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# NOTE ON SEARCH UNEMPLOYMENT AND WAGE DIFFERENTIALS

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ABSTRACT

This note presents a formula for the equilibrium ratio of the number of urban job seekers to the number of formal sector jobs when a non-competitive wage differential is maintained between the wage in the formal sector and earnings elsewhere in the economy. The loss of national product due to the wage differential is also examined. The loss attributable to the misallocation of labor is accentuated by the incentive which a high urban wage gives for wasteful job-seeking.

Note on Search Unemployment and Wage Differentials

The purpose of this note is to present a formula for the equilibrium ratio of the number of urban job-seekers U to the number of formal sector jobs J. It will be seen that this ratio depends on, among other things, the differential between the formal sector wage and earnings in agriculture. The social loss attributable to this wage differential can also be calculated.

In the modern or formal sector the number of jobs J is increasing continuously at a proportional rate n per year. Also, job-holders vacate their jobs, because of death, disability, or retirement, at a proportional rate of q per year. It is assumed that at the substantial income differentials with which we are here concerned, there are no other voluntary or involuntary withdrawals from lucrative formal employment. The same factor, q per year, applies to economic mortality in agriculture and informal employment.

The annual wage in the formal sector is y, the earnings per year in agriculture z, and the earnings per year of the unemployed in the urban informal sector x. We will denote $^{y}/_{z}$ as $\mu$ (>1) and $^{x}/_{z}$ as $\omega$(<1). The relative wage differential $\mu$ we take as a datum, determined by trade union bargaining power and political strength, by government policy, and by the acquiescence of the large international firms of the modern sector.

The central idea is that the number of job-seekers U adjusts until the expected value of urban income is equal to rural income. The expected value of urban income is a weighted average of income with a formal job and income without a formal job, the weights depending on the probability of getting one. That probability depends inversely on U. Urban-rural migration keeps U at the level where life prospects are equal in city and county.

The implementation of this simple idea is somewhat complicated, but the essential formula may be obtained as follows:

1. It is easier to deal in continuous time rather than discrete time. Thus we shall assume that people in the unemployment pool U at time t are continuously getting jobs at a rate of p per annum. In the short period of time dt, the number of placements in jobs is pUdt. From what was earlier assumed, it is also (n+q)Jdt. Therefore,

(1) $p = \frac{(n+q)J}{U}$ .

2. With a constant p, what is the probability that an individual in the unemployment pool at time zero will still be unemployed at time t? The answer is $e^{-pt}$. Ignoring new migrants, the original pool declines in size at a steady rate of attrition p. No matter how many of the original group are left, no matter how long the unemployed have been waiting, their chances of getting a job in the next instant remain the same. So the probability that an individual remains unemployed until time t and gets a job in the instant between t and t+dt is $pe^{-pt}dt$.

3. We assume that all incomes y,z,x are growing at an exponential rate $\gamma$, thanks to general technological progress. Our typical individual takes this growth into account, but he also discounts future incomes of all kinds by a rate r, For familiar theoretical reasons we do not explain here r is taken to be larger, or at least no less, than $n+\gamma$, the natural rate of growth of the economy. The typical individual also discounts future incomes by his survival chances. Here it does no harm, but greatly simplifies algebra, to make the biologically unrealistic assumption that economic mortality occurs probabilistically like exponential depreciation. The probability of sheer survival for a length of time t is $e^{-qt}$, and the probability of economic death during the next instant is q dt, independent of the previous length of life.

4. Let us now calculate the present value of an individual who joins the urban labor pool at time zero and gets a job at time t. It is:

(2) $$\int_0^t xe^{-(r+q-\gamma)u}du + \int_t^\infty ye^{-(r+q-\gamma)u}du$$

$$= \frac{x(1-e^{-(r+q-\gamma)t})}{r+q-\gamma} + \frac{y(e^{-(r+q-\gamma)t})}{r+q-\gamma}$$

The probability that this happens is, as we have seen, $pe^{-pt}$
So the expected value of the present value of urban income is

(3) $$\frac{px}{r+q-\gamma}\int_0^\infty (1-e^{-(n+q-\gamma)t})e^{-pt}dt + \frac{py}{r+q-\gamma}\int_0^\infty e^{-(n+q-\gamma)t}e^{-pt}dt$$

$$= \frac{px}{r+q-\gamma)\, p} + \frac{p\,(y-x)}{(r+q-\gamma)(r+q-\gamma+p)}.$$

This must be, in equilibrium, equal to the present value of staying the country, namely $\frac{z}{r + q - \gamma}$.

