Natural Convection with Localized Heating and Cooling on Opposite Vertical Walls in an Enclosure

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Abstract—This is a numerical study of turbulent natural convection flow in a rectangular enclosure. The flow of heat is one form of Newtonian motion. We consider natural convection in a three dimensional rectangular enclosure in the form of a room with heaters placed on opposite walls and two windows each on the adjacent opposite walls. The study of free convection in the past five decades focused mainly on two different simple geometries, first the single isothermal or constant flux vertical plate in isothermal stagnant surrounding, secondly, the enclosed rectangular cavity with heated floor and cooled walls. There has also been much emphasis on Reyleigh number as opposed to the Reynolds number used in this study. To analyze the flow and heat transfer rates, a complete set of non-dimensionalized equations governing Newtonian fluid and boundary conditions are recast into vector potential to eliminate the need for solving the continuity equations. A Boussinesq fluid motion in a three dimensional cavity is considered. The governing equations with the boundary conditions are descritized using three point central difference approximations for a non-uniform mesh. The resulting finite difference equations are solved using Matlab simulation software. The solutions are presented at the Reynolds number 5500, with Prandtl number 0.71. The results are presented on graphs to show velocity profiles and temperature distribution in the room. The room is divided into a number of regions with those near the heaters having high temperatures as those near the windows have low temperatures. Convective currents caused by buoyancy forces play a major role in determining the velocity profiles in the room.

Keywords—Heat Convection in an Enclosure; Heat Transfer; Natural Convection in a Rectangular Enclosure.

I. INTRODUCTION

A fluid is a substance whose constituent particles continuously change their positions relative to one another when subjected to energy change or force. In this concept a fluid is perceived to flow. Motion in fluids could be caused by different forces, one such force is heat. Natural convection is caused by local buoyancy differences brought about by the presence of hot and cold body surfaces. Natural convection occurs when the air in a space is changed with outdoor air without the use of mechanical systems, such as a fan. Most often natural convection is assured through operable windows but it can also be achieved through temperature and pressure differences between spaces. The flow pattern in fluids could also be turbulent or laminar. Laminar flow is where we have a steady flow while turbulent flow is irregular where various quantities show random variation with time and space co-ordinates.

In an enclosure, convection could be shown by movement of air due to buoyancy up and around the room subject to differential heating encountered in many practically important engineering problems. They include thermal insulation of buildings, heat transfer through open windows, refrigeration, etc. The study of free convection in the past five decades focused mainly on two different simple geometries, first the single isothermal or constant flux vertical plate in isothermal stagnant surrounding, secondly, the enclosed rectangular cavity with heated floor and cooled walls [Mynet & Duxbury, 1974; Olson & Glickman, 1991].

The motion of this study is to investigate heat distribution and velocity profiles brought about by heating two opposite vertical walls and cooling the other two vertical walls in an enclosure. A turbulent flow is considered and hence the use of high Reynolds number. Due to the elliptic nature of internal buoyant flows, natural convection flow requires a complete solution of the Navier-Stokes equation. Most of the fluid dynamics problems involving natural convection could be solved by experimental, theoretical and numerical methods. High Reynolds number natural convection has been identified as one of the main subjects for future research, extension to three dimensional calculations for practical and real life situations is considered of prime importance.

The objective of this numerical study is to investigate;

- The temperature distribution in the enclosure caused by non-uniformity in the boundary temperature as a result of heating the vertical opposite walls and cooling the two opposite adjacent vertical walls of the rectangular enclosure.
- Velocity profiles of air in the enclosure caused by the resulting buoyancy effects and energy gained from heater.

II. Literature Review

A number of researchers modeled a number of turbulent flows near vertical isothermal walls in their investigations. These include studies by Cebecii & Smith (1974), Leornardi (1984), de Vahl Davies (1986), Plumb & Kennedy (1984), Lankhorst (1991), Sigey (1999), Kipng'eno Joel (2006), among others. This led to a better understanding of turbulence in natural convection boundary layers.

Recent studies were undertaken by Sigey et al., (2004) who investigated in detail turbulent flow in a three dimensional enclosure in the form of a room with a convectional heater built into one of the walls and having a window in the same wall. Kipng'eno (2006), also studied turbulent natural convection with localized heating at the bottom wall (the floor) and two windows each on the vertical opposite walls of a rectangular enclosure.

