IMPACT OF LONGEVITY RISK WITH BEST FIT MORTALITY FORECASTING MODEL

BY:

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AN ACTUARIAL RESEARCH PROJECT SUBMITTED IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE AWARD OF THE DEGREE OF MASTER OF SCIENCE IN ACTUARIAL SCIENCE, SCHOOL OF MATHEMATICS, UNIVERSITY OF NAIROBI

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DECLARATION

This project is my original work and has not been presented for a degree to any University.

Signed..................................... Date.......................................

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This research has been submitted for examination with my approval as the University supervisor:

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A RESEARCH REPORT SUBMITTED IN PARTIAL FULFILMENT FOR THE REQUIREMENTS OF THE AWARD OF MASTER OF SCIENCE IN ACTUARIAL SCIENCE DEGREE OF THE UNIVERSITY OF NAIROBI.

DEDICATION

This thesis is dedicated to my family, especially Peter Rono, and friends.

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ABSTRACT

The data used in this research is from the Human Mortality database for the United States for the period 2000 - 2009. We first begin by fitting linear, Makeham, cubic smoothing spline and Lee Cater models to the data to establish which model best fits the data. Parameters are estimated using linear regression, graphical method and Singular Value Decomposition methods for the linear, Makeham and Lee Carter models respectively. By use of goodness of fit tests conducted using chi square, Cramer Von Mises criterion, Kolmogorov Smirnov and Anderson Darling tests, a best fit model to the data is established. Using standard error measures, best forecast model for the data is identified among the forecast methods, Cubic smoothing spline, ARIMA (Auto Regressive Integrated Moving Average) models with and without drift. Mortality rates based on the best fit model are forecasted to five year horizon using best forecasting method to the data. These rates are used to check if there has been a decline in mortality rates over the years. Impact of mortality decline (longevity risk) is then illustrated by calculation of Actuarial Present Values (APVs) of whole life annuity of a 60 year old male over the years.

Contents

CHAPTER ONE

1 INTRODUCTION

1.1 Background

Longevity risk is the risk of living beyond the expected time frame or estimated and is one of the prominent insurance risk. Unlike other risks in the market like inflation, taxes, lifestyles, fluctuation interest rates,etc. which are relatively stable over time and can be modeled deterministically using reasonable assumptions, longevity risk is dynamic and systematic and can not be diversified. Recent studies and research have indicates an increase in life expectancy and hence a reduction in the mortality rates. In fact, the future improvements of life expectancy are difficult to be predicted accurately but the general opinion from the experts tends to be that the trend of longevity improvements is certain, but deviations to both sides are possible. Improvement in mortality rates can be attributed majorly due to increased literacy and improved education level where people are more enlightened about their health issues and become focused on importance of dieting, regular exercise and non indulgence in drug abuse behaviors. Improved and accessibility to health facilities is also an important contributory factor for improvement of mortality rates.

Gains in life expectancy or improvements in mortality have been under estimated in most industrialized and developing countries in the past. In fact, (Oeppon,Vaupel 2002) in their report show striking evidences that the record life expectancy has been rising nearly three months per year in the past 160 years, and the asserted ceilings on life expectancy were surpassed

repeatedly in the past century.The revision of mortality projections leads to the requirement of additional funds to support increasing liabilities in terms of payments to retirees. The potential need for additional funds poses a serious threat to annuity providers and pension funds who have the responsibility of provision of survival benefits. This has been one of the major hindrance or slow down of research to investigate the trend in mortality and improve mortality rates.

Longevity risk has always been present but its significance has gained enormously in recent times. This is due to risk less yields in the financial markets are limited or have fallen considerably in many countries leaving only little funds for the provision of additional reserves. For assessment of longevity risk, maybe risk management for solvency purposes, a stochastic mortality modeling rather than a deterministic model is required since the mortality rates have been dynamic in the recent times.

However, the resulting risk capital charges are only credible if best estimate mortality is projected adequately. If best estimate liabilities are systematically underestimated, like an inappropriate mortality projection, the risk capital charges will never guarantee the desired risk safety level.

Thus mortality projections do not help in quantifying longevity risk but an adequate mortality projection can significantly reduce longevity risk. Therefore mortality projections are extremely important for practical actuarial work, hence insurance companies, pension funds and social security institutions should consistently look to improve their projections to minimize chances of solvency.

This research project is aimed at modeling mortality rates using an appropriate model to be able to forecast mortality. Further the project will

illustrate, using actuarial present values of whole life annuities at retirement ages, that there exists a decline in mortality over the years and how this decline affects pension schemes and annuity providers in terms of longevity risk.

Most researchers apply the use of one model to particular data under investigation, but this is not the case with this research where the appropriate model is determined by fitting commonly used models, linear, Makeham and Lee - Carter. Moreover the forecasting method is determined after evaluation of three models, cubic smoothing spline, auto regressive integrated moving average with and without drift, to be able to forecast with one that offers the best fit.

This research project is organized as follows; chapter two reviews the studies on mortality rate improvements previously done. Chapter three outlines the methodology of the study, that is how the model is fitted to the mortality data. The findings and results of the fit, investigation on how best the model fits and forecasting of mortality rates are discussed in chapter four. Chapter five concludes the research project and recommendations for further research studies are stated in this chapter.

1.2   Statement of the problem

The mortality data is examined to come up with a mortality model that best fits the data for forecast of mortality rates into the future. Using accurate mortality rates derived from the best fit model, investigate the impact of longevity risk to pension schemes and annuity providers.

1.3 Objectives

1.3.1 General Objective

The general objective of the study is to show the impact of longevity risk to pension schemes and annuity providers, using mortality rates derived or estimated from an appropriate mortality forecasting model that best fits mortality data under investigation.

1.3.2 Specific Objectives

The specific objectives of the study are;

(i) Fitting a mortality model that best fits the data.

(ii) Selection of an appropriate forecasting method.

(iii) Forecast mortality rates derived from best fit mortality model

(iii) Show the impact of longevity risk to pension schemes and annuity providers.

1.4   Justification/Significance of the study

Pension funds are the principal sources of retirement income for millions of people in the world. In Kenya, retirement income accounts for 68% of the total income of retirees. Annuity providers and pension schemes provides a life time income to pensioners with an expectation of a certain life expectancy. When retirees live beyond the expectations of these institutions, there is a risk of straining to the pension schemes and annuity providers to continue with the provision of this pension and may lead to their insolvency or fall. To avoid insolvency, some of these institutions may provide pension or annuities to retirees that is not proportional to their purchase prices, using random exaggerated mortality rates. The use of random mortality rates has led to this research which has an aim to examine mortality data to come up with a best fit mortality forecasting model . With these accurate mortality rates, we can be able to price appropriately longevity risk and hence provide annuities and pension to retirees for the value of their purchase price and at the same time making sure annuity providers do not lose money leading to insolvency.

CHAPTER TWO

2   LITERATURE REVIEW

Mortality data are often characterized by the presence of some outlier data. In particular, in the case of older ages, the high variability can be due to the small number of survivors in the population. This represents a common problem when estimating mortality rates for groups aged 90 and more. Techniques of smoothing/graduation have been implemented to avoid this shortage of data, because the heavy variance at older ages influences the fitting of mortality models (Delwarde et al 2007).

In the course of time graduation of data has been both extended and narrowed in its application, according to the particular interests of different writers. Th principles of graduation are of importance to a proper appraisal of the behavior of actuarial statistics and to the development of a sound judgment in the selection of a statistical basis for dealing with practical problems. Our concern is mainly with the principles and methods of adjusting a set of observed rates to provide a suitable basis for actuarial calculations of a practical nature.Further, the process of fitting mortality trend curves with the view to project into the future is our main concern.

Actuaries routinely face the problem of estimating a smoothed series of rates from crude rate data. The smoothed rates must estimate the probability of some outcome, such as death, for each point along an axis representing increasing age, length of service, or some other variable. Typically, there is some data for each point but prediction can be improved by making use of the surrounding observations. Techniques for performing such smoothing, or graduation, include graphical methods, curve fitting,

moving averages, Kings osculatory method, Whittaker-Henderson, as well as spline methods of graduations among others.

The methods of graduation that was imposed on the crude data vary from one researcher to the other. The methods may be based on three graduation methods. Firstly, Graphical methods where a hypothetical curve is drawn by inspection through the area bounded by the confidence intervals. With this method, there is clearly an element of objectivity judgment and use is made of experience as to what a curve of death may look like. Moreover, the method is subject to under graduation which is characterized by lack of smoothness.Secondly, Finite differences graduation methods depend on the principle that the standard error of the weighted mean of two or more independent or imperfectly correlated random errors is less than the sum of the correspondingly weighted individual standard errors. This is not usually the case. Lastly, curve fitting methods of graduation, based on mathematical formula, that are based on the assumption that the underlying values have a particular mathematical form whose parameters maybe estimated from the observed values. The mathematical formula were favored in mortality work because unlike summation formula they did not leave the rates for terminal ages to be dealt with by other and somewhat arbitrary means and they enabled prior experience of the natural shape of the force of mortality urve to be taken into account. In this research project, our main focus will be on the curve fitting methods of graduation by a mathematical formula and specifically cubic spline method of graduation. This section reviews the research works on graduation methods done previously.

The earlier statistical methods employed a two step procedure towards graduation. First, they calculated raw mortality rates as a ratio of the

number of deaths to the exposure. Second, they found a smooth set of graduated rates close to the raw mortality rates; this step is known as graduation. These early methods are are illustrated in details in books by (Benjamin, Pollard 1993) and (London 1985). The first step supposedly made suitable adjustments to account for the actuarial studies not being simple binomial experiments. The latter due to not only migration but also due to insured population dynamics. I say supposedly as contrary to the intent behind some of the exposure calculations the raw mortality rates turned out to be statistically biased and asymptotically inconsistent estimates of the underlying mortality rates, (Elveback 1958), (Breslow,Crowley 1974) and (Hoem 1984). More importantly violation of the likelihood principle forced them to make unrealistic assumptions on migration and resort to non-standard reasoning. A more direct approach to calculating raw mortality rates is via maximum likelihood (ML) estimation (Broffitt 1984). By adhering to the likelihood principle, ML estimation avoids the above mentioned problems in exposure calculations. Moreover, it indirectly derives the correct exposure but by subscribing to a sound statistical reasoning. This is skillfully expressed by (Boom 1984) in discussing (Broffitt 1984): This is intuitively superior to the concept of exposure familiar to us in the actuarial estimation/Balduccis assumption combination, since the awkwardness of having to expose the already dead still further to the risk of death is avoided.

