behind a horizon, and thus invisible to all distant observers. While no formal proof yet exists, numerical simulations offer supporting evidence of its validity.

5.4. Evolution Equations

Each solution of Einstein's equation encompasses the whole history of a universe — it is not just some snapshot of how things are, but a whole, possibly matter-filled, spacetime. It describes the state of matter and geometry everywhere and at every moment in that particular universe. Due to its general covariance, Einstein's theory is not sufficient by itself to determine the time evolution of the metric tensor. It must be combined with a coordinate condition, which is analogous to gauge fixing in other field theories.

To understand Einstein's equations as partial differential equations, it is helpful to formulate them in a way that describes the evolution of the universe over time. This is done in so-called "3+1" formulations, where spacetime is split into three space dimensions and one time dimension.

The best-known example is the ADM formalism. These decompositions show that the spacetime evolution equations of general relativity are well-behaved: solutions always exist, and are uniquely defined, once suitable initial conditions have been specified. Such formulations of Einstein's field equations are the basis of numerical relativity.

5.5. Global and Quasi-Local Quantities

The notion of evolution equations is intimately tied in with another aspect of general relativistic physics. In Einstein's theory, it turns out to be impossible to find a general definition for a seemingly simple property such as a system's total mass (or energy). The main reason is that the gravitational field—like any physical field—must be ascribed a certain energy, but that it proves to be fundamentally impossible to localize that energy. Nevertheless, there are possibilities to define a system's total mass, either using a hypothetical "infinitely distant observer" (ADM mass) or suitable symmetries (Komar mass). If one excludes from the system's total mass the energy being carried away to infinity by gravitational waves, the result is the so-called Bondi mass at null infinity. Just as in classical physics, it can be shown that these masses are positive. Corresponding global definitions exist for momentum and angular momentum. There have also been a number of attempts to define *quasi-local* quantities, such as the mass of an isolated system formulated using only quantities defined within a finite region of space containing that system. The hope is to obtain a quantity useful for general statements about isolated systems, such as a more precise formulation of the hoop conjecture.

CHAPTER 6: Quantum Theory and General Relativity

6.1. Relationship with Quantum Theory

If general relativity is considered one of the two pillars of modern physics, quantum theory, the basis of understanding matter from elementary particles to solid state physics, is the other. However, it is still an open question as to how the concepts of quantum theory can be reconciled with those of general relativity.

6.2. Quantum Field Theory in Curved Spacetime

Ordinary quantum field theories, which form the basis of modern elementary particle physics, are defined in flat Minkowski space, which is an excellent approximation when it comes to describing the behavior of microscopic particles in weak gravitational fields like those found on Earth. In order to describe situations in which gravity is strong enough to influence (quantum) matter, yet not strong enough to require quantization itself, physicists have formulated quantum field theories in curved spacetime. These theories rely on classical general relativity to describe a curved background spacetime, and define a generalized quantum field theory to describe the behavior of quantum matter within that spacetime. Using this formalism, it can be shown that black holes emit a blackbody spectrum of particles known as Hawking radiation, leading to the possibility that they evaporate over time. As briefly mentioned above, this radiation plays an important role for the thermodynamics of black holes.

6.3. Quantum Gravity

The demand for consistency between a quantum description of matter and a geometric description of spacetime, as well as the appearance of singularities (where curvature length scales become microscopic), indicate the need for a full theory of quantum gravity: for an adequate description of the interior of black holes, and of the very early universe, a theory is required in which gravity and the associated geometry of spacetime are described in the language of quantum physics. Despite major efforts, no complete and consistent theory of quantum gravity is currently known, even though a number of promising candidates exist.

Attempts to generalize ordinary quantum field theories, used in elementary particle physics to describe fundamental interactions, so as to include gravity have led to serious problems. At low energies, this approach proves successful, in that it results in an acceptable effective (quantum) field theory of gravity. At very high energies, however, the result are models devoid of all predictive power. One attempt to overcome these limitations is string theory, a quantum theory not of point particles, but of minute one-dimensional extended objects. The theory promises to be a unified description of all particles and interactions, including gravity; the price to pay is unusual features such as six extra dimensions of space in addition to the usual three. In what is called the second superstring revolution, it was conjectured that both string theory and a unification of general relativity and supersymmetry known as supergravity form part of a hypothesized eleven-dimensional model known as M-theory, which would constitute a uniquely defined and consistent theory of quantum gravity.

Another approach starts with the canonical quantization procedures of quantum theory. Using the initial-value-formulation of general relativity. the result is the Wheeler-deWitt equation (an analogue of the Schrödinger equation) which, regrettably, turns out to be ill-defined. However, with the introduction of what are now known as Ashtekar variables, this leads to a promising model known as loop quantum gravity. Space is represented by a web-like structure called a spin network, evolving over time in discrete steps.

Depending on which features of general relativity and quantum theory are accepted unchanged, and on what level changes are introduced, there are numerous other attempts to arrive at a viable theory of quantum gravity, some examples being dynamical triangulations, causal sets, twistor models or the path-integral based models of quantum cosmology.

All candidate theories still have major formal and conceptual problems to overcome. They also face the common problem that, as yet, there is no way to put quantum gravity predictions to experimental tests (and thus to decide between the candidates where their predictions vary), although there is hope for this to change as future data from cosmological observations and particle physics experiments becomes available.

6.4. Modern research: General Relativity and Beyond

General relativity is very successful in providing a framework for accurate models which describe an impressive array of physical phenomena. On the other hand, there are many interesting open questions, and in particular, the theory as a whole is almost certainly incomplete.

In contrast to all other modern theories of fundamental interactions, general relativity is a classical theory: it does not include the effects of quantum physics. The quest for a quantum version of general relativity addresses one of the most fundamental open questions in physics. While there are promising candidates for such a theory of quantum gravity, notably string theory and loop quantum gravity, there is at present no consistent and complete theory. It has long been hoped that a theory of quantum gravity would also eliminate another problematic feature of general relativity: the presence of spacetime singularities. These singularities are boundaries ("sharp edges") of spacetime at which geometry becomes ill-defined, with the consequence that general relativity itself loses its predictive power. Furthermore, there are so-called singularity theorems which predict that such singularities *must* exist within the universe if the laws of general relativity were to hold without any quantum modifications. The best-known examples are the singularities associated with the model universes that describe black holes and the beginning of the universe.

Other attempts to modify general relativity have been made in the ``-

context of cosmology. In the modern cosmological models, most energy in the universe is in forms that have never been detected directly, namely dark energy and dark matter. There have been several controversial proposals to obviate the need for these enigmatic forms of matter and energy, by modifying the laws governing gravity and the dynamics of cosmic expansion, for example modified Newtonian dynamics.

Beyond the challenges of quantum effects and cosmology, research on general relativity is rich with possibilities for further exploration: mathematical relativists explore the nature of singularities and the fundamental properties of Einstein's equations, ever more comprehensive computer simulations of specific spacetimes such as those describing merging black holes are run, and the race for the first direct detection of gravitational waves continues apace. More than ninety years after the theory was first published, research is more active than ever.

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