Sigey et al., (2011) also studied buoyancy driven free convection turbulent heat transfer in an enclosure. They investigated a three-dimensional enclosure in form of a rectangular enclosure containing a convectional heater built into one wall and having a window in the same wall. The heater is located below the window and the other remaining walls are insulated. The results were that the enclosure is stratified into three regions: a cold upper region, a hot region in the area between the heater and the window and a warm lower region. Sheremet (2011) also studied mathematical simulation of conjugate turbulent natural convection in an enclosure with local heat source.

Much work has been done but for even distribution of heat, a different approach that would increase efficiency especially in buildings is to be considered. In this study, the positioning of the heaters and simulation of high Reynolds heat transfer by Matlab simulation software creates a simplifying difference from the previous use of reyleigh number.

III. Mathematical Formulation

Turbulent natural convection in an enclosure arising from localized heating and cooling is encountered in a number of practical occurrences, such as the use of convectional heaters in rooms. The temperature and velocity fields in a room depend mainly on temperature of any heat source and window as well as air ventilation flow rate.

This is a numerical study of turbulent flow in a rectangular enclosure. The flow of heat is one form of Newtonian motion. We consider natural convection in a three dimensional rectangular enclosure in the form of a room with heaters placed on two opposite vertical walls and windows on the other two vertical walls.

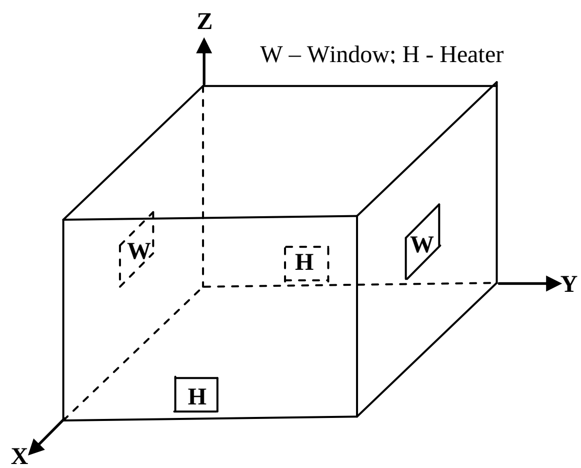


Figure 1: Model of a Room

Figure 1 above is a model of a room with heaters placed on two opposite walls (on the X-Z planes) and windows on the other two walls (Y-Z planes). The floor and the top planes are taken to be insulated (X-Y planes).

IV. Governing Equations

The fundamental equations of fluid dynamics are based on the following universal laws of conservation; conservation of mass (continuity), momentum and energy.

Consider a fluid in which the density ρ is a function of position x_j (j=1, 2, 3), let u_j (j=1, 2, 3) denote the components of the velocity. Hence in writing the various equations, use of the notation of Cartesian tensors with the usual summation convection is applied Currie (1974).

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_j}(\rho u_j) = 0 \tag{1}$$

$$\frac{\partial}{\partial x_t}(\rho u_i) + \frac{\partial}{\partial x_j}(\partial u_j u_i) = -\frac{\partial \rho}{\partial x_i} + \rho g_i + \frac{\partial}{\partial x_j}\left[\mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) + \mu_s \delta_{ij}\frac{\partial u_k}{\partial x_k}\right] \tag{2}$$

$$\frac{\partial}{\partial t}\left(\rho c_p T\right) + \frac{\partial}{\partial x_j}\left(\rho c_p u_j T\right) = \frac{\partial}{\partial x_j}\left(\lambda \frac{\partial T}{\partial x_j}\right) + \beta T\left(\frac{\partial p}{\partial t} + \frac{\partial u_j p}{\partial x_j}\right) + \Phi \tag{3}$$

The equations (1), (2) and (3) for the continuity, mean momentum and mean energy respectively are however in the most general form, hence non-dimensionalization, Reynolds

