

# STATISTICAL PROPERTIES OF THE DORFMAN-STERRETT GROUP SCREENING PROCEDURE WITH ERRORS IN DECISION

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**Summary:** Methods that reduce the cost and time involved in detecting defective or nonconformal members of a large population have been explored extensively in the quality control literature. These methods have also found extensive application in insect-vector, rodent-bacterium and blood screening. Group-screening designs are plans that identify defect factors in a large population by initially pooling factors together and then classifying each pool as nonconformal (NC) or conformal (C). Individual testing is then carried out only amongst individual factors in pool that are found to be nonconformal. A modification of this strategy, suggested by Sterrett (1957), proposes a reversion to a group test, in a group declared defective, upon detection of the first nonconformal factor and then carrying out individual testing only if the new group is nonconformal. This procedure is referred to as the Dorfman-Sterrett procedure in the literature.

The statistical properties of the restricted Dorfman-Sterrett procedure, where the number of reversion to a group test is predetermined, has found little discussion in the literature. This study uses a testing of hypothesis approach to compare the performance of the Dorfman-Sterrett procedure with the Dorfman procedure assuming that factors or groups can be misclassified. Under the testing of hypothesis approach, using a  $2^8$  fractional factorial design, cost functions which are linear functions of expected total number of incorrect decisions and the expected number of tests, are derived and used as a basis for comparing the procedures of interest

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## 1. Introduction

Inspection of factors in quality control can be split into two complementary areas: the acceptance sampling plans and plans that classify factors as conformal (non-defective) or nonconformal (defective). The fundamental difference between the two approaches is essentially the form of decision

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to be reached. In the first case, the decisions to accept or reject a whole population are based on a sample or samples. In the second case, the decisions are based on individual factors, each factor being classified as nonconformal (NC) or conformal (C). It is the second category that is of interest to us.

Instead of testing each factor individually, Dorfman (1943) proposed putting the factors into groups at the first stage and testing only those factors in nonconformal groups at the second stage of analysis. If the group test has a negative result, indicating that the group is C, then all the factors in that group are declared C and the group undergoes no further investigation. This two-stage procedure shall herein be referred to as the Dorfman procedure.

A modification to the Dorfman procedure where factors in an NC group are tested individually until an NC factor is found has also been proposed in the literature (Sterrett, 1957). Once this NC factor has been identified, the remaining factors are pooled together to form a new sub-group. If the sub-group is C, there is no further investigation. Otherwise, individual testing is restarted amongst the untested factors, until another NC factor is found. On detecting such an NC factor, the remaining factors are tested as a sub-group, and so on until a decision is reached regarding to each factor. This procedure consists of testing individual factors in an NC group till a first NC is found and then reverting to a Dorfman procedure. This reversion takes place until a final C sub-group is identified. This modification has been referred to in the literature as the Step-wise group-screening procedure (Patel and Manene, 1987) and elsewhere as the Dorfman-Sterrett procedure (Huang, Johnson and Kotz, 1989; Johnson, Kotz and Rodriguez, 1989).

The original Sterrett plan does not pre-assign the number of times we shall revert to a Dorfman procedure. It seems therefore, that we are faced with a plan that may stop after tests and that may not adequately reduce the number of tests, and hence cost of inspection. A modification to Sterrett plan, referred to in the literature as the (Restricted) Dorfman-Sterrett procedure, extends Sterrett's plan to include a stopping rule (Johnson et al., 1989). Under this plan, the number of reversions to a Dorfman procedure is pre-assigned. If, for instance, only one reversion to a Dorfman procedure is allowed, the procedure is referred to as a single-step Dorfman-Sterrett procedure. If two reversions are allowed, we have a two-step Dorfman-Sterrett procedure; and in general, if  $\gamma$ -reversions are allowed, we have an  $\gamma$ -step (or multi-step) Dorfman-Sterrett procedure ( $\gamma \leq k - 2$ ), where  $k$  denotes group size. The Dorfman procedure can be viewed as a zero-step restricted Dorfman-Sterrett procedure as there is no reversion to a group testing upon detecting a first NC factor.

The group-screening procedure has been used by various researchers to estimate the proportion of NCs or infected individuals in a given population. The approach has been used to test for presence of the aster-yellows virus (Thompson, 1962), bacteria (Sobel and Elashoff, 1975), HIV (Gastwirth and Hammick, 1989; Kline, Brothers, Brookmeyer, Zeger and Quinn, 1989) and post-transfusion hepatitis (Davis, Grizzle and Bryan, 1973).

In this study we focus on the statistical properties of the restricted Dorfman-Sterrett procedure. Some aspects of this plan have been discussed elsewhere (Johnson et al., 1989) assuming, however, that probabilities of misclassification were constant and not functions of either group size or the number of NC factors in a group. Expected Proportional Reduction (EPR) in testing, along with probabilities of correctly classifying factors, formed the basis of discussing the efficiency of the procedure. In this paper, we use orthogonal fractional factorial plans of the type given by Plackett and Burman (1946) to interrogate properties of the procedure. We refer to this approach as the testing

of hypothesis approach, in line with assumptions presented in Watson (1961). Cost functions, which are linear functions of expected number of tests and expected number of incorrect decisions, shall form the basis for comparing procedures of interest.

In Section 2, we present the assumptions and notation used in this study. Section 3 and Section 4 present derivations of expected number of tests, and expected number of incorrect decisions, for the Dorfman procedure and the  $\gamma$ -step Dorfman-Sterrett procedures, respectively. In Section 5, we discuss the performance of the Dorfman-Sterrett procedure in terms of relative costs in testing and present examples of optimal designs for illustration. Section 6 presents a summary of our findings within the context of existing literature.

## 2. Notations and Assumptions

In this section we present the assumptions based on which expressions for the expected number of tests and the expected number of incorrect decisions shall be derived.

### 2.1. Notation

Assume that we have a population of  $f$  factors divided into  $g$  groups of size  $k$ . That is,  $f = kg$ . Each of these groups has two levels: 1 if they are NC and 0, otherwise.

Let  $D_i$  be a dichotomous random variable taking a value 1 if the  $i$ -th group is truly NC and 0, otherwise,  $i = 1, \dots, g$ . To address the possibility of misclassification, we shall further let  $T_i$  be a dichotomous random variable taking a value 1 if during screening the  $i$ -th group is declared NC (correctly or incorrectly) and 0 otherwise.

The symbol  $r$  denotes the total number of groups declared NC. This number can be expressed as the sum  $r = \sum_{i=1}^g T_i$ . It is these  $r$  groups that shall be subject to individual testing. Items in the remaining  $g - r$  groups shall be declared C without being tested.

We shall let  $S_i$  denote the number of NC factors in the  $i$ -th group of size  $k$  and assume that  $S_i \sim \text{Binomial}(n, p)$ , where  $p$  denotes the a-priori probability of a factor being truly NC,  $i = 1, \dots, g$ .