This gives the basic equation:

$$\frac{p(y-x)}{r + q - \gamma + p} = z - x, \text{ or}$$

$$py - px = (r + q - \gamma)(z - x) + pz - px, \text{ or}$$

(4)
$$p = (r + q - \gamma) . \frac{z - x}{y - z}$$

Putting (4) together with the definition (1) of p in terms of U and J, we have

(5)
$$\frac{U}{J} = \frac{n + q}{r + q - \gamma} \frac{y - z}{z - x} = \frac{n + q}{r + q - \gamma} . \frac{\mu - 1}{1 - \omega}$$

5. From (5) can be derived some interesting implications.

The unemployment rate, in terms of formal jobs, is (a) greater the larger is formal wage, relative to agricultural income, (b) greater the larger is income while unemployed, relative to agricultural income, (c) greater the faster the rate of job creation in the formal sector.

This last implication may appear paradoxical, as it would seem intuitively that a rapid expansion of J is the eventual solution to the unemployment problem. Of course it is, in the long run sense that if J grows faster than the labor force eventually everyone will be employed in the modern sector. But in the long intermediate run, before the traditional rural sector is ecompletely phased out, there will be unemployment, and it will be a larger rather than smaller fraction of J the faster J is growing.

The unemployment problem can only be alleviated if expansion of J is accompanied by a reduction of the wage differential y - z.

6. Let us consider now the loss of national product due to the wage differential. This loss arises from the fact that the marginal product of labor is kept higher in the modern sector than in agriculture, and also higher than the marginal product of informal urban underemployment. Society would gain by shifting labor until the marginal products are equated, or until all labor is moved out of occupations with lower marginal product.

For convenience of illustration, assume the marginal product in agriculture is constant at z. This implies that agricultural output is not limited in any degree by shortage of land or other cooperating factors, a situation that may be approximated in the subsistence agricultural sectors of many developing economies. Likewise assume the marginal product of informal urban employment is constant at x.

In the modern sector, however, the marginal product of labor varies inversely with the amount of employment. Specifically, let output Q of modern sector be $J^{\alpha}$, so that marginal productivity is $\alpha J^{\alpha-1}$. We are to compare two situations: the optimum with output $Q^*$ and employment $J^*$, against the alternative with output $\hat{Q}$ and employment $\hat{J}$. The relevant comparisons are as follows:

Table 1

| | Optimum | Alternative |
|---|---|---|
| modern wage | $\alpha J^{*\alpha-1} = z$ | $\alpha \hat{J}^{\alpha-1} = \mu z$ |
| modern employment | $J^* = (\frac{z}{\alpha})^{\frac{1}{\alpha-1}}$ | $\hat{J} = (\frac{\mu z}{\alpha})^{\frac{1}{\alpha-1}}$ |
| modern output | $Q^* = (\frac{z}{\alpha})^{\frac{\alpha}{\alpha-1}}$ | $\hat{Q} = (\frac{\mu z}{\alpha})^{\frac{\alpha}{\alpha-1}}$ |

$$J^* = (\frac{1}{\mu})^{\frac{1}{\alpha-1}} \hat{J}$$

$$Q^* = (\frac{1}{\mu})^{\frac{\alpha}{\alpha-1}} \hat{Q}$$

| | Optimum | Alternative |
|---|---|---|
| unemployment | 0 | $\hat{U} = \lambda \frac{\mu-1}{1-\omega} \hat{J}$ |
| output of unemployed | | $\lambda \frac{\mu-1}{1-\omega} \hat{J}x = \lambda \frac{\mu-1}{1-\omega} \hat{J}\omega z$ |
| output of agriculture | $(N - J^*)z$ | $(N - \hat{J} - \hat{U})z$ |
| total output | $(\frac{1}{\mu})^{\frac{\alpha}{\alpha-1}}\hat{Q} + Nz - (\frac{1}{\mu})^{\frac{1}{\alpha-1}} \hat{J}z$ | $\hat{Q} + Nz - \hat{J}z + \lambda \frac{\mu-1}{1-\omega} \hat{J}z(\omega-1)$ |