(1976) decomposition, Boussinesq (1903) approximation and appropriate boundary conditions together with these equations are necessary for determining the velocity component u_j and the fluid properties such as the density ρ, pressure P and the temperature T which apply turbulent flows. The final set of equations then become;

$$\frac{\partial U_j}{\partial t} + U_j \frac{\partial U_i}{\partial x_j} = -\frac{M_1}{\rho_R}\frac{\partial \rho}{\partial x_j} - M_2 \Theta g_i + M_3 \frac{\partial^2 U_i}{\partial {x_j}^2} \quad (4)$$

$$\frac{\partial \Theta}{\partial t} + 2U_j \frac{\partial \Theta}{\partial x_j} = T_2 \frac{\partial^2 \Theta}{\partial {x_j}^2} \quad (5)$$

Equations (4) for momentum and (5) for energy are descritized using three point central difference approximation as shown below;

V. METHOD OF SOLUTION

Equation (4) with respective substitutions of M_1, M_2 and M_3 can be written as;

$$\frac{\partial u}{\partial t} = \frac{1}{\mathrm{Re}}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) - v\left(\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y}\right) - \frac{Eu}{\rho_R}\frac{\partial \rho}{\partial y} - \frac{\Theta g_i}{(Fr)^2} \quad (6)$$

Using Taylor's central difference approximation;

$$u_{i,j}^{n+1} = \left(1 - \frac{4k}{h^2 \mathrm{Re}}\right)u_{i,j}^n + \left(\frac{k}{h^2 \mathrm{Re}} - \frac{vk}{2h}\right)u_{i+1,j}^n + \left(\frac{k}{h^2 \mathrm{Re}} + \frac{vk}{2h}\right)u_{i-1,j}^n + \left(\frac{k}{h^2 \mathrm{Re}} - \frac{vk}{2h}\right)u_{i,j+1}^n + \left(\frac{k}{h^2 \mathrm{Re}} + \frac{vk}{2h}\right)u_{i,j-1}^n - \left(\frac{Eu}{\rho_R} + \frac{\Theta g_i}{(Fr)^2}\right) \quad (7)$$

Equation (5) with respective substitutions T_2 can also be written as;

$$\frac{\partial \Theta}{\partial t} = \frac{1}{\mathrm{Pr}\,\mathrm{Re}}\left(\frac{\partial^2 \Theta}{\partial x^2} + \frac{\partial^2 \Theta}{\partial y^2}\right) - 2v\left(\frac{\partial \Theta}{\partial x} + \frac{\partial \Theta}{\partial y}\right) \quad (8)$$

Using Taylor's central difference approximation;

$$\frac{\Theta_{i,j}^{n+1} - \Theta_{i,j}^n}{k} = \frac{1}{\mathrm{Pr}\,\mathrm{Re}\,h^2}\left\{\left(\Theta_{i+1,j}^n - 2\Theta_{i,j}^n + \Theta_{i-1,j}^n\right) + \left(\Theta_{i,j+1}^n - 2\Theta_{i,j}^n + \Theta_{i,j-1}^n\right)\right\} - \frac{v}{h}\left\{\left(\Theta_{i+1,j}^n - \Theta_{i-1,j}^n\right) + \left(\Theta_{i,j+1}^n - \Theta_{i,j-1}^n\right)\right\} \quad (9)$$

Equations (7) and (9) together with boundary conditions are used in a code with Matlab software to simulate the expected results.

VI. RESULTS AND DISCUSSIONS

In an enclosure, each boundary is considered impermeable and capable only of motion in its own place (static motion). This implies that the normal component of velocity at each boundary is zero. $u(x, y, z, 0) = \sin \pi x + \cos \pi y$. In the simulation of the graphical results the following parameters were used; Reynolds number = 5,500 Prandtl number = 0.71, Euler number = 2.71828 and Frounde number = 0.01.

The Reynolds Number (Re) is an important parameter of forced viscous flow. If the Reynolds number of the system is small, the viscous force is predominant and the effects of viscosity are important, while if it is large, the inertia force is predominant and the viscous effects are only important in the narrow layer near the solid boundary. The Prandtl Number (Pr) represent the ratio of momentum diffusivity (v) to thermal diffusivity (k) that gives the measure of effectiveness of energy and momentum transport of diffusion in thermal and velocity boundary layer respectively.

6.1. *Temperature Distribution*

Figure 2(a), and figure 4(a) shows the vertical temperature fields in the planes x = 0.1 and x = 0.9 respectively. Being close to the heaters there are higher temperatures in the lower regions where the heaters are placed. The temperature decreases as you move up towards the top of the enclosure as a result of the warm fluid rising and mixing up with the cold fluid from the window areas. The lowest temperatures are recorded on the walls containing the windows. It is warmest at these two planes in the enclosure.