Let  $\delta_{ij}$  be a dichotomous random variable taking a value 1 if the  $j$ -th individual or factor in the  $i$ -th group is truly NC and 0, otherwise,  $j = 1, \dots, k; i = 1, \dots, g$ . Clearly,  $\delta_{ij} \sim \text{Binomial}(1, p)$ . Also let  $\tau_{ij}$  be a dichotomous random variable taking a value 1 if the  $j$ -th individual or factor in the  $i$ -th group is declared NC (correctly or incorrectly) and 0 otherwise,  $j = 1, \dots, k; i = 1, \dots, g$ .

Further, for each group declared NC,  $M$  shall denote the trial at which the first factor declared NC is found and  $X$  shall denote the number of truly NC factors among these  $M$  factors. We shall further let  $J$  be a dichotomous random variable taking value 1 if the  $m$ -th tested factor is truly defective and 0, otherwise.

To compute the expected number of incorrect decisions in an  $r$ -step Dorfman-Sterrett procedure, we shall let  ${}^rG_{ik}$  be random variables that denote the number of factors correctly classified NC in the  $i$ -th group of size  $k$  and  ${}^rG'_{ik}$  be random variables that denote the number of factors incorrectly classified NC the  $i$ -th group of size  $k$ .

In subsequent discussion,  ${}^rEI(NC)_k$  shall denote the expected number of incorrectly classified NC factors,  ${}^rEI(NC)_{k|s}$  shall denote the expected number of incorrectly classified NC factors conditional on  $s$ , the number of NC factors in the group, and  ${}^rEI(NC)_{k|m,x,j,t,s}$  the expected number

of incorrectly classified NC factors conditioned on the variables  $M$ ,  $X$ ,  $J$ ,  $T_i$ , and  $S_i$ . The expected number of correctly classified C factors shall be denoted by  ${}^rEI(C)_k$ , with its conditional variants expressed as  ${}^rEI(C)_{k|s}$  and  ${}^rEI(C)_{k|m,x,j,t,s}$ .

## 2.2. Group-screening assumptions

In this study we shall employ the same two-stage group-screening assumptions made by Watson (1961). These assumptions are:

- all factors have, independently, the same prior probability of being NC,  $p$ ;
- NC factors have the same effect,  $\Delta > 0$ ;
- there are no interactions present;
- the required designs exist;
- the directions of possible effects are known; and
- the errors of all observations are independently normal with a constant known variance  $\sigma^2$ .

### 2.2.1. Screening with errors at stage one

Each of the  $g$  groups under consideration are tested at the initial stage. We assume that any of these groups can be misclassified as NC or C. These groups are then tested for significance using a  $2^8$  factorial design of type given by Plackett and Burman (1946).

Under Plackett and Burman (1946) designs, assuming interactions to be absent,  $4 \left[ \frac{g}{4} \right]$  runs are required to test the significance of the main effects of the  $g$  groups orthogonally. That is,

$$4 \left[ \frac{g}{4} \right] = g + h$$

where  $h = 1, 2, 3$  and  $4$ , and by definition  $\left[ \frac{g}{4} \right]$  is the least integer greater than  $\frac{g}{4}$  except that  $\left[ \frac{g}{4} \right] = 0$  when  $g = 0$ .

If  $\hat{A}$  is the estimate of the main effect of any group with  $s$  NC factors, each with effect  $\Delta > 0$  for  $s = 0, 1, \dots, k$  and  $\sigma$  is the error in observation then  $E(\hat{A}) = s\Delta$  and  $\text{Var}(\hat{A}) = \frac{\sigma^2}{4 \left[ \frac{g}{4} \right]} = \frac{\sigma^2}{g+h}$  for  $h = 1, 2, 3$ , and  $4$ .

Assuming normality in observations, it then follows that

$$Z = \frac{\hat{A} - E(\hat{A})}{\sqrt{\text{Var}(\hat{A})}} = \frac{\hat{A} - s\Delta}{\sqrt{\frac{\sigma^2}{g+h}}} = y - s\phi_1,$$

where  $y = \frac{\hat{A}}{\sqrt{\frac{\sigma^2}{g+h}}}$  and  $\phi_1 = \frac{\Delta}{\sqrt{\frac{\sigma^2}{g+h}}}$ , is standard normal.

Testing the hypothesis that a group is C is equivalent to testing the null hypothesis  $H_0 : s\phi_1 = 0$  against the alternative  $H_a : s\phi_1 \neq 0$ .

The normal deviation test can be used if  $\sigma^2$  is known. If however,  $\sigma^2$  is unknown, the test will

reduce to a  $t$ -test.

The power of the test is thus given by

$$\pi_1(s\phi_1, \alpha_1) = \int_{A_{\alpha_1}}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{(y - s\phi_1)^2}{2}\right] dy \quad (1)$$

where  $\alpha_1$  is defined as

$$\alpha_1 = \int_{A_{\alpha_1}}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{Z^2}{2}\right] dZ$$

which is the level of significance of the test.

In terms of the random variables proposed in Section 2.1,  $\pi_1(s\phi_1, \alpha_1) = \mathbb{P}(T_i = 1 | S_i = s)$  and  $\alpha_1 = \mathbb{P}(T_i = 1 | S_i = 0)$ .

### 2.2.2. Screening with errors in observation at the second stage

At the second stage of the group screening plan, we focus on the classification of individual factors amongst the groups declared NC. To capture the possibility of misclassifying items, we shall let  $\alpha_2$  denote the probability of classifying a C as NC.

If there are  $r$  groups each of size  $k$  declared NC, then we have  $rk$  factors to observe.

If  $\hat{B}$  is the estimate of the main effect of a factor, with effect  $\Delta > 0$ , then  $E(\hat{B}) = \Delta$  and  $Var(\hat{B}) = \frac{\sigma^2}{rk}$ . Now define

$$V = \frac{\hat{B} - \Delta}{\sqrt{\sigma^2/rk}} = \frac{\hat{B}}{\sqrt{\sigma^2/rk}} - \frac{\Delta}{\sqrt{\sigma^2/rk}} = U - \phi_2,$$

where  $U = \frac{\hat{B}}{\sqrt{\sigma^2/rk}}$  and  $\phi_2 = \frac{\Delta}{\sqrt{\sigma^2/rk}}$ .

So testing the hypothesis of significant factor effects at the second stage is equivalent to testing the hypothesis  $H_0 : \phi_2 = 0$  against  $H_1 : \phi_2 \neq 0$ .

Assuming normality, the power of the test in the second stage, denoted by  $\pi_2(\phi_2, \alpha_2)$  according to (Watson, 1961), is given by

$$\pi_2(\phi_2, \alpha_2) = \int_{\omega_{\alpha_2}}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{(U - \phi_2)^2}{2}\right] dU,$$

where  $\alpha_2 = \int_{B_{\alpha_2}}^{\infty} \exp\left[-\frac{Z^2}{2}\right] dZ$  is the size of the critical region for testing the factors in the group declared NC.

In terms of the random variable proposed in Section 2.1,  $\pi_2(\phi_2, \alpha_2) = \mathbb{P}(\tau_{ij} = 1 | \delta_{ij} = 1)$  and  $\alpha_2 = \mathbb{P}(\tau_{ij} = 1 | \delta_{ij} = 0)$ .