When the data from a given population are scanty but are known or suspected to come from an experience similar to that for which a standard graduated table already exists, it may be possible to use this standard table as a base curve for graduating the new data. Any one of the standard graduation methods can be used for this purpose (Benjamin, Pollard 1986).

16

Since the standard table is believed to be similar to that underlying the experience, no value of the ratio should vary too much from unity. With this in mind, the choice of the standard table is important since any special feature in its graduation will be reproduced, even exaggerated, in the graduation of the new data. This method of graduation by reference to a standard table is not considered often as it is not always possible to find a suitable standard table, so that even if the constants in the graduation formula are chosen properly, the adherence of the results to the rough data is satisfactory.

Six point Lagrangian interpolation formula (Elandt,1998) provides good approximations for adult mortality but it is less accurate for early childhood ages. However the application of Lagrangian formula is limited to the ages less than seventy five hence it is not a viable graduation method at this point as our main aim is to smooth mortality data available and forecast for older ages. Bayesian method of graduation is another method that has been employed by some of the actuaries across the world. Bayesian statistics provide a coherent method for combining prior information and current observation. Also, parameters are viewed as being uncertain and hence this approach requires the quantification of the uncertainty in the form of a prior probability distribution (Hickman, Miller, 1977). The most persistent impediment to the application of the Bayesian methods is the difficulty that arises in specifying a prior distribution. (Kimeldorf, Jones, 1977) managed this problem by using conjugate multinormal distributions for the prior and the sampling distributions. That is, the solution involved defining classes of matrices from which a covariance matrix of the prior distribution might be conveniently selected. An improvement of this problem was developed by operating in a new metric. That is, by

transforming the observations and the prior distribution by application of the arc sine transformation, the graduator does not have to specify individual variances for the prior distribution. Instead, more easily interpreted past sample sizes are specified (Hickman, Miller, 1977).

Benjamin Gompertz argued that on physiological grounds the intensity of mortality gained equal proportions in equal intervals of age, giving rise to an increasing force of mortality (Gompertz,1825). Makeham(1860) made a development on Gompertz law by introduction of a constant as well as the exponentially increasing component of the force of mortality to reflect the division of causes of death into two kinds, those due to chance and those due to deterioration. Makeham's formula was found to give satisfactory adherence to data for a number of experiences in the late 19th and early 20th centuries, and several standard tables were graduated by its use. This method has led to convenient calculation of monetary functions that its adoption has been justified at the expense of some departure from the degree of adherence to data that would otherwise be required. it has been found increasingly difficult, however, to obtain satisfactory graduations by this simple formula and more complicated formula have therefore been developed. Curves allied to Makeham were introduced in the early years of the 20th century when it became evident that changes in basic age pattern of mortality meant that it was rarely possible to graduate successfully by Makeham's formula. The main changes were a substantial improvement in mortality at the younger ages and a relative 'heaping up' of mortality at middle age. Allied formula were tried without any very great success since a single curve will not fit the whole range of the table. This arises form the fact that the observed mortality experience is heterogeneous and mixes generations.

King's osculatory interpolation was introduced by George King in 1914 to reduce the effects of age misstatement (preferences for even ages or multiples of five) and provide mild graduation. This method was used to produce English Life Tables 7 to 10. The method is applicable for intervals of five years of age and requires the pivotal value and osculatory interpolation to be computed (Rose, Zailan, 2011). Kings method can be summarized as the exposed to risk, also applied directly to crude mortality rates, being grouped into quinquennial age groups, then a pivotal exposed to risk value is calculated for the central age of each group. Graduated exposed to risk values at the remaining ages are then found by osculatory interpolation and finally graduated mortality rates are found by division. Osculatory interpolation overcomes the discontinuity difficulty associated with ordinary interpolation. King's pivotal value formula gives the value of a single central ordinate in terms of sums of equally spaced ordinates and its derivation commences with consideration of a third degree polynomial. This formula is the same as the well known Gauss backward formula as far as second differences. The osculatory effect is obtained by changing the third difference term. This method assumes that the underlying curve can be represented adequately by a cubic, at least over sections of its length. The graduated rates at the intermediary ages calculated by means of osculatory are not very smooth, owing to the variation in the crude rates and the limited graduation power of the method (Benjamin,Pollard, 1986). This leads to further thought of a better method of graduation that will take into account this problem.

(Schuette, 1978) did a variation of the Whittaker-Henderson technique of graduation proposed in 1914, which uses squared deviations for the terms relating to fit and squares of differences in both directions, by replacing the

squares of absolute values in the measures of both fit and smoothness and finding the solution via linear programming. Standing in the way of practical use of Schuette's method are lack of appreciation for the rationale and perceived computational difficulties. Recent research works in parametric programming has made it possible to streamline the computations so that Schuette's problem can be solved completely as quickly as one can generate solutions to the Whittaker-Henderson problem for a few different parameter values. Schuette's graduation technique has been extended to generalization to bivariate graduation(Portnay, 1994). Portnay suggests that two dimensional data can be smoothed by minimizing a combination of the sum of weighted absolute deviations for initial estimates and the sums of absolute values of differences in the two directions. Further, the techniques of parametric programming make it feasible to explore the difference results obtained by varying the emphasis given to fit versus smoothness. Portnay found that measuring fit by absolute deviation rather than by squared deviation results in a graduation that is less influenced by outliers. A single graduation by this method corresponds in a natural way to a median estimate: by changing the fit measure we can obtain say 25th and 75th percentiles. Another one of the major problems of Whittaker-Henderson graduation method is the amount of subjectivity that enters into the determination of the degree of smoothness required. That is, Whittaker left the analysis of how smooth the graduated series should be, choosing smoothing coefficients, to the intuition of the analyst. In a bid to rectify this problem, (Giesecke,1981) investigated the use of chi square statistic to set the smoothing coefficients when using the Whittaker-Henderson graduation procedure. In theory, a Whittaker-Henderson solution using an analyst-set smoothing coefficient

can yield better predictions than a Whittaker-Henderson where the smoothing coefficient is iteratively set to yield a 50th percentile chi square. If by chance an ungraduated series is already smooth, the chi square iterations would enhance peaks and valleys more than they should be. If the ungraduated series is ragged, the 50th percentile graduation may not be smooth enough. It is remarkable then that 50th percentile graduation does as well as the graduation with intuitively set coefficients. He concludes by stating that at the very least, it can be said that the 50th percentile solution provides a good initial solution.

Although osculatory interpolation formula were known last century, increasing interest has been shown in them in recent years. Attention has been focused on a particular kind of osculatory polynomials known as spline functions. Spline functions are obtained by joining together a sequence of polynomial arcs, the polynomials being chosen in such a way that derivatives up to and including the order one less than the degree of the polynomial used are continuous everywhere (Benjamin, Hollard 1986). One of the advantages of the spline functions is that one can avoid the marked undulatory behavior (curve being smooth in a local sense) commonly encountered when a single polynomial is fitted exactly through a large number of data points. At the same time, greater smoothness is obtained than with traditional interpolation procedures, which gives rise to discontinuities in the first derivative of the interpolating function. The degree of the spline is selected. Usually, the degree is assumed to be as small as possible. Typically a second degree spline is tested (That is, a quadratic regression) first, if fit is not good enough, a cubic or higher order regression can be tested (Bruce et al, 2013). Penalized splines functions is part of the spline family of functions. P splines introduces a penalty for

21

lack of smoothness and by adjusting the penalty, we can balance between the fit to the data and smoothness. Higher penalties produce poorer fit but are much more smoother, whereas no penalty produces a curve that fits the data well. That is, fits every data point but very rough (CMI, 2005). P splines have an edge effect, that is to say, that the final trend or slope of mortality improvements may not be stable and can produce unreasonable values. When a spline curve is fitted to a set of data, the resulting curve fitted through the points is not unique. Hence it is clear that a particular spline function chosen arbitrarily is unlikely to produce a satisfactory interpolation (Benjamin, Pollard, 1986).Therefore, a constraint must be introduced to eliminate the undulation problem. (Greville 1974) has shown that the determination of the smoothest interpolating spline for given data points takes a particularly simple form in the case of a cubic spine. It was seen that when a natural cubic spline is fitted through points, any undulatory behavior witnessed before disappears (Benjamin, Pollard, 1986).

CHAPTER THREE

3   METHODOLOGY

3.1   The Data

The data used in this research project is obtained from the Human
Mortality Database. The period of the mortality data used was from 2000
to 2009 for the United States of America. Both raw data and for the life
tables data was utilized accordingly. The assumptions made with the data
are; it is similar to the Kenyan data.; there are no migration in the
population after age sixty five and that age of an individual is defined as
the completed age in years. (Depoid,1973) notes that death statistics are
more reliable than census data in countries where a birth registration
system has been in operation for a long time. Hence the use of mortality
data rather than census data.

The empirical mortality rate at age x in calendar year, Y can be calculated
as;
$$q_x^Y = \frac{l_x^Y - l_{x+1}^Y}{l_x^Y} = 1 - \frac{l_{x+1}^Y}{l_x^Y} = \frac{d_x^Y}{l_x^Y}$$
where;

l_x^Y = Number of people aged x living at the beginning of year, Y

d_x^Y = Number of people dying between ages x and $x+1$ in calendar year, Y.

3.2   Statistical models for the force of mortality

3.2.1   Makeham model

(Gompertz,1825) assumed that the force of mortality μ_x at age x was an exponential function of age,

$$\mu_x = Bc^x \qquad (3.1)$$

An extra parameter was added to this model to take into account the force of accidental death, assumed to be a constant independent of age (Makeham, 1860), and obtained the model

$$\mu_x = A + Bc^x \qquad (3.2)$$

(Bowers et al, 1997) states that if the lifetime, X, of a person is Gompertz distributed and the time, Y, to a fatal accident has an exponential distribution, and the random variables X and Y are independent, then the minimum of X and Y has a Makeham distribution. Based on this, (Marshall, Olkin, 1997) mention that Makeham model can be considered as a shock model. Gompertz and Makeham models were used for over a century mostly by insurance companies to project the mortality tables beyond age 65.