Figure 3(a) shows the isotherms at the vertical plane x = 0.5. It is the region coinciding with the windows and equidistant from the two windows. At this plane, the temperatures are low. This is because of the distance from the heaters but there are higher temperatures at bottom wall of the enclosure due to the effect of the heaters. The walls here recorded the lowest temperatures due to the cooling from the windows.

The horizontal plane z = 0.5 is shown in figure 5 coinciding with the windows and close to the heaters, the plane records mixed temperatures with high temperatures on the region close to the heaters and low temperatures in the middle of the room due to the effect of the windows.

The room is divided into a number of regions with those near the heaters having high temperatures as those near the windows have low temperatures. Mixing of hot and cold fluid and turbulence makes the other areas warm. Natural convection also plays a key role in the variation of temperature distribution in the room.

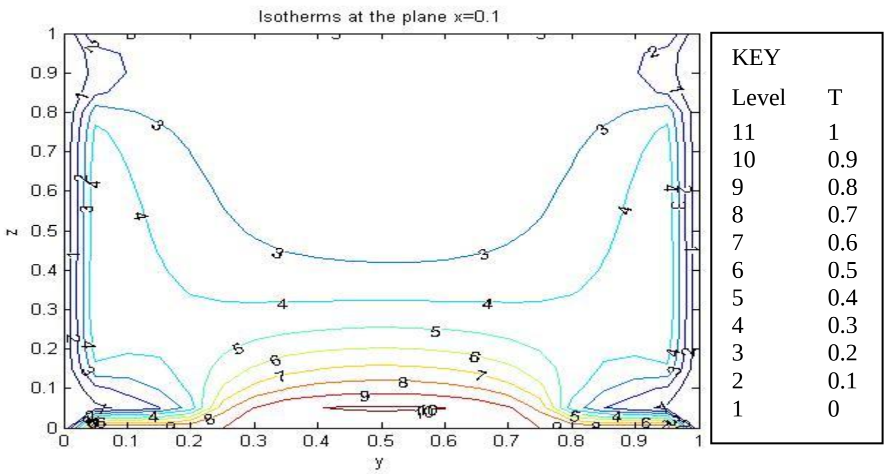


Figure 2(a): Isotherms at the Plane x = 0.1 for Re = 5500

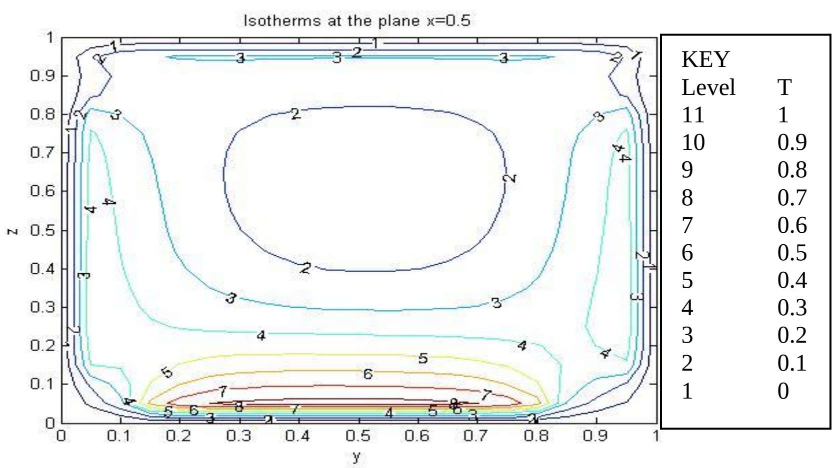


Figure 3(a): Isotherms at the Plane x = 0.5 for Re = 5500

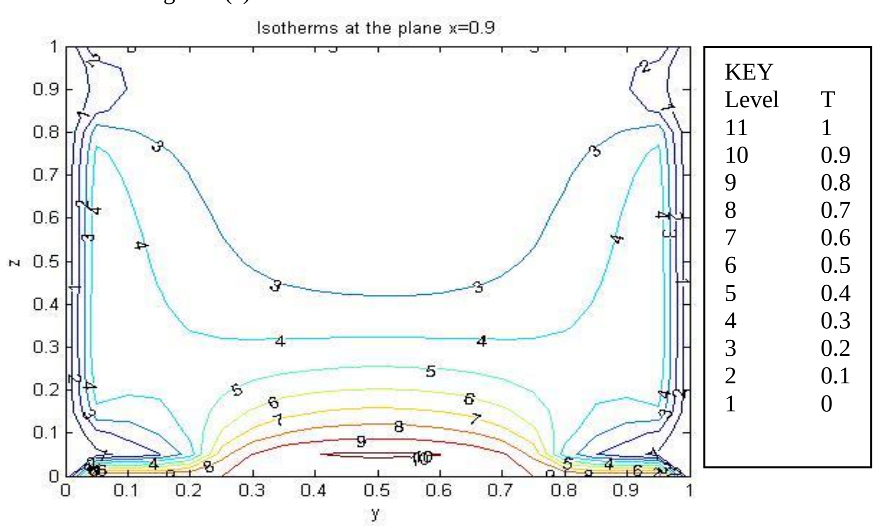
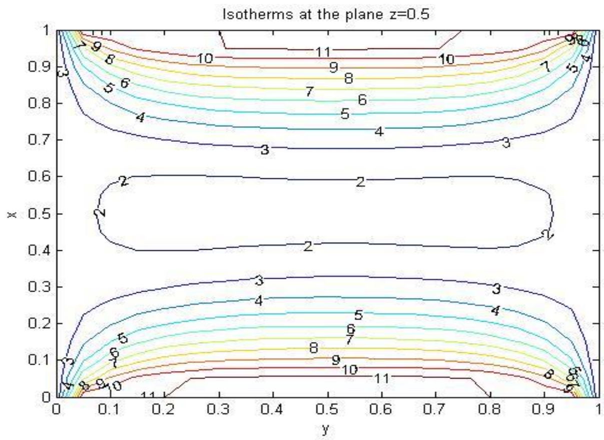


Figure 4(a): Isotherms at the Plane x = 0.9 for Re = 5500



KEY	
Level	T
11	1
10	0.9
9	0.8