## 2.3. Key probabilities

The computation of expected number of runs and the expected number of incorrect decisions in this study are generalisations of the results by Johnson, S. and Rodriguez (1988) and Johnson et al. (1989) to the hypothesis testing paradigm used by Watson (1961), Curnow (1965) and Patel and Ottieno (1984).

In line with the results of Watson (1961), the probability that a group of size  $k$  is declared NC is given by

$$\pi_1^* = \sum_{s=0}^k \pi_1(s\phi_1, \alpha_1) \binom{k}{s} p^s q^{k-s}. \quad (2)$$

The probability that a group of size  $k$  is declared NC given that a factor within the group is NC shall take the form presented in Curnow (1965), which is

$$\pi_1^+ = \sum_{s=1}^k \pi_1(s\phi_1, \alpha_1) \binom{k-1}{s-1} p^{s-1} q^{k-s}. \quad (3)$$

In terms of the random variables described in Section 2.1, the expressions in equation (2) and equation (3) are simply  $\pi_1^* = \mathbb{P}(T_i = 1)$  and  $\pi_1^+ = \mathbb{P}(T_i = 1 | \delta_{ij} = 1)$ , respectively.

Taking into account the fact that the iterative reversion to group testing after a first NC factor is identified, whether correctly or incorrectly, Johnson et al. (1989) derived the following two probability distributions which we now present under the testing of hypothesis framework by (Watson, 1961). The first is the probability that the  $m$ -th factor is the first (correctly) classified NC factor while there are  $x$  truly NC factors among the first  $m$  tested factors in the group of size  $k$  containing  $s$  NC factors is

$$p_{NC}(m, x | k, s) = \begin{cases} \frac{\binom{m-1}{x-1} \binom{k-m}{s-x}}{\binom{k}{s}} \pi_2 [1 - \pi_2]^{x-1} [1 - \alpha_2]^{m-x} & \text{if } x \geq 1, \\ 0 & \text{if } x = 0, \end{cases} \quad (4)$$

and the second is the probability that the  $m$ -th factor is the first (incorrectly) classified NC factor while there are  $x$  truly NC factors among the first  $m$  tested factors in the group of size  $k$  containing  $s$  NC factors is

$$p_C(m, x | k, s) = \begin{cases} \frac{\binom{m-1}{x} \binom{k-m}{s-x}}{\binom{k}{s}} \alpha_2 [1 - \pi_2]^x [1 - \alpha_2]^{m-x-1} & \text{if } x \leq m-1 \\ 0 & \text{if } x = m, \end{cases} \quad (5)$$

The sum of equation (4) and equation (5) leads to the probability of finding a first NC factor at the  $m^{\text{th}}$  trial in a group of size  $k$  containing  $s$  NC factors while there are  $x$  NC factors among the first  $m$  tested factors which is given by

$$p(m, x | k, s) = p_{NC}(m, x | k, s) + p_C(m, x | k, s). \quad (6)$$

The probabilities mentioned in equation (4), equation (5) and equation (6) are the conditional probabilities  $p_{NC}(m, x | k, s) = \mathbb{P}(M = m, X = x, J = 1 | S_i = s)$ ,  $p_C(m, x | k, s) = \mathbb{P}(M = m, X = x, J = 0 | S_i = s)$ , and  $p(m, x | k, s) = \mathbb{P}(M = m, X = x | S_i = s)$ , respectively.

### 3. The Dorfman (or zero-step Dorfman-Sterrett) procedure

In this section we present expressions for the expected number of tests and expected number of incorrect decisions for a Dorfman procedure.

### 3.1. The expected number of runs

Let  ${}^0R_{ik}$  be a random variable that denotes the number of tests required to classify all factors in the  $i$ -th group,  $i = 1, \dots, g$ , of size  $k$  in a Dorfman procedure. This variable takes a value  $1 + k$  if the group is declared NC and 1 otherwise. Thus

$${}^0R_{ik} = 1 + kT_i. \quad (7)$$

**Lemma 1** The expected number of tests needed to classify all the factors in the  $i$ -th group of size  $k$  given that it contains  $s$  NC factors using a zero-step Dorfman-Sterrett procedure is given by

$${}^0E_{k|s} = 1 + k\pi_1(s\phi_1, \alpha_1).$$

**Proof.** The desired results follows immediately as the conditional expectation

$${}^0E_{k|s} = \mathbb{E}({}^0R_{ik}|S_i = s) = 1 + k\mathbb{E}(T_i|S_i = s) = 1 + k\mathbb{P}(T_i = 1|S_i = s).$$

■

**Lemma 2** The expected number of tests needed to classify all the factors in the  $i$ -th group of size  $k$  using a zero-step Dorfman-Sterrett procedure is given by

$${}^0E_k = 1 + k\pi_1^*.$$

**Proof.** This result is the expected value of the random variable given in equation (7) and follows immediately on substituting equation (2). That is,  ${}^0E_k = \mathbb{E}({}^0R_{ik}) = 1 + k\mathbb{E}(T_i)$ . ■

### 3.2. Expected number of correct (and incorrect) decisions

Having discussed the expected number of runs, we now derive expressions for the expected number of incorrect decisions. We let  ${}^0G_{ik}$  denote the number of correctly classified NC factors,  ${}^0G'_{ik}$  be the number of C factors incorrectly declared NC, and  $S_i^*$  be the number of factors declared NC.

The conditional density of  ${}^0G_{ik}$  given that the group of size  $k$  is declared NC,  $T_i = 1$ , and that it contains  $s$  NC factors,  $S_i = s$ , is  $\text{Binomial}(s, \pi_2)$ . Similarly, the conditional density of  ${}^0G'_{ik}$  given that  $T_i = 1$  and  $S_i = s$  is  $\text{Binomial}(k - s, \alpha_2)$ . Since  $S_i^* = {}^0G_{ik} + {}^0G'_{ik}$ , the conditional density of  $S_i^*$  given  $T_i = 1$  and  $S_i = s$  is a convolution of the conditional densities of  $G_i$  and  $G'_i$  given  $T_i = 1$  and  $S_i = s$  and can therefore be expressed as

$$P_k[S_i^* = z|S_i = s, T_i = 1] = \sum_m \binom{s}{m} \pi_2^m (1 - \pi_2)^{s-m} \binom{k-s}{z-m} \alpha_2^m (1 - \alpha_2)^{k-s-m}. \quad (8)$$

**Lemma 3** The expected number of factors declared NC in a group of size  $k$  that contains  $s$  NC factors is given by

$$\mathbb{E}[S_i^*|S_i = s] = \pi_1(s\phi_1, \alpha_1)[(k-s)\alpha_2 + s\pi_2(\phi_2, \alpha_2)].$$

**Proof.** This results is simply the conditional expectation  $\mathbb{E}(S_i^*|S_i = s) = \mathbb{E}\mathbb{E}(S_i^*|T_i = t, S_i = s)$  and is a consequence of equation (8). ■