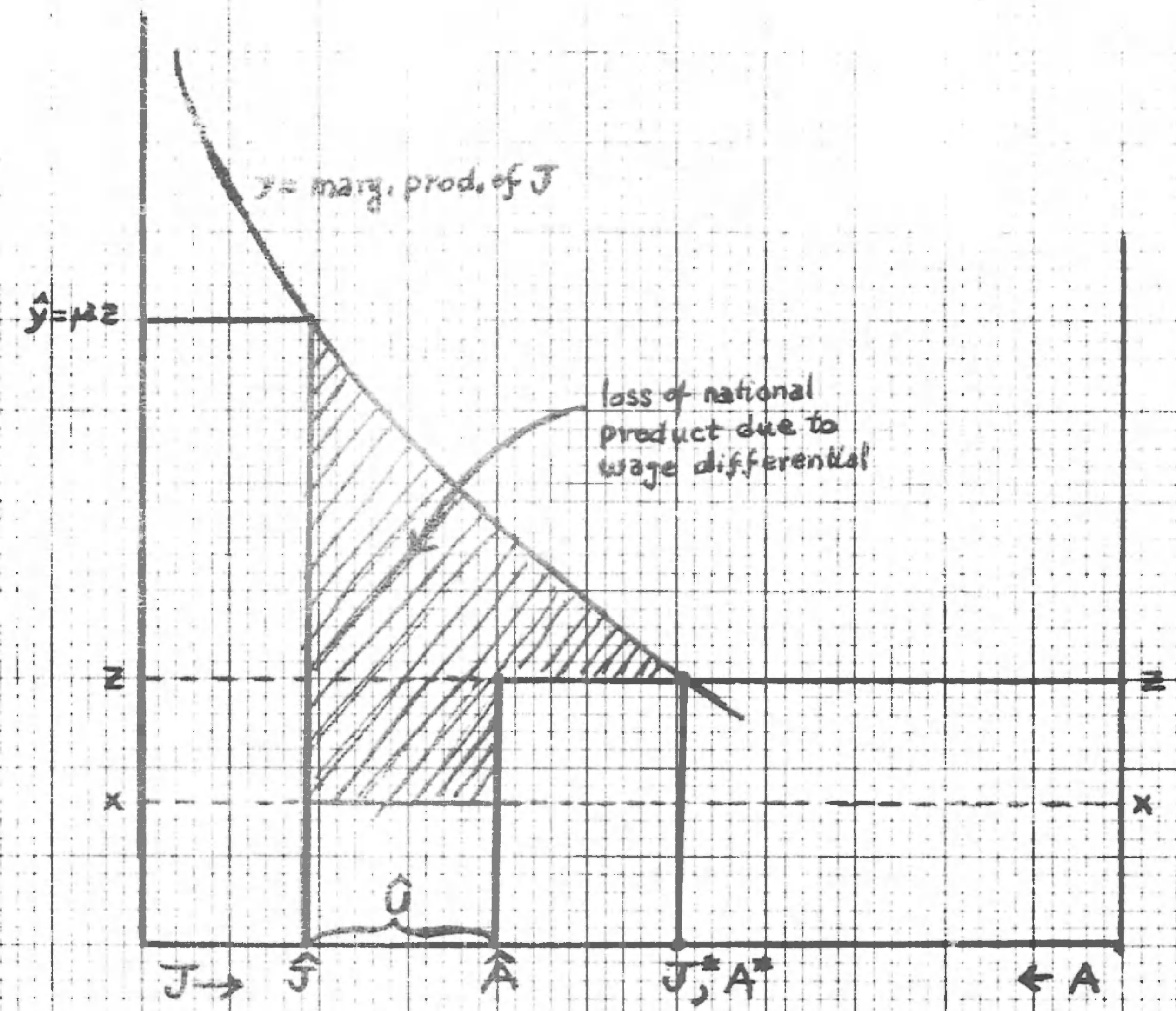
$$z = \frac{\alpha}{\mu} \hat{J}^{\alpha-1}, \quad \hat{J}z = \frac{\alpha}{\mu} \hat{Q}$$