8	0.7
7	0.6
6	0.5
5	0.4
4	0.3
3	0.2
2	0.1
1	0

Figure 5: Isotherms at the Plane z = 0.5 for Re = 5500

6.2. Velocity Flow Fields

The structure of the flow can easily be seen in the vector plots which have been selected from the Y-Z plane. The vector plots are in the planes x = 0.1, x = 0.5 and x = 0.9 (see figure 2(b), figure 3(b) and figure 4(b)).

In figure 3(b) where the plane is at x = 0.5, the structure of the flow shows two circular motions in different directions; one in the clockwise direction (to the right) while the other (to the left) is anti-clockwise. This is due to the strong convective motion developed by the heating and high Reynolds number.

The warm fluid gains energy becomes less dense and gains in velocity resulting in an upward motion at the centre where the effect of the heaters is felt, while on the sides the cold fluid descends. This is due to buoyancy effects. The velocity of the descending fluid is strongest near the windows while the rising velocity is strongest near the heaters (at the bottom).

Figure 2(b) and figure 4(b) shows the flow patterns in the planes x = 0.1 and x = 0.9 respectively. They have similar patterns since they have the same temperature effects with more strength in upward motion since they are close to the source of heat. On the sides there is mixing up of warm and cold fluid resulting into slow movement of the fluid particles.