**Corollary 1** The expected number of incorrectly classified NC factors given that the group of size  $k$  contains  $s$  NC factors can be expressed as

$${}^0EI(NC)_{k|s} = \begin{cases} (k-s)\pi_1(s\phi_1, \alpha_1)\alpha_2 & \text{if } s > 0, \\ k\alpha_1\alpha_2 & \text{if } s = 0. \end{cases}$$

**Proof.** This result follows from the fact that the expected value in Lemma 3 is a sum of the expected number of correctly and incorrectly classified NC factors. Clearly,  $(k-s)\pi_1(s\phi_1, \alpha_1)\alpha_2 = k\alpha_1\alpha_2$  when  $s = 0$ . ■

**Corollary 2** The expected number of incorrectly classified C factors is

$${}^0EI(C)_{k|s} = \begin{cases} s - s\pi_1(s\phi_1, \alpha_1)\pi_2(\phi_2, \alpha_2) & \text{if } s > 0, \\ 0 & \text{if } s = 0. \end{cases}$$

**Proof.** This result also stems from the fact that the expected value in Lemma 3 is a sum of the expected number of correctly and incorrectly classified NC factors. ■

Similar results are given in Johnson et al. (1988), Patel and Ottieno (1984) and Odhiambo and Manene (1987).

The result we have presented in this section are similar to those of Watson (1961), Patel and Ottieno (1984) and Johnson et al. (1988). These results shall be used in the subsequent discussion as the iterative nature of the multistep Dorfman-Sterrett uses these results at the final step.

## 4. The multi-step Dorfman-Sterrett procedure

In this section we present expressions for the expected number of tests and the expected number of incorrect decisions that arise when using a multi-step Dorfman-Sterrett procedure.

### 4.1. The expected number of runs

Let  ${}^\gamma R_{ik}$  be a random variable that denotes the number of tests required to classify all the factors in the  $i$ -th group,  $i = 1, \dots, g$ , of size  $k$  using a  $\gamma$ -step Dorfman-Sterrett procedure.

The expected value of this random variable given that the group has been declared defective ( $T_i = 1$ ) and contains  $s$  defective factors ( $S_i = s$ ) is given by

$$\begin{aligned} \mathbb{E}[{}^\gamma R_{ik}|S_i = s, T_i = 1] &= \sum_{m=1}^k \sum_x \sum_{j=0}^{(s)} \mathbb{E}[{}^\gamma R_{ik}|M = m, S_i = s, X = x, J = j, T_i = 1] \\ &\quad \times P_k[M = m, X = x, J = j|S_i = s, T_i = 1], \\ &= \sum_{m=1}^k \sum_x \sum_{j=0}^{(s)} {}^\gamma E_{k|m,s,x,j,1} p(m,x,j|k,s), \end{aligned} \tag{9}$$

where  $p(m,x,j|k,s)$  is defined in equation (6) and  ${}^\gamma E_{k|m,s,x,j,1}$  can be defined as

$${}^\gamma E_{k|m,s,x,j,1} = \begin{cases} 1 + m + {}^{\gamma-1} E_{k-m|s-x} & \text{if } m \leq k-2, j = 0, 1, s > 0, \\ 1 + m + {}^{\gamma-1} E_{k-m|0} & \text{if } m \leq k-2; j = 0, 1; s = 0; x = 0, \\ 1 + k & \text{if } m = k-1, k; j = 0, 1; \forall x. \end{cases} \tag{10}$$



The expression in equation (10) arises from the following considerations:  $1 + m + \gamma^{-1} E_{k-m|s-x}$  tests are required if a first NC factor is found at the  $m$ -th trial and the remaining factors are subjected to a  $(\gamma - 1)$ -step Dorfman-Sterrett procedure in a group of size  $k - m$  containing  $s - x$  NC factors;  $1 + m + \gamma^{-1} E_{k-m|0}$  tests are required if the group contains no defective factor, a first NC factor is found at the  $m$ -th trial and the remaining factors are subjected to a  $(\gamma - 1)$ -step Dorfman-Sterrett procedure in a group of size  $k - m$  containing no NC factors; and  $1 + k$  tests are required if the first factor declared NC in the group is not found on or before the  $(k - 1)$ -st trial.

**Lemma 4** The expected number of tests on a group of size  $k$  in a  $\gamma$ -step Dorfman-Sterrett procedure given that the group contains  $s$  NC factors is

$${}^\gamma E_{k|s} = 1 + k\pi_1(s\phi_1, \alpha_1) - \pi_1(s\phi_1, \alpha_1) \sum_{m=1}^{k-2} \sum_x^{(s)} \{k - m - \gamma^{-1} E_{k-m|s-x}\} p(m, x|k, s).$$

**Proof.** This expected value is defined as

$${}^\gamma E_{k|s} = \mathbb{E}[{}^\gamma R_{ik}|S_i = s] = \sum_{t=0}^1 \mathbb{E}[{}^\gamma R_{ik}|T_i = t, S_i = s] \mathbb{P}(T_i = t|S_i = s),$$

where  $\mathbb{E}[{}^\gamma R_{ik}|T_i = 1, S_i = s]$  is given in equation (9). The desired result follows upon substituting equation (1). ■

**Corollary 3** The expected number of tests on a group of size  $k$  in a  $\gamma$ -step Dorfman-Sterrett procedure given that the group contains no NC factor is

$${}^\gamma E_{k|0} = 1 + k\alpha_1 - \alpha_1 \left[ \frac{k\alpha_2 - 1 + [k(1 - \alpha_2) - 1](1 - \alpha_2)^{k-2} - (k-2)(1 - \alpha_2)^{k-1}}{\alpha_2} \right] + \alpha_1 \alpha_2 \sum_{m=1}^{k-2} \gamma^{-1} E_{k-m|0} (1 - \alpha_2)^{m-1}.$$

**Proof.** This expected value follows on substituting  $s = 0$  in Lemma 4. ■

**Corollary 4** The expected number of tests required to test all the factors in a group of size  $k$  in a  $\gamma$ -step Dorfman-Sterrett procedure is given by

$${}^\gamma E_k = 1 + k\pi_1^* - \alpha_1 \sum_{m=1}^{k-2} (1 - \alpha_2)^{m-1} \alpha_2 \{k - m - \gamma^{-1} E_{k-m|0}\} q^k - \sum_{s=1}^k \pi_1(s\phi_1, \alpha_1) \sum_{m=1}^{k-2} \sum_x^{(s)} (k - m - \gamma^{-1} E_{k-m|s-x}) p(m, x|k, s) \binom{k}{s} p^s q^{k-s}.$$

*Proof.* This expectation is simply the sum

$${}^\gamma E_k = {}^\gamma E_{k|0} q^k + \sum_{s=1}^k {}^\gamma E_{k|s} \binom{k}{s} p^s q^{k-s},$$

where  ${}^\gamma E_{k|0}$  and  ${}^\gamma E_{k|s}$  are given by Corollary 3 and Lemma 4, respectively. □