3.2.2 Lee - Carter Model

The LeeCarter model(1992) has become widely used and there have been various extensions and modifications proposed to attain a broader interpretation and to capture the main features of the dynamics of the mortality intensity.The Lee Carter methodology is a milestone in the mortality projections actuarial literature. The model describes the logarithm of the observed mortality rate for age x and year t, $m_{x,t}$ as the

sum of an age specific component, α_x, that is independent of time and another component that is the product of time-varying parameter, K_t, reflecting the general level of mortality and an age-specific component, β_x, that represents how mortality at each age varies when the general level of mortality changes (D'Amato et al, 2011).

$$\ln m_{x,t} = \alpha_x + \beta_x K_t + \epsilon_{x,t} \tag{3.3}$$

where

α_x describes the general age shape of the age specific death rates, $m_{x,t}$

β_x coefficients describes the tendency of mortality at age x to change when the general level of mortality K_t changes

K_t is an index that describes the variation in the level of mortality to t

$\epsilon_{x,t}$ is an error term and it is normally distributed

(Girosi, King,2007) showed that to produce mortality forecasts, Lee and Carter assume that β_x remains constant over time and they use forecasts of $\hat{K}_t$ from a standard univariate time series.

Due to the bilinear multiplicative construct $(\beta_x K_t)$ present in in the Lee Carter model, there is a clear identifiability problem that is traditionally resolved by ensuring that these parameters satisfy a pair of specified constraints, given by

$$\sum_t K_t = 0, \sum_x \beta_x = 1$$

3.2.3 Other Models

Earlier models for mortality rates were deterministic, and include the Gompertz curve (1825), which provides a satisfactory fit to adult mortality, but overestimates death rates at ages greater than 80. The Perks (1932) logistic model (a generalization of the Gompertz curve) gives a relatively good fit to mortality rates over the entire adult range. The Heligman and Pollard (1980) curve provides a relatively good fit to mortality rates over all ages. Progresses in computational algorithms help handling complexes models, and the number of parameters is no longer an issue. (Pitacco, 2004), (Haberman, 2004) and (Boe 1998) reviews earlier contributions to mortality forecast and recent models. Some recent studies shown that the mortality rates predicted from the classic parametric formulas were erratic (Stoto 1983; Murphy 1995). Stochastic models seem more appealing, because they associate a confidence error to each estimate. In 1992, Lee and Carter presented a stochastic model, based on a factor analytic approach, to fit and predict mortality rates for United States. Since then, because of its simplicity and relatively good performance, the Lee-Carter (LC) model has been widely used for demographic and actuarial applications in various countries. For example, the LC model was used for Japan (Wilmoth 1993), the seven most economically developed countries (G7) (Tuljapurkar et al. 2000), Austria (Carter, Prskawetz 2001), Australia (Booth et al. 2002), Belgium (Brouhns et al. 2002), and the Nordic countries (Lundstrm, Qvist 2004; Koissi et al. 2006b).

Logistic model developed by an actuary, (Perks, 1932) has not received much attention as the Gompertz and Makeham models. With this model, the force of mortality at age x is given by the four parameter function

26

$$\mu_x = \frac{A + Be^{\mu x}}{1 + Ce^{\mu x}} \tag{3.4}$$

By assuming that the parameter $A = 0$ in the logistic model, (Beard, 1963) obtained the 3-parameter model

$$\mu_x = \frac{Be^{\mu x}}{1 + Ce^{\mu x}} \tag{3.5}$$

(Kannisto, 1992) used the simple 2-parameter model

$$\mu_x = \frac{Be^{\mu x}}{1 + Be^{\mu x}} \tag{3.6}$$

Note that the Gompertz ($A = 0; C = 0$), Makeham ($C = 0$), Beard ($A = 0$) and Kannisto ($A = 0; B = C$) models are all special cases of the logistic model; therefore, after successfully fitting the logistic and another one of the above models, a likelihood ratio test could be performed to check whether a model more parsimonious than the logistic one would be appropriate considering its smaller number of parameters.

(Beard, 1971) showed that the logistic model can arise in a heterogeneous population where each member has a Makeham force of mortality and where the parameter B varies among individuals according to a gamma distribution. This Makeham-gamma model is a frailty model. (Thatcher et al, 1998) also mention two ways in which the logistic model could karise from stochactic processes. The logistic model can also be considered as a shock model: if the lifetime X follows a Beard distribution, the time to an accident Y is exponentially distributed and the random variables X and Y are independent, then $\min(X; Y)$ follows a Perks distribution.

3.3 Estimation of parameters

3.3.1 Makeham Model

Let q_x represent the probability that a person aged x dies before attaining age $x + 1$. In terms of μ_x, q_x equals (Bowers et al, 1997)

$$q_x = 1 - exp\left(-\int_0^1 \mu_y dy\right)$$

$$= 1 - e^{-\mu_x}$$

The complement of q_x, $p_x = 1 - q_x$ represents the probability that a person aged x survives at least to age $x + 1$.

We begin by transforming the Makeham model into a linear equation using the log linear transformation, and fit the curve using linear regression.

$$(1 - q_x) = exp - \left(\int_0^1 \mu_{x+t} dt\right)$$

$$= \exp^{-\mu_x}$$

$$\mu_x = -ln(1 - q_x)$$

Taking natural logarithm on both sides we get;

$$ln\mu_x = ln(-ln(1 - q_x))$$

Thus the log linear transformation of the force of mortality would be given by

$$ln\mu_x = ln(-ln(1 - q_x))$$

We use the linear equation

$$y = \alpha + \beta x \qquad (3.7)$$

, where $y = lnq_x$, $x =$ the specific ages from age 20 to age 100, to transform Makeham equation into a linear form. The parameters α and β are then estimated using the linear regression;

$$\hat{\beta} = \frac{\sum xy - n\bar{x}\bar{y}}{\sum x^2 - n\bar{x}}$$

.

$$\hat{\alpha} = \bar{y} - n\bar{x}$$

where $\bar{y}$ and $\bar{x}$ are means of y and x respectively. The linear equation, $y = \alpha + \beta x$, is then used to estimate the new value of y. A trend line is then fitted using Microsoft excel software between the ages of 20 and 100.

The new value of y, y^1, and hence q_x^1, that has been estimated is used for log linear transformation. Thus the log linear transformation of the force of mortality would be given by;

$$ln\mu_x = ln(-ln(1 - q_x^1))$$

That is,

$$y = ln(-ln(1 - q_x^1))$$

A trend line is then fitted using excel between ages 20 and 100. This is because on investigation of the log of the force of mortality between those ages, it was identified that a straight line can be used to estimate the data points. This is consistent with the Gompertz model that is accurate for middle ages.

The parameters of the Makeham model, B and c, are estimated by sketching the straight line and noting its slope and y - intercept. Suppose the trend line is given by the equation

$$y = mx + d$$

, then

$$\hat{c} = e^m$$

and

$$\hat{B} = \frac{log\hat{c}}{(\hat{c} - 1)}e^d$$

The term A was obtained by taking a representative younger age 30 and applying the formula;

$$\hat{A} = e^{-y_m} - e^{-y_g}$$

where

y_m = height of the graduating curve at a representative younger age 30 in the experience

and

y_g = height of the straight line

Makehams' survival rates are then calculated as;

$$\hat{p}_x = \exp\left[-\hat{A} - \frac{\hat{B}(\hat{c} - 1)}{log\hat{c}}\hat{c}^x\right]$$

Therefore,

$$\hat{q}_x = 1 - \exp\left[-\hat{A} - \frac{\hat{B}(\hat{c} - 1)}{log\hat{c}}\hat{c}^x\right] \tag{3.8}$$

3.3.2   Lee - Carter Model

For the Lee - Carter model, Lee and Carter used Singular Values Decomposition (SVD) of the matrix $Z_{x,t} = [ln(m_{x,t}) - \hat{\alpha}_x]$ to obtain estimates of β_x and K_t (Lawson,Hanson, 1974): where

$$\hat{\alpha}_x = \frac{1}{n} \sum_{t=t_1}^{t_n} log(m_{x,t})$$

Applying singular value decomposition on the matrix, Z, we get;

$$SVD(Z) = USV'$$

where U represents the age component, V represents the time component and the S are the singular values which are arranged in a descending order.

The LC parameter $\hat{\beta}$ is derived from the first vector of age component matrix. That is,

$$\hat{\beta} = U_{1,1}U_{2,1}...U_{x,1} = U_{x,1}$$

Parameter K_t is derived from the first vector of time component multiplied by the first singular value. That is,

$$\hat{K}_t = S_1 * [V_{1,1}V_{2,1}...V_{t,1}]$$

But we need to adjust the parameters to fulfill LC model constraints that,

$$\sum_t K_t = 0$$

,

$$\sum_x \beta_x = 1$$

Hence we use quotient transformation to achieve this.

Parameter $\hat{\beta}$ becomes,

$$\hat{\beta} = \frac{1}{\sum_x} U_{x,1} * U_{x,1}$$

and $\hat{K}_t$ becomes,

$$\hat{K}_t = \sum_x U_{x,1} * S_1 * V_{t,1}$$

Central death rate is then given by;

$$m_{x,t} = e^{\hat{\alpha}_x + \hat{\beta}_x \hat{K}_t}$$

and since

$$m_x = -ln(1 - q_x)$$

then

$$q_x = 1 - e^{-m_x}$$

Hence mortality rate will be given by;

$$\hat{q}_x = 1 - e^{-\left(e^{\hat{\alpha}_x + \hat{\beta}_x \hat{K}_t}\right)} \tag{3.9}$$

The parameters of the LC model can also be estimated using the Weighted Least Squares (WLS) suggested by (Wilmoth, 1993):

Minimize $\sum_{x=1}^{X} \sum_{t=1}^{T} W_{x,t} \left[ln(m_{x,t}) - \alpha_x - \beta_x \kappa_t\right]^2$

$W_{x,t} = d_{x,t}$, where $d_{x,t}$ is the observed number of deaths for age - group x in year t. If this does not hold, an alternative choice is $W_{x,t} = d_{x,t}/(\sum_x \sum_t d_{x,t})$ for $d_{x,t} \neq 0$ and $W_{x,t} = 0$ for $d_{x,t} = 0$, which gives $\sum_x \sum_t W_{x,t} = 1$ as expected.