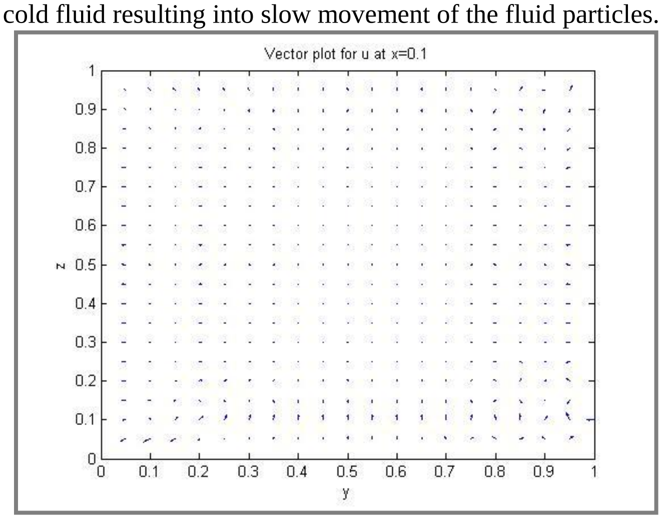


Figure 2(b): Velocity Vector Plot at the Plane x = 0.1 for Re = 5500

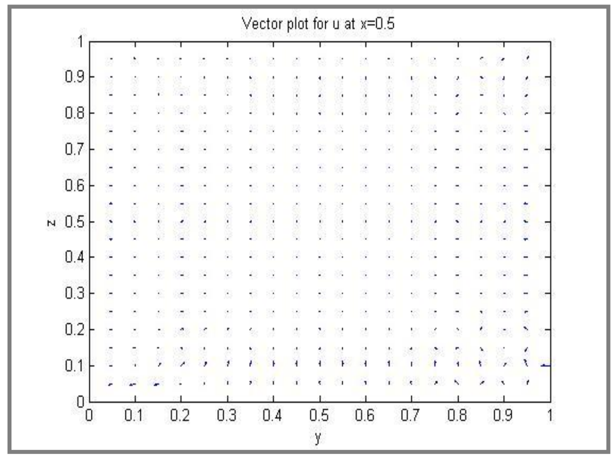


Figure 3(b): Velocity Vector Plot at the Plane x = 0.5 for Re = 5500

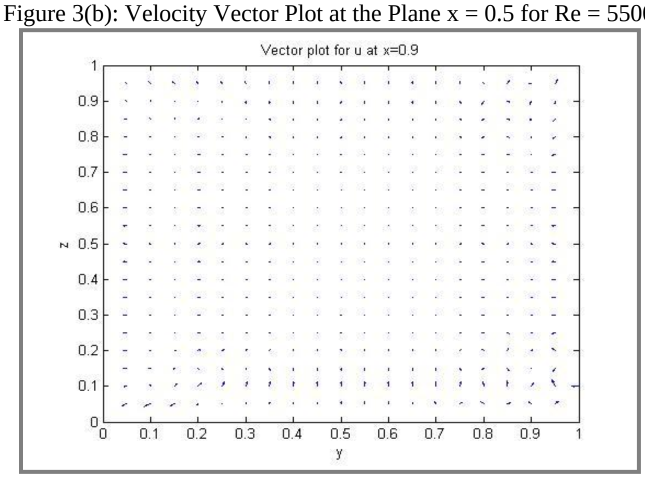


Figure 4(b): Velocity Vector Plot at the Plane x = 0.9 for Re = 5500

VII. CONCLUSION

In this study we investigate temperature distribution and velocity profiles in an enclosure brought about by heat transfer. Natural convection played a major role in the study. A three dimensional rectangular enclosure in the form of a room with heaters placed on opposite walls and two windows each on the adjacent opposite walls was considered for the purposes of the study. The flow was considered turbulent hence the solutions were obtained for Reynolds number 5,500 and Prandtl number 0.71.

To analyze the flow and heat transfer rates, a complete set of non-dimensionalized equations governing Newtonian fluid and boundary conditions are recast into vector potential to eliminate the need for solving the continuity equations. A Boussinesq fluid motion in a three dimensional cavity is considered. The governing equations with the boundary conditions are descritized using three point central difference approximations for a non-uniform mesh. The resulting finite difference equations are then solved using Matlab simulation software.

The results show that turbulent natural convection plays a major role in temperature distribution in an enclosure. The room is divided into a number of regions with those near the heaters having high temperatures as those near the windows have low temperatures. This helps in keeping some of the items at stated temperatures. Convective currents caused by buoyancy forces due to temperature differences between room air and air in contact with hot and cold surfaces also play a major role in determining the velocity profiles. The study has a wide range of applications in engineering. A good example is in an oven where direct heating may not be required.

VIII. Recommendations / Future Work

- There is need to investigate forced convection and its effect on temperature distribution and velocity profiles in a room.
- A more practical approach in engineering would reduce theoretical assumptions made in this work.
- Investigation of buoyancy driven natural convection in non-rectangular enclosures.

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