**Theorem 1** The expected number of tests required to analyse all the  $f$  factors in a  $\gamma$ -step Dorfman-Sterrett procedure is given by

$$\begin{aligned} \mathbb{E}[\gamma R] &= h + \frac{f}{k} + f\pi_1^* - \frac{f\alpha_1}{k} \sum_{m=1}^{k-2} (1 - \alpha_2)^{m-1} \alpha_2 \{k - m - \gamma^{-1} E_{k-m|0}\} q^k \\ &\quad - \frac{f}{k} \sum_{s=1}^k \pi_1(s\phi_1, \alpha_1) \sum_{m=1}^{k-2} \sum_x^{(s)} (k - m - \gamma^{-1} E_{k-m|s-x}) p(m, x|k, s) \binom{k}{s} p^s q^{k-s}. \end{aligned}$$

*Proof.* The number of tests on a group of size  $k$  in a  $\gamma$ -step Dorfman-Sterrett procedure is given by

$$\gamma R = h + \sum_{i=1}^g \gamma R_{ik},$$

where  $h = 1$  or  $2$  or  $3$  or  $4$  is the control. The desired result will follow immediately from  $\mathbb{E}(\gamma R) = h + g\gamma E_k$ .  $\square$

## 4.2. The expected number of incorrect decisions

We now present expressions for the expected number of incorrect decisions in a  $\gamma$ -step Dorfman-Sterrett procedure. This expectation is the sum of the number of NC factors declared C and the number of C factors declared NC.

### 4.2.1. Expected number of incorrectly classified Cs

We start by deriving an expression for the expected number of incorrectly classified C factors in a group of size  $k$ . This expected value is the difference between the actual number of NCs in the group and those that are correctly classified as NC.

Letting  $\gamma G_{ik}$  be a random variable that denotes the number of factors correctly declared NC in the  $i$ -th group,  $i = 1, \dots, g$  of size  $k$ , we determine  $\gamma EC(NC)_k = \mathbb{E}(\gamma G_{ik})$ , the expected number of correctly classified NC factors in the group.

In a  $\gamma$ -step Dorfman-Sterrett procedure there are six ways in which we may correctly classify NC factors in a group of size  $k$ . There can be  $1 + \gamma^{-1} E(NC)_{k-m|s-x}$  correctly classified NC factors if the first correctly classified NC factor is found at the  $m$ -th trial while there are  $x$  truly NC factors among these  $m$  factors. The remaining  $(k - m)$  factors are then pooled together to form a group, containing  $(s - x)$  NC factors, that is subjected to a  $(\gamma - 1)$ -step Dorfman-Sterrett procedure. It is also possible to have  $\gamma^{-1} E(NC)_{k-m|s-x}$  correctly classified NC factors if the  $m$ -th tested factor, ( $m \leq k - 2$ ), is the first incorrectly classified NC factor while there are  $x$  truly NC factors among these  $m$  factors. The remaining  $(k - m)$  factors are pooled together to form a group containing  $(s - x)$  NC factors and subjected to a  $(\gamma - 1)$ -step Dorfman-Sterrett procedure. There may also be  $1 + \pi_2(\phi_2, \alpha_2)$  correctly classified NCs if the  $(k - 1)$ -st tested factor is the first correctly classified NC factor while in fact there are  $(s - 1)$  truly NC factors among the first  $(k - 1)$  factors tested. We could also have  $\pi_2(\phi_2, \alpha_2)$  correctly classified NCs if the  $(k - 1)$ -st factor is incorrectly classified NC and the  $k$ -th factor are correctly classified as NC, or have a single correctly classified NC factors if the  $(k - 1)$ -st tested factor is the first correctly classified NC factor while in fact there are  $s$  truly NC factors among

the first  $(k-1)$  factors tested. Finally, we could have a single correctly classified NC factor if the  $k$ -th tested factor is the first correctly classified NC factor while in fact there are  $s$  truly NC factors among the first  $k$  factors tested. With these assumptions, the conditional expectation

$${}^\gamma E(NC)_{k|m,x,j,s,t} = \mathbb{E}({}^r G_{ik} | M = m, X = x, J = j, T_i = t, S_i = s)$$

can be expressed as

$${}^\gamma E(NC)_{k|m,x,j,s,t} = \begin{cases} 1 + \gamma^{-1} E(NC)_{k-m|s-x} & \text{if } 1 \leq m \leq k-2, \max(0, s-k+m) \leq x \leq \min(m, s) \\ & j = 0, t = 1 \text{ and } s > 0, \\ \gamma^{-1} E(NC)_{k-m|s-x} & \text{if } 1 \leq m \leq k-2, \max(0, s-k+m) \leq x \leq \min(m, s) \\ & j = 1, t = 1 \text{ and } s > 0, \\ 1 + \pi_2(\phi_2, \alpha_2) & \text{if } m = k-1, x = s-1, j = 0, t = 1 \text{ and } s > 0, \\ \pi_2(\phi_2, \alpha_2) & \text{if } m = k-1, x = s-1, j = 1, t = 1 \text{ and } s > 0, \\ 1 & \text{if } m = k-1, k, x = s, j = 1, t = 1 \text{ and } s > 0, \\ 0 & \text{otherwise.} \end{cases} \quad (11)$$

**Lemma 5** The expected number of correctly classified NC factors given that the group of size  $k$  contains  $s$  NC factors is given by

$${}^\gamma EC(NC)_{k|s} = \pi_1(s\phi_1, \alpha_1) \left[ \sum_{m=1}^{k-2} \sum_x^s \{ p_{NC}(m, x|k, s) + p(m, x|k, s) \gamma^{-1} EC(NC)_{k-m|s-x} \} \right. \\ \left. + p_{NC}(k-1, s-1|k, s) + p_{NC}(k-1, s|k, s) \right. \\ \left. + p(k-1, s-1|k, s) \pi_2(\phi_2, \alpha_2) + p_{NC}(k, s|k, s) \right].$$

**Proof.** This expected value is simply

$${}^r EC(NC)_{k|s} = \mathbb{E}({}^r G_{ik} | S_i = s), \\ = \sum_m \sum_x \sum_j \sum_t \sum_s^r EC(NC)_{k|m,x,j,t,s} \mathbb{P}(M = m, X = x, J = j | T_i = t, S_i = s) \mathbb{P}(T_i = t | S_i = s),$$

where  $\mathbb{P}(M = m, X = x, J = j | T_i = t, S_i = s)$  can be obtained from equation (4) if  $j = 1$  or equation (5) if  $j = 0$  and further, from  $\mathbb{P}(T_i = t | S_i = s)$  which can be obtained directly from equation (1). ■

**Corollary 5** The expected number of incorrectly classified C factors in a  $\gamma$ -step Dorfman-Sterrett procedure given that the group under consideration has  $s$  NC factors is given by

$${}^\gamma EI(C)_{k|s} = s - \pi_1(s\phi_1, \alpha_1) \left[ \sum_{m=1}^{k-2} \sum_x^s \{ p_{NC}(m, x|k, s) + p(m, x|k, s) \gamma^{-1} EC(NC)_{k-m|s-x} \} \right. \\ \left. + p_{NC}(k-1, s-1|k, s) + p_{NC}(k-1, s|k, s) \right. \\ \left. + p(k-1, s-1|k, s) \pi_2(\phi_2, \alpha_2) + p_{NC}(k, s|k, s) \right].$$