(Wilmoth, 1993) and (Alho, 2000) proposed using Maximum Likelihood Estimation (MLE) to find the parameters in the LC model. This approach

is based on a Poisson approximation of the number of deaths, $D_{x,t}$ presented by (Brillinger, 1986):

$$D_{x,t} \hookrightarrow Poisson(m_{x,t}E_{x,t})$$

, where $m_{x,t} = exp(\alpha_x + \beta_x \kappa_t)$

The coefficients α_x, β_x and κ_t are estimated by maximizing the full log likelihood (Wilmoth, 1993)

$$l = \sum_{x=x_1}^{x_A} \sum_{t=t_1}^{t_1+T-1} [D_{x,t}ln(m_{x,t}E_{x,t}) - E_{x,t}exp(\alpha_x + \beta_x \kappa_t) - ln(D_{x,t}!)]$$

The loglikelihood function, denoted $l(\theta)$, defined as $l(\theta) = \ln L(\theta)$, equals

$$l(\theta) = \sum_{65}^{100} d_x ln q_x(\theta) + (l_x - d_x)ln p_x(\theta)$$

. We can find the MLE $\hat{\theta}$ numerically, either by maximizing directly the log-likelihood function $l(\theta)$ or by solving the system of equations

$$\frac{\delta l(\theta)}{\delta \theta_j} = 0, j = 1, ..., p,$$

where θ_j is the jth component of θ.

3.3.3 Other Models

For the logistic model, we find that

$$q_x = 1 - exp\left(-\int_x^{x+1} \frac{A + Be^{\mu y}}{1 + Ce}dy\right)$$

$$= 1 - \left(e^{-A}\right)\left(\frac{1+Ce^{\mu x}}{1+Ce^{\mu(x+1)}}\right)^{B-AC/C\mu}$$

For Beard model,

$$q_x = 1 - \left(\frac{1 + Ce^{\mu x}}{1 + Ce^{\mu(x+1)}}\right)^{B/C\mu}$$

While for Kannisto model,

$$q_x = 1 - \left(\frac{1 + Be^{\mu x}}{1 + Be^{\mu(x+1)}}\right)^{1/\mu}$$

And for the Makeham model we have,

$$q_x = 1 - exp\left[-A + (B/\mu)(1 - e^{\mu})e^{\mu x}\right]$$

In the Kannisto model, as x tends to ∞, q_x tends to $1 - e^{-1} = 0.632$, while it tends to $1 - e^{-B/C}$ in the Beard and logistic models. In the Gompertz and Makeham models, since μ_x is unbounded, q_x tends to 1.

Demographers often use the approximation

$$q_x \cong 1 - e^{-\mu_{x+1/2}}$$

obtained with the use of the mid point rule. That is, $\int_x^{x+1} \mu_y dy \cong \mu_{x+1/2}$. This approximation is very close to the true value. (Thatcher et al, 1998) showed that for Kannisto model, the relative difference between the exact value of q_x and the approximate value obtained using the mid point rule is minimal (only 0.03 percent for a male aged 80 and 0.0008 percent at age 100). The advantage of using the exact formula for q_x instead of the approximate one lies in the fact that an analytical expression can be found for the probability $_np_x$ that a person aged x survives to age $x + n$,

$$_np_x = exp\left[-\int_x^{x+n} \mu_y dy\right] = (e^{nA})\left(\frac{1 + Ce^{\mu x}}{1 + Ce^{\mu(x+n)}}\right)^{(B-AC)/C\mu}$$

for the logistic model and

$$_np_x = \left(\frac{1 + Be^{\mu x}}{1 + Be^{\mu(x+n)}}\right)^{1/\mu}$$

for Kannisto model.

The use of the mid-point rule

$$_{n}p_{x} = exp\left[-\int_{x}^{x+n}\mu_{y}dy\right] \cong exp\left[-\mu_{x+n/2}\right]$$

would become less accurate as n gets larger, while using the mid point rule over successive one year intervals

$$_{n}p_{x} = \prod_{i=0}^{n-1}p_{x+i} \cong \prod_{i=0}^{n-1}exp\left[-\mu_{x+i+1/2}\right]$$

would be more cumbersome.

Let θ be the vector of parameters of dimension p to be estimated, and $q_{x}(\theta)$ the value of q_{x} calculated with a parametric model. For a logistic model $p = 4$ and $\theta = (\mu, A, B, C)$; for the Beard model $p = 3$ and $\theta = (\mu, B, C)$; Makeham model $p = 3$ and $\theta = (\mu, A, B)$ while for the Kannisto model, $p = 2$ and $\theta = (\mu, B)$ We will use the method of maximum likelihood to estimate the parameter vector θ. The likelihood function equals

$$L(\theta) = \prod_{65}^{100}q_{x}(\theta)^{d_{x}}p_{x}(\theta)^{l_{x}-d_{x}}$$

where d_{x} is the number of deaths between ages x and $x + 1$ and l_{x} is the number of people living at age x.

3.4   Goodness of fit tests

3.4.1   Chi-Squared Test

To test whether the fit of a model is appropriate to the data, χ^2 goodness of fit test statistic for males and females was used and compared with the critical value of the χ^2 distribution with the appropriate number of degrees of freedom. The χ^2 test statistic is defined as

$$\chi^2 = \sum \frac{(q_x^{obs} - q_x^{exp})^2}{q_x^{exp}} \tag{3.10}$$

where q_s^{obs} is the observed mortality rate at age x, q_x^{exp} is the expected mortality rate at age x according to the parametric model used.

3.4.2 Cramer Von Mises Criterion

This method is used to test goodness of fit of a cumulative distribution function, F^*, compared to a given empirical distribution function, F_n, or for comparing two empirical distributions. It is also used as a part of other algorithms such as minimum distance estimation. This criterion was used to test goodness of fit. It is given by;

$$\omega^2 = \int_{-\infty}^{\infty} [F_n(x) - F^*(x)]^2 \, dF^*(x) \tag{3.11}$$

where, F^* = theoretical distribution and F_n = empirically observed distribution

3.4.3 Kolmogorov Smirnov (KS) two sample test

This method is used to calculate the largest absolute difference between the cumulative distributions for the observed data and the fitted distribution according to the equations;

$$D = max(D^+, D^-) \tag{3.12}$$

where,

$D^+ = \max \left(\frac{i}{n} - F(x) \right)$, i = 1...,n

$$D^- = \max \left(F(x) - \frac{(i-1)}{n} \right), \text{ i} = 1,...,\text{n}$$

where D = KS statistic, x = value of the ith point out of n data points and $F(x)$ = fitted cumulative distribution.

3.4.4 Anderson Darling Test

The goodness of fit test by Anderson darling test has the statistic;

$$W^2 = n \int_{-\infty}^{\infty} \frac{[F_n(x) - F(x)]^2}{F(x)[1 - F(x)]} dF(x) \tag{3.13}$$

where W^2 = AD statistic, n = number of data points, $F(x)$ = fitted cumulative distribution and $F_n(x)$ = cumulative distribution of the input data.

This test calculates the integral of the squared difference between the input data and the fitted distribution, with increased weighting for the tails of the distribution.

Normality tests were done using the Shapiro test for normality.

3.5 Description of graduation method

In this research project cubic spline graduation method will be used. Although osculatory interpolation formula were known last century, increasing interest has been shown in them in recent years. Attention has been focused on a particular kind of osculatory polynomials known as spline functions. Particularly, we focus on the cubic spline method of graduation. A spline of odd degree $2k - 1$ with knots at x_1, $x_2,...x_n$ is called a natural cubic spline if it is given in each of the two intervals $(-\infty, x_1)$ and

37

(x_n, ∞) by a polynomial of degree $k - 1$ or less. The polynomials of degree $k - 1$ in the two tails are not in general the same. The smoothest interpolation cubic spline through a given set of data points is linear before the first data point and linear after the final point. When natural cubic spline method is fitted through some points, any undulatory (curve being smooth in a local sense) behavior witnessed by use of quadratic fit through the same points disappears (Benjamin, Pollard, 1986). For the interval $(-\infty, x_1)$, it is linear and can be expressed in the form;

$$y = a_o + a_1 x, x < x_1 \tag{3.14}$$

At the point (x_1, y_1), the curve becomes cubic with the same first and second derivatives as the initial straight line. We can add the term $b_1(x - x_1)^3$ to obtain a suitable cubic term. We have,

$$y = a_o + a_1 x + b_1(x - x_1)^3, x_1 \leq x < x_2 \tag{3.15}$$

Where the first two derivatives of (3.15) are the same as in (3.14) above at the point (x_1, y_1) because the first two derivatives of $b_1(x - x_1)^3$ are the zero at that point. We may define

$$\phi_1(x) = \{0, x < x_1; (x - x_1)^3, x \geq x_1$$

Combining (3.14) and (3.15) we have,

$$y = a_o + a_1 x + b_1 \phi_1(x)$$

The cubic spline over the range $x_2 \leq x < x_3$ will be different from the cubic over the interval $x_1 \leq x < x_2$ although the two must have the same first and second derivatives at x_2. We may add $b_2(x - x_2)^3$ and define,

$$\phi_2(x) = \{0, x < x_2; (x - x_2)^3, x \geq x_2$$

38

Natural cubic spline for $x < x_3$ therefore can be written in single simple form as,

$$y = a_o + a_1 x + b_1 \phi_1(x) + b_2 \phi_2(x) \tag{3.16}$$

Hence the complete natural cubic spline passing through the n data points, valid for the whole range $(-\infty, \infty)$, can be written in the form;

$$y = a_o + a_1 x + \sum_{j=1}^{n} b_j \phi_j(x) \tag{3.17}$$

where

$$\phi_j(x) = \{0, x < x_j; (x - x_j)^3, x \geq x_j$$

For $x \geq x_n$, the natural cubic spline reduces to a straight line. This can only be true provided the cubic and quadratic terms in

$$\sum_{j=1}^{n} b_j (x^3 - 3x^2 x_j + 3x x_j^2 - x_j^3)$$

are zero. Hence the following constraints are imposed.

$$\sum_{j=1}^{n} b_j = \sum_{j=1}^{n} b_j x_j = 0$$

It follows that;

$$b_{n-1} = -\left\{ \frac{1}{x_n - x_{n-1}} \right\} \sum_{j=1}^{n-2} b_j (x_n - x_j)$$

$$b_n = \left\{ \frac{1}{x_n - x_{n-1}} \right\} \sum_{j=1}^{n-2} b_j (x_{n-1} - x_j)$$

and we deduce the following form for the natural cubic spline over the entire interval $(-\infty, \infty)$

$$y = a_o + a_1 x + \sum_{j=1}^{n-2} b_j \Phi_j(x) \tag{3.18}$$

where

$$\Phi_j(x) = \phi_j(x) - \left\{ \frac{x_n - x_j}{x_n - x_{n-1}} \right\} \phi_{n-1}(x) + \left\{ \frac{x_{n-1} - x_j}{x_n - x_{n-1}} \right\} \phi_n(x)$$

The use of k knots requires the estimation of k regression coefficients $(a_o, a_1, b_1, b_2, ..., b_{k-2})$. If k exceeds the number of data points,n, the system will be over determined. Therefore, k must be less than or equal to n. If $k = n$ and the knot ages coincide with the data ages, the spline function will pass through all the original data points and a perfect fit will be obtained, hence there will be no graduation. The use of a small number of knots will usually result in a bold smooth graduation and the effect of increasing the number is to improve adherence to data (possibly at the expense of smoothness).