total output optimum less alternative

$$(\frac{1}{\mu})^{\frac{\alpha}{\alpha-1}}\hat{Q} - (\frac{1}{\mu})^{\frac{1}{\alpha-1}}\frac{\alpha}{\mu} \hat{Q} - \hat{Q} + \frac{\alpha}{\mu} \hat{Q} + \lambda (\mu-1)\frac{\alpha}{\mu} \hat{Q}$$

$$= \hat{Q} \ \{(\frac{1}{\mu})^{\frac{\alpha}{\alpha-1}} - (\frac{1}{\mu})^{\frac{1}{\alpha-1}}\frac{\alpha}{\mu} - 1 + \frac{\alpha}{\mu} + \lambda (\mu-1)\frac{\alpha}{\mu} .\}$$

Figure 1



y = marg. prod. of J
ŷ=μz
loss of national product due to wage differential
z
x
z
x
Û
J→
Ĵ
Â
J*, A*
← A

For example, take the following values of the parameters: $\alpha = \frac{1}{3}$, $\mu = 4$, $\lambda = \frac{1}{2}$. The difference in output is $\frac{13}{24}\hat{Q}$.

Figure 1 shows graphically the qualitative nature of the social loss.

7. The calculation would be modified if realized net incomes in the three categories differed from marginal productivities. This would happen if, for example, high wage workers were taxed to pay transfers to their less fortunate countrymen, or if they were informally taxed to help less fortunate kinsmen. Transfers to unemployed raise the attractiveness of membership in the urban labor force. They increase the amount of unemployment and therefore add to the social loss due to income differentials. Transfers to agriculture, on the other hand, diminish the amount of unemployment and therefore mitigate the social loss.

It is possible, but tedious, to work out these modifications for any specified transfer system. For example, suppose that high-wage beneficiaries are taxed sufficiently to narrow their differential advantage over the rest of the population by $\frac{1}{25}$. Suppose also that $\omega$ (which now becomes relevant) is $\frac{1}{2}$, and that in the alternative situation agricultural employment is 10 times modern employment. The social gain from moving to the optimum is $\frac{634}{1320}\hat{Q}$ instead of $\frac{13}{24}\hat{Q}$ $(=\frac{715}{1320}\hat{Q})$.

8. It is also possible to introduce the possibility of monopoly in the modern sector, so that the wage rate there is only a fraction of marginal product. Suppose this fraction is f, so that $z = f\frac{\alpha}{\mu}\hat{J}^{\alpha-1} = f\alpha J^{*\alpha-1}$.

None of the other calculations in Table 1 are affected.

All we need to do is to substitute $f\alpha/\mu$ for $\alpha/\mu$ in the bottom line. If f were $\frac{1}{2}$, this would add $\frac{11}{48}\hat{Q}$ to the original illustrative calculations of $\frac{26}{48}\hat{Q}$.

The reason for an additive correction is that with monopoly the wage differential understates the differential of marginal productivities. Every marginal shift of labor increases output by more than the wage differential.

A further gain could be made by eliminating monopoly, of course. Although the existence of monopoly in the modern sector may be an argument offered for union and government support of high wages in that sector, high wages are not an effective way to cancel the effect of monopoly. Instead they pile one monopoly on to another, and in a sense make modern employers and employees joint monopolists at the expense of the rest of the populace. The proper antidote to monopoly is competition, either from new domestic firms or from imports.

9. It should be emphasized that this note is an exercise in comparative statics. The ratio $\frac{U}{J}$ calculated here is an equilibrium ratio. The dynamics of migration between county and city have not been specified or studied, and it is possible that on some dynamic assumptions the equilibrium is not stable. On this subject the reader should consult the well known work of Todaro and a forthcoming paper by Richard Porter of IDS.