**Proof.** This expected value is the difference between the total number of NCs in the group and the number of factors declared NC in that group. That is,  ${}^\gamma EI(C)_{k|s} = s - {}^\gamma EC(NC)_{k|s}$ . ■

**Theorem 2** The expected number of incorrectly classifying a C factor in a group of size  $k$  in a  $\gamma$ -step Dorfman-Sterrett procedure is

$${}^\gamma EI(C)_k = kp - \sum_{s=1}^k \pi_1(s\phi_1, \alpha_1) \left[ \sum_{m=1}^{k-2} \sum_x^s \{ p_{NC}(m, x|k, s) + p(m, x|k, s) \gamma^{-1} EC(NC)_{k-m|s-x} \} \right. \\ \left. + p_{NC}(k-1, s-1|k, s) + p_{NC}(k-1, s|k, s) \right. \\ \left. + p(k-1, s-1|k, s) \pi_2(\phi_2, \alpha_2) + p_{NC}(k, s|k, s) \right] \binom{k}{s} p^s q^{k-s}.$$

**Proof.** This results follows from Corollary 5 and the fact that

$${}^r EI(C)_k = \sum_{s=0}^k {}^r EI(C)_{k|s} \mathbb{P}(S_i = s) = \sum_{s=0}^k {}^r EI(C)_{k|s} \binom{k}{s} p^s (1-p)^{k-s}.$$

■

#### 4.2.2. Expected number of incorrectly classified NCs

Let  ${}^\gamma G'_{ik}$  be a random variable that denotes the number of factors incorrectly declared NC in the  $i$ -th group,  $i = 1, \dots, g$  of size  $k$ .

In a  $\gamma$ -step Dorfman-Sterret procedure there are several ways in which we may incorrectly classify NC factors in a group of size  $k$ . There can be  ${}^{\gamma-1} EI(NC)_{k-m|s-x}$  incorrectly classified NC factors if the first correctly classified NC factor is the  $m$ -th tested while in fact there are  $x$  truly NC factors among the first  $m$  tested factors and that all the  $(s-x)$  NC factors in the subsequent Dorfman procedure are incorrectly classified, where  $\max(0, s-k+m) \leq t \leq \min(m, s)$ . We would have  $1 + {}^{\gamma-1} EI(NC)_{k-m|s-x}$  incorrectly classified NC factors given that the  $m$ -th tested factor, ( $m \leq k-2$ ), is the first incorrectly classified NC factor, where  $\max(0, s-k+m) \leq t \leq \min(m, s)$ . There could be  $\alpha$  incorrectly classified NC factors if the  $(k-1)$ -st tested factor is the first correctly classified NC factor while in fact there are  $s$  truly NC factors among the first  $(k-1)$  factors tested. We could have a single incorrectly classified NC factors given that the  $k$ -th tested factor is the first incorrectly classified NC factor while in fact there are  $s$  truly NC factors among the first  $k$  factors tested; We could have  $\alpha_2$  incorrectly classified NC factors given that the  $(k-1)$ -st tested factor is the first incorrectly classified NC factor while in fact there are  $x$  truly NC factors among the first  $(k-1)$  factors tested. It is also possible that we can have  $(1 + \alpha_2)$  incorrectly classified NC factors given that the  $(k-1)$ -st tested factor is the first incorrectly classified NC factor while in fact there are  $s$  truly NC factors among the first  $(k-1)$  factors tested. There will a single incorrectly classified NC factors given that the  $(k-1)$ -st tested factor is the first incorrectly classified NC factor while in fact there are  $(s-1)$  truly NC factors among the first  $(k-1)$  factors tested. We will also have a single incorrectly classified NC factors given that the  $k$ -th tested factor is the first incorrectly classified NC factor while in fact there are  $s$  truly NC factors among the first  $k$  factors tested.

With these assumptions, the conditional expectation

$${}^\gamma EI(NC)_{k|m,x,j,s,t} = \mathbb{E} \mathbb{E} \mathbb{E} \mathbb{E} ({}^\gamma G'_{ik} | M = m, X = x, J = j, T_i = t, S_i = s)$$

can be expressed as

$${}^\gamma EI(NC)_{k|m,x,j,s,1} = \begin{cases} {}^{\gamma-1} EI(NC)_{k-m|s-x} & \text{if } 1 \leq m \leq k-2 \max(0, s-k+m) \leq x \leq \min(m, s) \\ & j = 0 \text{ and } s > 0, \\ 1 + {}^{\gamma-1} EI(NC)_{k-m|s-x} & \text{if } 1 \leq m \leq k-2 \max(0, s-k+m) \leq x \leq \min(m, s) \\ & j = 1 \text{ and } s > 0, \\ 1 + \alpha_2 & \text{if } m = k-1, x = s, j = 1 \text{ and } s > 0, \\ \alpha_2 & \text{if } m = k-1, x = s, j = 0 \text{ and } s > 0, \\ 1 & \text{if } m = k, x = s, j = 1 \text{ and } s > 0 \\ 0 & \text{otherwise.} \end{cases}$$

**Lemma 6** It also follows that the expected number of incorrectly classified NC factors given that the group of size  $k$  contains  $s$  NC factors is given by

$$\begin{aligned} {}^\gamma EI(NC)_{k|s} = \pi_1(s\phi_1, \alpha_1) & \left\{ \sum_{m=1}^{k-2} \left\{ \sum_{x=1}^{(s-1)} [p_C(m, x|k, s) + {}^{\gamma-1} EI(NC)_{k-m|s-x} p(m, x|k, s)] \right\} \right. \\ & + p_C(k-1, s-1|k, s) + p_C(k-1, s|k, s) \\ & \left. + \alpha_2 p(k-1, s|k, s) + p_C(k, s|k, s) \right\}. \end{aligned}$$

**Proof.** This expected value is follows from where equation (4), equation (5), equation (1) and the fact that

$$\begin{aligned} {}^r EC(NC)_{k|s} &= \mathbb{E}({}^r G'_{ik} | S_i = s), \\ &= \sum_m \sum_x \sum_j \sum_t \sum_s {}^r EI(NC)_{k|m, x, j, t, s} \mathbb{P}(M = m, X = x, J = j | T_i = t, S_i = s) \mathbb{P}(T_i = t | S_i = s). \end{aligned}$$

■

The following result is a consequence of substituting  $s = 0$  in Lemma 6.