3.6   Mortality forecast

The estimated mortality rates, q_x from the best fit model will be forecast by Cubic smoothing spline, Auto Regressive Integrated Moving Average (ARIMA) model, (p,d,q), with and without a drift. The values of p,d and q will be established by an automatic ARIMA function. The model which has the lowest AIC will be selected as the best fit for the forecast. The forecast will have a five year horizon.

CHAPTER FOUR

4 DATA ANALYSIS AND RESULTS

4.1 Linear model

The data for both the male and female lives were investigated for linearity. That is, checking if the mortality rates follows a linear function. This was done using log linear transformation and the graph below illustrates the results;
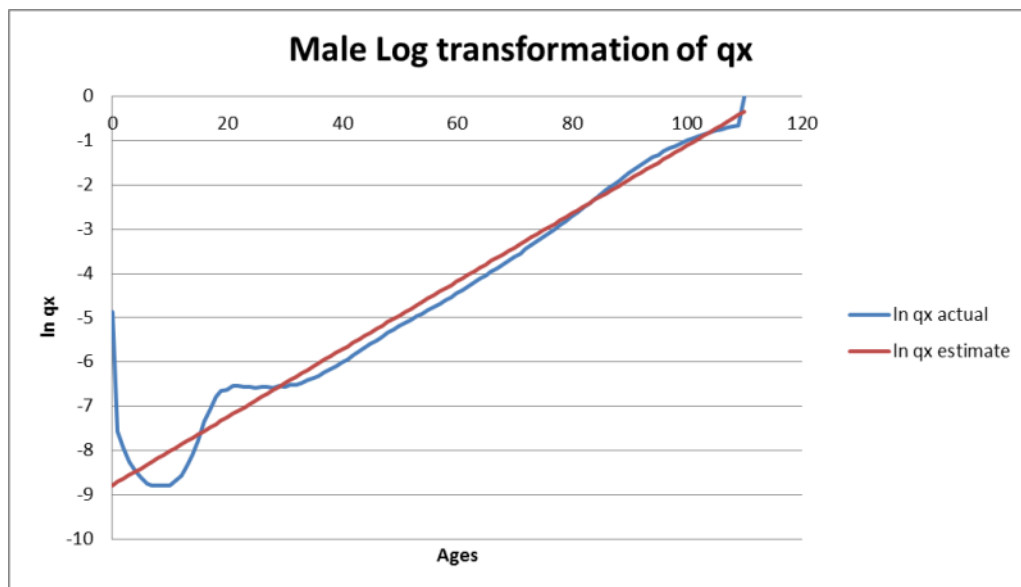
Figure 4.1
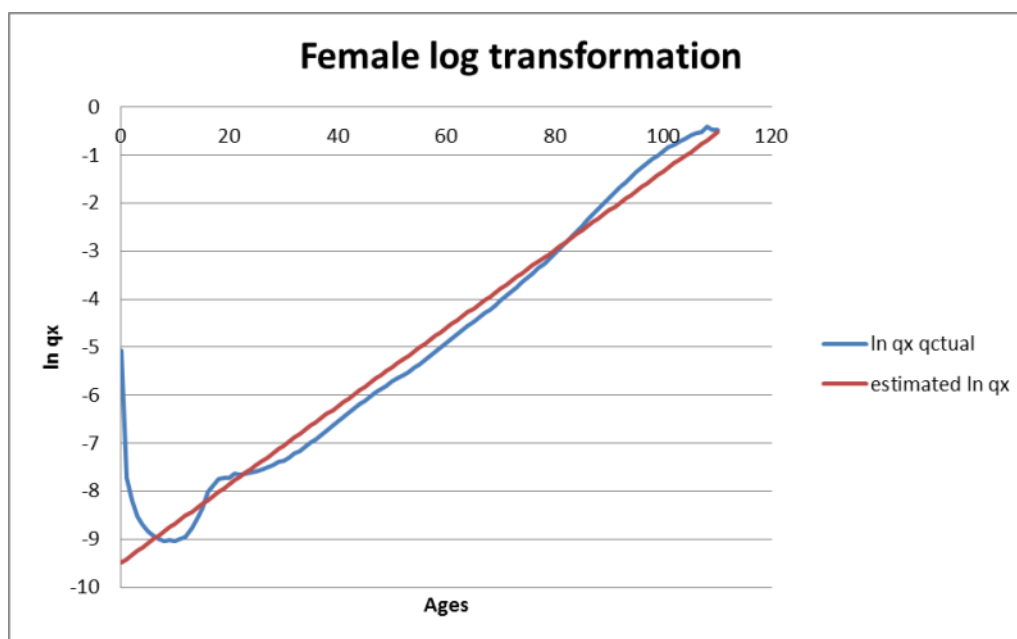
Figure 4.2

We are able to see from figures 4.1 and 4.2 that the mortality rates are increasing at the younger ages, form age 0 to 5 years. This may be attributed due to child disease, lack of proper care or ignorance and proneness of child to some diseases. Mortality rates start decreasing at about age 10, there is a dip, to about age 20 where it takes an upward turn.We can explain this upward turn by the lifestyle (e.g. fast and reckless driving, indulgence in drug abuse,etc) led by this age group.The female lives have less accident hump at the than the male lives due to controlled life style. Mortality trend reduces slightly and stabilizes to a linear trend in the middle ages from about age 40. From the investigations, mortality rates for both males and females are not linear. Hence the need to look for alternative models of fitting the data.

4.2   Makeham model

Model fit

Makeham model commences with fitting the Gompertz curve to middle ages. The trend line that resulted from the fit in excel software for male and female respectively was;

$$y = 0.0791x - 8.8613 \qquad\qquad (4.1\text{a})$$

$$y = 0.0832x - 9.5459 \qquad\qquad (4.1\text{b})$$

and R squared of 0.999049723 and 0.99950578 for male and female respectively. This means that 99.904% and 99.95% of male and female mortality rates respectively can be explained by the age, hence age affects mortality significantly.
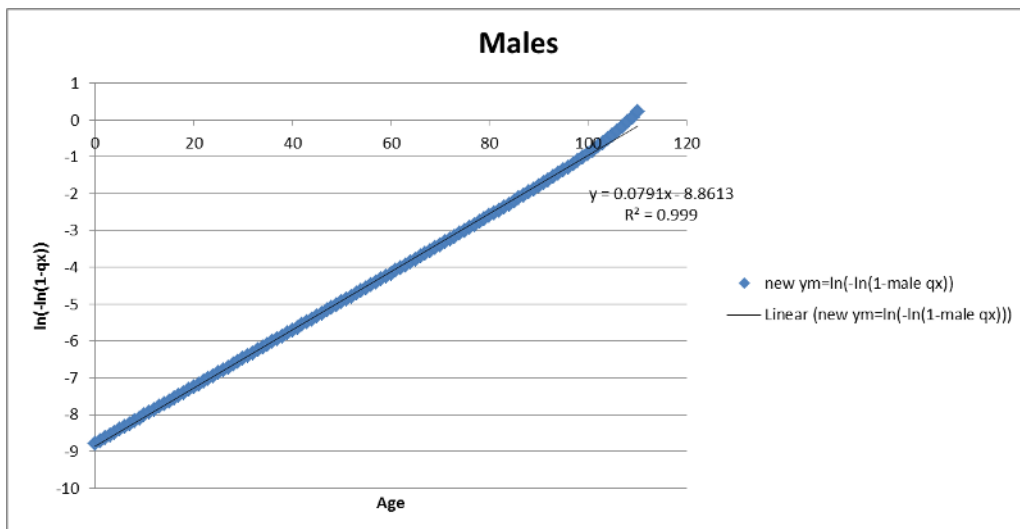
Figure 4.3
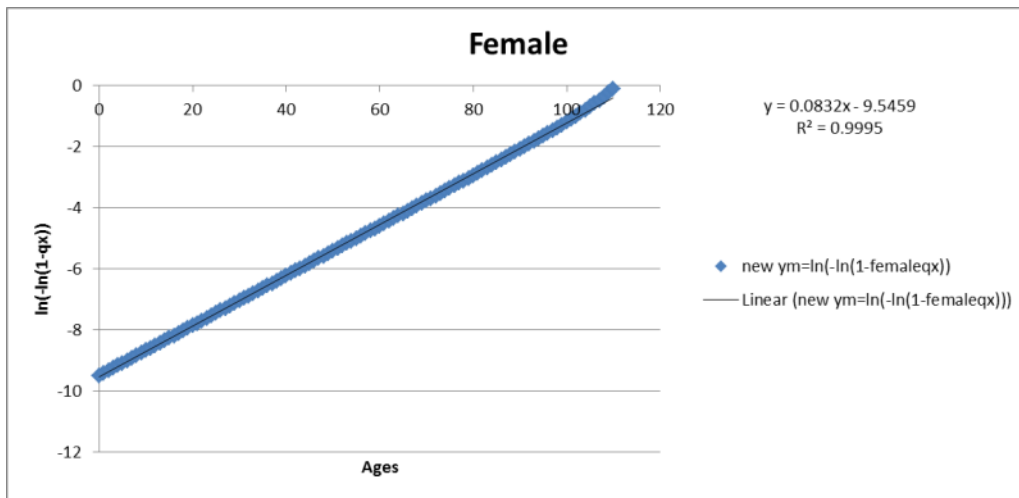
Figure 4.4

Figures 4.3 and 4.4 have a linear trend. That is, the line represents an asymptotic straight line. Parameters B and C are estimated from this line. Parameter A is estimated from this line and the graduating curve.

$$A_m = 0.010285469$$

$$A_f = 0.007459156$$

$$B_m = 0.000136239$$

$$B_f = 0.0000685578$$

$$C_m = 1.082295009$$

$$C_f = 1.08678545$$

where subscript m represents the male lives and subscript f represents the female lives.