**Corollary 6** The expected number of incorrectly classified NC factors given that the group of size  $k$  contains no NC factors is also given by

$${}^\gamma EI(NC)_{k|0} = \alpha_1 \left[ \sum_{m=1}^{k-2} [1 + {}^{\gamma-1} EI(NC)_{k-m|0}] \alpha_2 (1 - \alpha_2)^{m-1} + 2\alpha_2 (1 - \alpha_2)^{k-2} \right].$$

**Theorem 3** The expected number of incorrectly classified NC factors in a  $\gamma$ -step Dorfman-Sterrett procedure is

$$\begin{aligned} {}^\gamma EI(NC)_k &= \alpha_1 \left[ \sum_{m=1}^{k-2} [1 + {}^{\gamma-1} EI(NC)_{k-m|0}] \alpha_2 (1 - \alpha_2)^{m-1} + 2\alpha_2 (1 - \alpha_2)^{k-2} \right] q^k \\ &+ \sum_{s=1}^{k-1} \pi_1(s\phi_1, \alpha_1) \left[ \sum_{m=1}^{k-2} \left\{ \sum_{x=1}^{(s-1)} [p_C(m, x|k, s) + {}^{\gamma-1} EI(NC)_{k-m|s-x} p(m, x|k, s)] \right\} \right. \\ &\quad + p_C(k-1, s-1|k, s) + p_C(k-1, s|k, s) \\ &\quad \left. + \alpha_2 p(k-1, s|k, s) + p_C(k, s|k, s) \right] \binom{k}{s} p^s q^{k-s}. \end{aligned}$$

**Proof.** This results follows from Lemma 6, Corollary 6 and the fact that

$${}^r EI(NC)_k = \sum_{s=0}^k {}^r EI(NC)_{k|s} \mathbb{P}(S_i = s) = \sum_{s=0}^k {}^r EI(NC)_{k|s} \binom{k}{s} p^s (1-p)^{k-s}.$$

■

### 4.2.3. Total number of incorrect decisions

**Lemma 7** The expected total number of incorrect decisions made on a group of size  $k$  in a  $\gamma$ -step Dorfman-Sterrett procedure is

$$\begin{aligned}
{}^\gamma EI_k &= \alpha_1 \left[ \sum_{m=1}^{k-2} [1 + (k-m)^{\gamma-1} PI(NC)_{k-m|0}] \alpha_2 (1-\alpha_2)^{m-1} + 2\alpha_2 (1-\alpha_2)^{k-2} \right] q^k \\
&+ \sum_{s=1}^{k-1} \pi_1(s\phi_1, \alpha_1) \left[ \sum_{m=1}^{k-2} \left\{ \sum_{x=1}^{(s-1)} [p_C(m, x|k, s) + {}^{\gamma-1} EI(NC)_{k-m|s-x} p(m, x|k, s)] \right\} \right. \\
&\quad \left. + p_C(k-1, s-1|k, s) + p_C(k-1, s|k, s) \right. \\
&\quad \left. + \alpha_2 p(k-1, s|k, s) + p_C(k, s|k, s) \right] \binom{k}{s} p^s q^{k-s} \\
&+ kp - \sum_{s=1}^k \pi_1(s\phi_1, \alpha_1) \left[ \sum_{m=1}^{k-2} \sum_x^s \{ p_{NC}(m, x|k, s) + (s-x)p(m, x|k, s) {}^{\gamma-1} p_C(NC)_{k-m|s-x} \} \right. \\
&\quad \left. + p_{NC}(k-1, s-1|k, s) + p_{NC}(k-1, s|k, s) \right. \\
&\quad \left. + p(k-1, s-1|k, s) \pi_2(\phi_2, \alpha_2) + p_{NC}(k, s|k, s) \right] \binom{k}{s} p^s q^{k-s}
\end{aligned}$$

**Proof.** This result is a sum of the expected number of incorrectly classified C factors in the group of size  $k$  and the expected number of incorrectly classified NC factors in the group of size  $k$  as given in Theorems 2 and 3, respectively. ■

**Corollary 7** The expected total number of incorrect decisions made on the population of size  $f$ , that is divided into  $g$  groups of size  $k$  each, under the  $\gamma$ -step Dorfman-Sterrett procedure, assuming  $p$  is small and that  $\frac{\Delta}{\sigma} \rightarrow \infty$ , is

$$\begin{aligned}
{}^\gamma I_k &\approx fp - \frac{fp}{k\alpha_2} \left[ 1 - (1-2\alpha_2)(1-\alpha_2)^{k-2} \right] - \frac{fp\alpha_2}{k} \sum_{m=1}^{k-2} (k-m)^{\gamma-1} EC(NC)_{k-m|1} (1-\alpha_2)^{m-1} \\
&+ \frac{f\alpha_1}{k} (1-kp) + \frac{f}{k} (1-kp)(2\alpha_2 - \alpha_1)(1-\alpha_2)^{k-2} + \frac{f\alpha_1\alpha_2}{k} (1-kp) \sum_{m=1}^{k-2} {}^{\gamma-1} EI(NC)_{k-m|0} (1-\alpha_2)^{m-1} \\
&+ \frac{fp}{k} \left[ \frac{k\alpha_2 - 1 + [k(1-\alpha_2) - 1](1-\alpha_2)^{k-2} - (k-2)(1-\alpha_2)^{k-1}}{\alpha_2^2} \right] \\
&+ \frac{2fp\alpha_2}{k} (1-\alpha_2)^{k-2} + \frac{fp}{k} \sum_{m=1}^{k-2} (k-m)^{\gamma-1} EI(NC)_{k-m|1} (1-\alpha_2)^{m-1}.
\end{aligned} \tag{12}$$

**Proof.** The result follows by multiplying the result of Lemma 7 by  $g$  and assuming that  $p$  is small and  $\frac{\Delta}{\sigma} \rightarrow \infty$ . ■

### 4.3. Cost functions

In Sections 4.1 and 4.2, we extended the results given in Johnson et al. (1989) to include the group screening assumptions of Watson (1961) to derive our results. With these results, we now present an expression for cost function based on which the performance of the  $\gamma$ -step Dorfman-Sterrett procedure shall be evaluated.

Let  $c_1$  be the cost of inspection per run and  $c_2$  be the loss incurred per incorrect decision. The expected total cost in a  $\gamma$ -step Dorfman-Sterrett procedure is given by

$$C = c_1 E({}^\gamma R) + c_2 {}^\gamma I, \tag{13}$$

where  $E({}^\gamma R)$  is given in Theorem 1 and  ${}^\gamma I$  is given in Lemma 7.

**Table 1:** The minimum cost  $C$  relative to  $c_1$  assuming that  $\frac{\Delta}{\sigma} \rightarrow \infty$  and  $p$  is small with  $\frac{c_1}{c_2} = 0.1$  and  $\alpha_1 = \alpha_2 = 0.01$  for  $\gamma$ -step Dorfman-Sterrett procedure with  $f = 100$ .