Comparison of the actual and estimated mortality rates using the Makeham model for both male and female lives are shown in the graphs below;
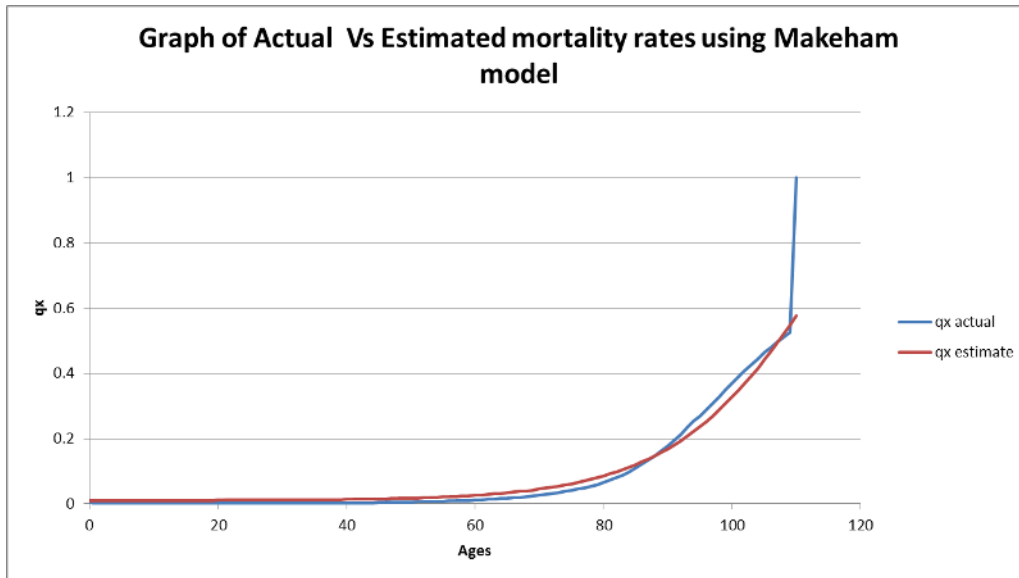
Figure 4.5
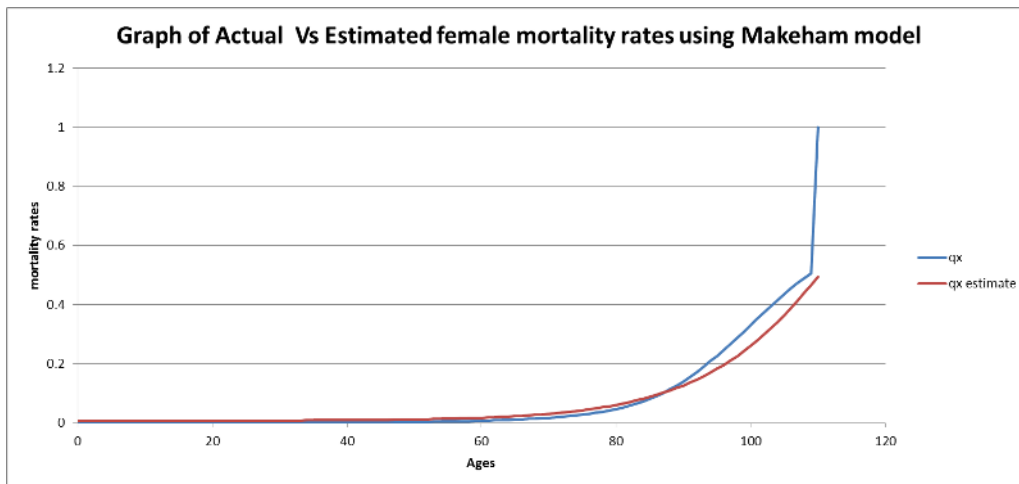
Figure 4.6

Figures 4.5 and 4.6 shows the fitted Makeham curve which shows slight variation in middle and older ages. The Makeham trend is similar to the actual data except in the advanced ages from age 100. The actual mortality

trend has a sharp bend assuming mortality rate of 1 from ages 110 onward, unlike the estimated mortality rates trend which is gradual throughout.

Model goodness of fit results

The P value = 1 for both male and female lives, as calculated using the chi square test in R programming indicates that the null hypothesis is not rejected. Hence the estimated mortality rates are similar to the observed ones. However due to the small values of the mortality rates used, there is a chance that the test is incorrect. This prompts the use of other tests of goodness.

The Cramer Von mises criterion test conducted in R shows that at 95% confidence interval,the P value is 0.08691309 and 0.09090909 for male and female lives respectively. This means that the null hypothesis is not rejected. That is, the estimated mortality rates are as distributed as the actual mortality rates.

Makeham is a good fit as indicated by the Kolmogorov Smirnov test conducted in R. For the male lives, the test showed that the D value is 0.027027 and P value is 0.9221, meaning that there is 92% chance that the highest difference between the fitted and the actual mortality rates is 0.027027. Similarly, for the female lives whose D value is 0.036036 and P value of 0.8658, makeham model is a good fit.

Tests for normality were conducted using Shapiro Wilk test. For both male and female lives, the population form which the data comes from, is not normally distributed.

4.3 Cubic spline model

Model fit

Cubic spline was conducted for the representative ages 20, 30, 40, 50, 60, 70, 80, 90 and 100 for both the male and female lives. The graphs below illustrates the mortality rates estimated using the model as compared to the actual rates.
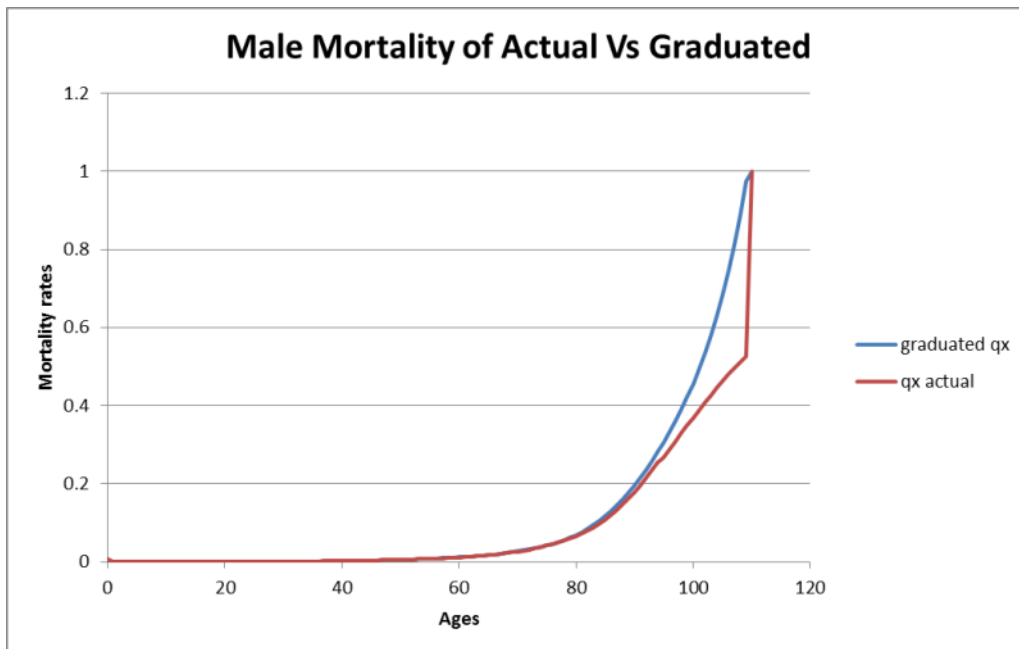
Figure 4.7
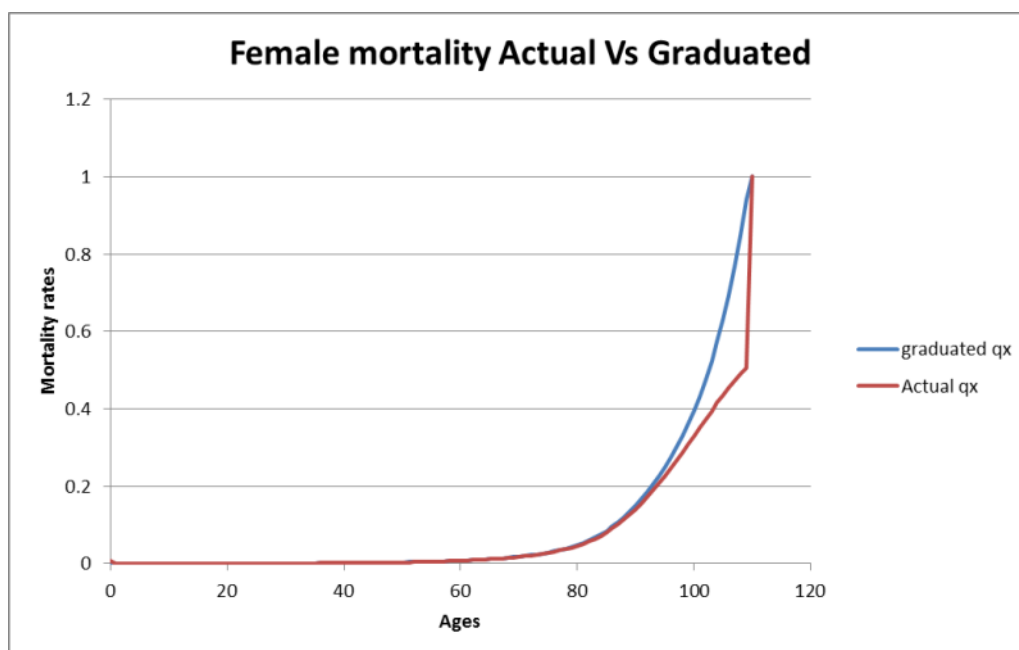
Figure 4.8

Figures 4.7 and 4.8 shows that cubic spline is a good fit especially in the earlier ages. There is a slight variation in the older ages for both male and female lives. The cubic spline curve is smooth from the younger ages to the maximum age of 110 at the mortality rate of 1 as opposed to the actual rates which has a sharp turn to have mortality rate of 1 at age 110. Hence , we can conclude from the figures that cubic spline has a better fit than the makeham model.

Goodness of fit of the Model

Similarly as in the Makeham model, the chi square test P value for both male and female lives is 1. This implies a good fit although it might not be correct given the small values involved. Other tests are used to verify the fit of goodness.

Cramer Von Mises criterion test indicates a P value of 0.7362637 for the male lives and 0.8631369 for the female lives. This implies that the

distribution of the graduated mortality rates is as the actual mortality rates. Further, the females lives graduated values are better estimated than the male lives.

The Kolmogorov Smirnov test indicates D value of 0.063063 and P value of 0.9801 for the female lives. This indicates that the graduated and the actual mortality rates are from the same distribution hence a good fit. Similarly, for the male lives, D value is 0.09009 and P value is 0.4062.

Anderson Darling test shows a p values of 0.20187 when there are ties and 0.19448 after adjustment for ties, for the male lives. Similarly, for the female lives, p values are 0.81405 when there are ties and 0.84942 after tie adjustment. The results imply that the null hypothesis (all samples come from a common population) is not rejected.

The population from which the data was obtained and the graduated data are not normally distributed as indicated by the Shapiro Wilk Test for normality. This was done on both male and female lives.

4.4   Lee Carter model

Parameter estimation

The Lee Carter model has three parameters to be estimated. Male lives data was used in the estimation of the parameters, α_x, β_x and K_t. Results of estimated parameters, goodness of fit tests and estimated mortality rates based on the model are shown in the appendix. The figures of the parameters are shown below;
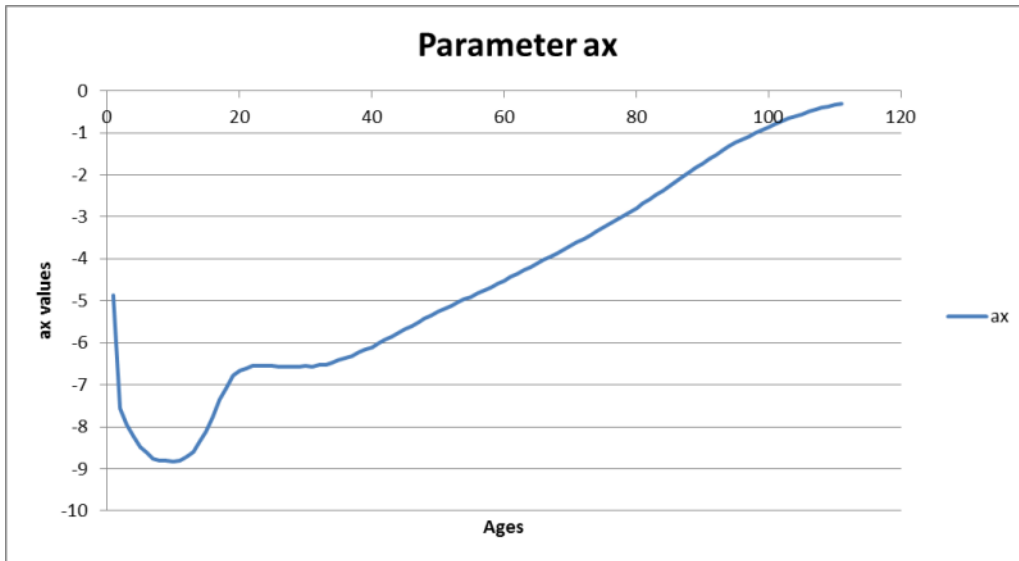
Figure 4.9

Parameter $\hat{\alpha}_x$ represents age specific pattern of mortality. From figure 4.9, we find that we have an upward trend of mortality in general. The younger ages have a high mortality which reduces gradually to the twenties where we have a mortality hump. After the hump, the figure shows that mortality increases gradually to the older ages.
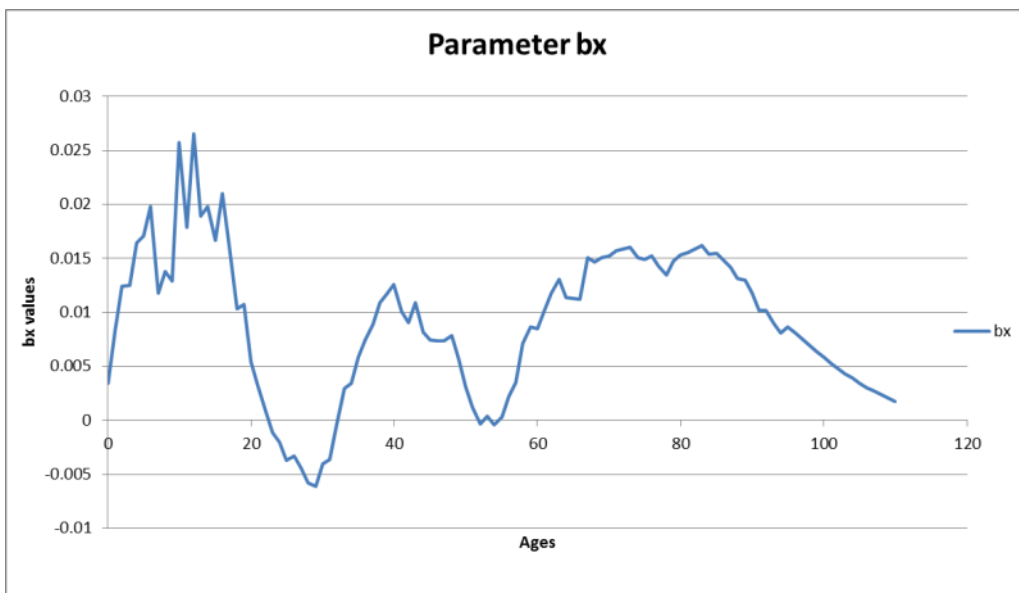
Figure 4.10

$\hat{\beta}_x$ represents decline in mortality at particular age x. The parameter describes the tendency of mortality at age x to change as the general level of mortality changes. When $\hat{\beta}_x$ is large for some age x, it means the death rate varies a lot than the general level of mortality and when it is small, death rate varies a little. From figure 4.10, the parameter has higher values in the younger ages than the older ages, which then declines steadily from about age 80.
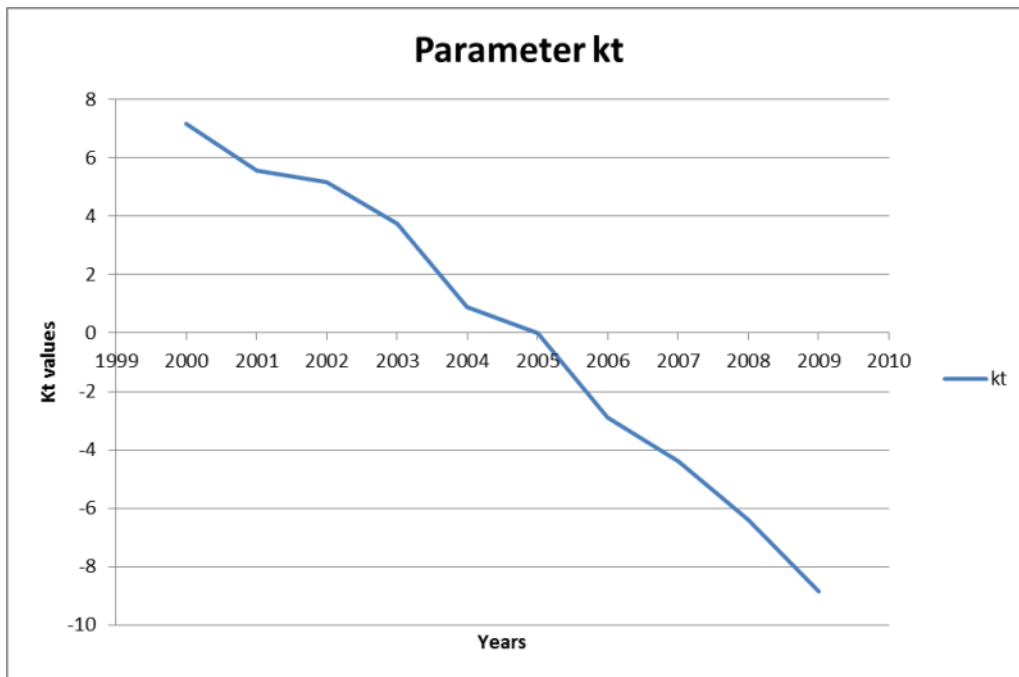
Figure 4.11

Index of general level of mortality is represented by parameter, K_t. It captures the main time trend on the logarithmic scale in death rates at all ages. Figure 4.11 indicates a general decrease in the level of mortality from year 2000 to 2009.

Model fit

51

Comparison between the actual mortality rates and the estimated ones from the Lee Carter model, using the above estimated parameters, are shown in the graph below.
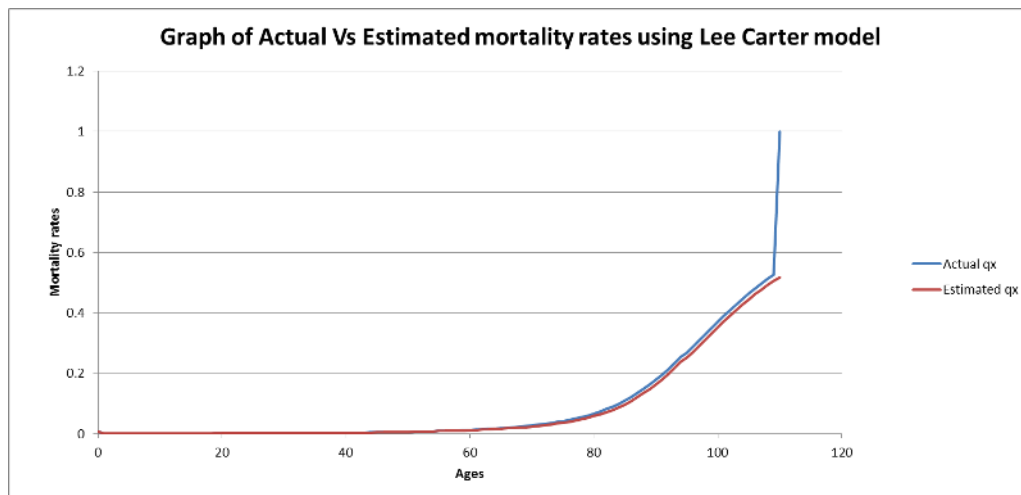
Figure 4.12

From the figure 4.12, we see that the actual mortality rates from ages 0 to 110 follows the mortality rates estimated using the Lee Carter model almost to perfection. In the Lee Carter model, mortality rates increase gradually even in the older ages as opposed to the actual data where there is sharp bend at age 110 with mortality rate of 1. LC model, implies the maximum age might not be actually 110 but more than that. Hence this model is best fit for the data in comparison to the linear and Makeham models.