$p$	$\gamma = 0$		$\gamma = 1$		$\gamma = 2$	
	$k_{(min)}$	$RC_{opt}$	$k_{(min)}$	$RC_{opt}$	$k_{(min)}$	$RC_{opt}$
0.001	32	8.299663	43	8.934004	45	7.182395
0.002	22	10.908135	31	10.933457	32	9.192659
0.005	14	16.079304	19	14.958687	20	13.285924
0.008	11	19.809261	15	17.946076	16	16.337235
0.010	10	21.906204	14	19.659005	14	18.093743
0.020	7	30.151702	10	26.657969	10	25.252122
0.050	5	46.754948	6	41.703213	6	40.632172
0.080	4	58.684548	5	53.433659	5	52.590277
0.100	3	65.038037	4	60.326312	4	59.624490
0.120	3	70.977644	4	66.676220	4	66.076869
0.150	3	79.887055	3	75.720358	3	75.323974
0.180	2	87.641436	3	83.571563	3	83.275509
0.200	2	91.601040	3	88.805700	3	88.576533
0.220	2	95.560644	3	94.039837	3	93.877557
0.250	2	101.500050	3	101.891042	3	101.829092

### 5. Optimum screening procedures

The ultimate purpose of this study was to provide a theoretic basis for computing optimum group size,  $k_{(min)}$ , for given prevalence rate  $p$ . The relative efficiency of any two designs can then be assessed on the basis of the relative cost in testing corresponding to the value  $k_{(min)}$  obtained in each procedure. The relative cost in testing shall be defined as the ratio of  $C$  to  $c_1$ . That is,

$$RC = E(\gamma R) + \frac{c_2}{c_1} \gamma I.$$

We shall let  $RC_{opt}$  denote the optimum relative cost in testing that corresponds to the  $k_{(min)}$  obtained for a given prevalence rate. There being no explicit expression for  $RC$ , the values  $k_{(min)}$  and  $RC_{opt}$  cannot be obtained analytically and are computed using a computer search algorithm.

For selected prevalence rates in the interval (0.001, 0.22), Table 1 presents minimum relative costs in testing,  $RC_{opt}$ , and the corresponding minimum group size,  $k_{(min)}$ , for 0, 1 and 2-step Dorfman-Sterrett procedures assuming that significance levels at stage 1 and 2 of testing are both 0.01. The results indicate that the two-step Dorfman-Sterrett procedure performs better than both the Dorfman procedure and the single-step Dorfman-Sterrett procedure for all the prevalence rates considered.

Similarly, Table 2 presents minimum relative costs in testing,  $RC_{opt}$ , and the corresponding minimum group size,  $k_{(min)}$ , for 0, 1 and 2-step Dorfman-Sterrett assuming that the level of significance at stage 1 is smaller than that at stage 2. In this case, the two-step procedure performs better than the zero- and single-step procedure for prevalence rates lower than 15%. For prevalence rates greater than 15%, the single-step procedure performs best.

In Table 3, the minimum relative costs in testing,  $RC_{opt}$ , and the corresponding minimum group size,  $k_{(min)}$ , for 0, 1 and 2-step Dorfman-Sterrett are presented assuming that the level of significance at stage 1 is greater than that at stage 2. The single-step Dorfman-Sterrett procedure performs better than the zero- and two-step procedure for all prevalence rates considered.

**Table 2:** The minimum cost  $C$  relative to  $c_1$  assuming that  $\frac{\Delta}{\sigma} \rightarrow \infty$  and  $p$  is small with  $\frac{c_1}{c_2} = 0.1$ ,  $\alpha_1 = 0.01$  and  $\alpha_2 = 0.05$  for  $\gamma$ -step Dorfman-Sterrett procedure with  $f = 100$ .

$p$	$\gamma = 0$		$\gamma = 1$		$\gamma = 2$	
	$k_{(min)}$	$RC_{opt}$	$k_{(min)}$	$RC_{opt}$	$k_{(min)}$	$RC_{opt}$
0.001	32	8.32980	36	9.192741	37	7.494578
0.002	23	10.93669	27	11.279743	28	9.441341
0.005	14	16.10659	18	15.467835	19	13.528757
0.008	11	19.83578	14	18.530043	15	16.607432
0.010	10	21.93203	13	20.284007	14	18.390383
0.020	7	30.17625	9	27.360293	10	25.643953
0.050	5	46.77519	6	42.440443	6	41.197337
0.080	4	58.70306	5	54.246021	5	53.345213
0.100	3	65.05698	4	60.968410	4	60.407058
0.120	3	70.99505	4	67.435733	4	67.008607
0.150	3	79.90214	3	76.220792	3	76.292861
0.180	2	87.64718	3	84.156617	3	84.429016
0.200	2	91.60520	3	89.447167	3	89.853119
0.220	2	95.56322	3	94.737717	3	95.277222
0.250	2	101.50025	3	102.673542	3	103.413377

**Table 3:** The minimum cost  $C$  relative to  $c_1$  assuming that  $\frac{\Delta}{\sigma} \rightarrow \infty$  and  $p$  is small with  $\frac{c_1}{c_2} = 0.1$ ,  $\alpha_1 = 0.05$  and  $\alpha_2 = 0.01$  for  $\gamma$ -step Dorfman-Sterrett procedure with  $f = 100$ .

$p$	$\gamma = 0$		$\gamma = 1$		$\gamma = 2$	
	$k_{(min)}$	$RC_{opt}$	$k_{(min)}$	$RC_{opt}$	$k_{(min)}$	$RC_{opt}$
0.001	32	12.17168	33	8.024422	47	10.92051
0.002	23	14.72428	25	10.010886	33	12.93920
0.005	15	19.79775	17	14.116281	21	17.01488
0.008	11	23.45730	14	17.177988	16	20.03053
0.010	10	25.50624	12	18.944068	14	21.76975
0.020	7	33.59176	9	26.080625	10	28.80597
0.050	5	49.75505	6	41.391025	6	43.91881
0.080	4	61.40468	5	53.382764	5	55.58896
0.100	3	67.83816	4	60.174282	4	62.55073
0.120	3	73.53779	4	66.739667	4	68.78262
0.150	3	82.08723	3	75.638158	3	77.95574
0.180	2	90.20158	3	83.648123	3	85.62793
0.200	2	94.00120	3	88.988100	3	90.74273
0.220	2	97.80082	3	94.328077	3	95.85752
0.250	2	103.50025	3	102.338042	3	103.52971



## 6. Conclusion

This paper has discussed statistical properties of the restricted Dorfman-Sterrett designs assuming that the power of the test at the initial stage of screening is a function of group size and the number of NC factors in the group. We have used orthogonal fractional factorial plans of the type given by Plackett and Burman (1946) to study the procedure. Cost functions, which are linear functions of expected total number of incorrect decisions and the expected number of tests, have been derived giving a basis for comparing procedures of interest.

The Dorfman-Sterrett procedure introduces a stopping rule to the plan suggested by Sterrett (1957) and implemented by others in the literature (Manene, 2007; Manene, Rotich and Simwa, 2002; Manene and Simwa, 2004; Odhiambo and Manene, 1987; O'Geran and Wynn, 1992; Patel and Manene, 1987; Patel and Manene, 1992). Other studies, unlike Johnson et al. (1989), have interrogated the performance of these designs using the orthogonal fractional factorial plans assuming equal group sizes and with error in observations. We have in this study generalized the results of Johnson et al. (1989) and discussed the performance of their plan under the testing of hypothesis paradigm. Our results can also be viewed as extensions to the results of Achia, Manene and Ottieno (2010), allowing for misclassification of factors or groups. Our findings support the use of the single-step Dorfman-Sterrett procedure when assuming error in observation. This finding may be important in practical situations where numerous reversions to the Dorfman screening may not be very appealing.

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