Goodness of fit of the model

The goodness of fit tests conducted in R programming showed p value of 1 for the chi square test, 0.999001 for the Cramer Von Mises criterion test which implies the mortality rates estimated are distributed as the actual ones observed in the mortality tables given. Kolmogorov Smirnov test had a D value of 0.063063 and p-value = 0.9801. That is, there is 98% percent

chance that the difference in CDF of the estimated and actual mortality rates is 0.063063. Anderson darling test had a p value of 0.96844 when there are ties and 0.94905 after adjustment for ties. This implies the Lee Carter model is the best fit model for this data. Shapiro Wilk test indicated that both the actual and estimated mortality rates do not come form a normal population.

4.5   Forecast of mortality

The forecast of mortality rates estimated using the Lee Carter model can be reduced to forecast of the parameter associated with general level of mortality, K_t, as it is time based. This parameter is forecast using cubic smoothing spline, time series model with and without a drift, Auto Regression Integrated Moving Average (ARIMA) of (0,1,0) in R programming. Horizon of the forecast is five years. Results of R are displayed in the appendices.

From the analysis of the three methods of forecast, cubic smoothing spline and ARIMA model with drift give almost similar results. ARIMA model without drift assumes stationarity from the last year's value of parameter, K_t. ARIMA model with drift, has a Mean Absolute Standard Error (MASE) of 0.3485736 and 0.4175512 for cubic smoothing spline. The smaller the value of the MASE, the better the fit, hence ARIMA model with drift give a better forecast. Further, The Root Mean Square Error (RMSE) was 0.7688246 and 1.040821 for ARIMA model with drift and cubic smoothing spline respectively. RMSE that is less than one is better than more than one, therefore ARIMA model with drift is a better forecast method. The Mean Absolute Percentage Error (MAPE) was 1060.53 and

63.81906 for ARIMA with drift and cubic smoothing spline respectively. Hence cubic smoothing spline offers a better fit in regard to MAPE. On further application to the data and comparing to the actual mortality rates, cubic smoothing spline was found to give a closer estimate to the actual data. Hence in this research, we shall use cubic smoothing spline for forecasting mortality into the horizon. The results are shown in the appendix.

From the tables 14 and 15 in the appendix, we can deduce that mortality has had a downward trend, in our case from the year 2000 to the last forecast(using both ARIMA with drift and cubic smoothing spline models) year 2014 for both. We can therefore calculate the percentage change in mortality using the formula,

$$\%change = \left[\left(\frac{F}{S}\right)^{\frac{1}{Y}} - 1\right] * 100 \tag{4.2}$$

where,

F = current year mortality

S = previous year mortality

Y = the number of years

The results are illustrated in the table below.

Table 1: Percentage mortality change

Age	2000-2004	2005-2009	2010-2014 (ARIMA)	2010-2014 (Cubic)
60	-1.3167%	-1.6095%	-1.3830%	-1.6699%
65	-1.7502%	-2.1392%	-1.8395%	-2.2202%
70	-2.3307%	-2.8491%	-2.4531%	-2.9593%
75	-2.2614%	-2.7670%	-2.3844%	-2.8771%
80	-2.2954%	-2.8132%	-2.4282%	-2.9307%

From the table above, we can deduce that as much as the mortality declines with both the ARIMA and the cubic smoothing spline forecasting models, ARIMA model tends to underestimate the mortality rates. Therefore annuity providers should be careful in the selection of forecasting models. In the study we focus on mortality rates forecast using the cubic smoothing spline.

4.6   Impact of mortality change

Our results indicates a decrease in the mortality rates over the years by approximately 1.7% using cubic smoothing spline models for a 60 year old male. This decrease may seem small but has a major impact on pension schemes and annuity providers. we take an example of a whole life annuity for a male aged 65 years. Assumption is made that the sum assured is 1 is payable continuously .
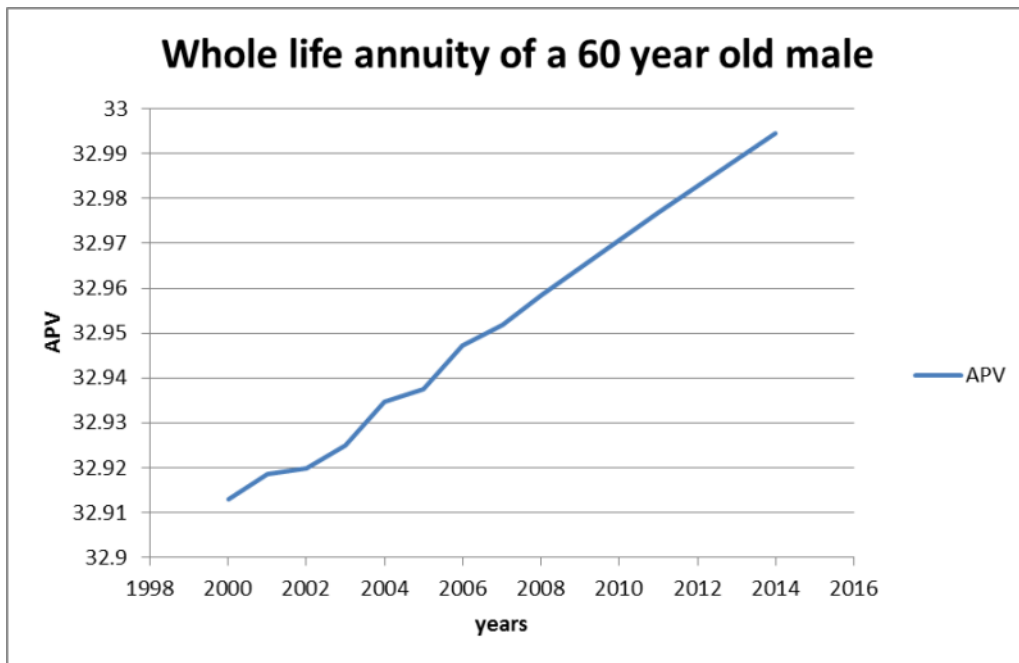
Figure 4.13

The actuarial present values of the whole life annuity has an upward trend. This implies that the present values increase with decrease in mortality.

We can illustrate using a numerical example. APV of a whole life annuity of a 60 year old male payable continuously for the year 2009. Assume amount 1 is paid at an interest of 3%. The APV using actual mortality rates is 32.74567 and using mortality rates based on Lee carter model is 32.9646. We can observe difference in the present values due to difference in mortality rates.

Hence, pension schemes and annuity providers should incorporate longevity risk (decline in mortality rates) when pricing annuities to avoid insolvency in future. This can be done by using appropriate mortality rates from best fit model, which in our case is the Lee Carter model.

56

CHAPTER FIVE

5 CONCLUSION

5.1 Summary

We began by establishing the best fit model to the data. We fitted linear model based on log transformation, makeham model, cubic spline and Lee Carter models. Parameters of the were estimated using methods, linear regression, graphical method and Singular Value Decomposition for the linear model, Makeham model and Lee Carter model respectively. There was no estimation of parameters for the cubic spline as it non parametric. Goodness of fit techniques used for all models were chi square goodness of fit, Cramer Von Mises criterion, Kolmogorov smirnov and Anderson Darling tests. In the analysis we concluded that the Lee Carter model was the best fit model to our data.

Selection of a forecast model was done using the results of the parameter K_t of the Lee Carter model. This is because forecast of mortality rates based on the Lee carter model is reduced to forecast of the parameter K_t. Cubic smoothing spline, ARIMA models with a drift and without a drift forecasting methods were tested using standard errors. We concluded that cubic smoothing spline model with a drift was the best fit model. Therefore, mortality rates were forecasted to a five year horizon.

Actuarial Present Values (APVs) of a whole life annuity for a 60 male life was calculated using the forecast mortality rates based on the Lee Carter model. The results obtained indicated an upward trend in the APVs of the

annuity from previous years, implying that APVs increase with a decrease in mortality rates .This implication may have a high cost to pension schemes and annuity providers to the tune of insolvency if the decline of mortality is not taken care of properly. Hence in calculating the present values of annuities, annuity providers and pension schemes should take into account the changing declining patterns of mortality in recent times to accommodate longevity risk.

5.2 Recommendations

In this research we considered three models, this can be extended to include other models in the investigation of best fit model to the data. Moreover, we assume forecasting parameter K_t of the Lee Carter model is enough to forecast mortality rates. This may not be true as the other parameters, α_x and β_x may be dynamic and this needs to investigated. Further research can be conducted in modifications of the Lee carter model, for example, combining expert opinion and subjective indicators to the model. Methods of forecasting also need to be reviewed and studied in details. Experts should closely monitor the rates of mortality decline and re-calibrate the model from time to time.

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7 APPENDIX

7.1 Makeham Results

Male lives

¿chisq.test(malesmakeham) pearson's chi square

Pearson's Chi-squared test

data: malesmakeham X-squared = 0.70801, df = 110, p-value = 1

Warning message: In chisq.test(malesmakeham) : Chi-squared

approximation may be incorrect

cvm¡-cramer.test(qx.actual,qx.estimate) ¿ cvm

1 -dimensional nonparametric Cramer-Test with kernel phiCramer (on

equality of two distributions)

x-sample: 111 values y-sample: 111 values

critical value for confidence level 95 observed statistic 0.1426729 , so that

hypothesis ("x is distributed as y") is ACCEPTED . estimated p-value =

0.08691309

[result based on 1000 ordinary bootstrap-replicates]

¿ ks.test(qx.actual,qx.estimate,alternative = "two.sided",exact=NULL)

Kolmogorov two sided

Two-sample Kolmogorov-Smirnov test

data: qx.actual and qx.estimate D = 0.53153, p-value = 4.807e-14

alternative hypothesis: two-sided

Warning message: In ks.test(qx.actual, qx.estimate, alternative =

"two.sided", exact = NULL) : p-value will be approximate in the presence

of ties ¿ ks.test(qx.actual,qx.estimate,alternative = "less",exact=NULL)

kolmogorov one sided

Two-sample Kolmogorov-Smirnov test

data: qx.actual and qx.estimate $D^- = 0.027027, p-value = 0.9221$ alternative hypothesis : the CDF of x lies below that of y

Warning message: In ks.test(qx.actual, qx.estimate, alternative = "less", exact = NULL) : p-value will be approximate in the presence of ties

¿ ad.test(qx.actual,qx.estimate)

Anderson-Darling k-sample test.

Number of samples: 2 Sample sizes: 111, 111 Number of ties: 7

Mean of Anderson-Darling Criterion: 1 Standard deviation of

Anderson-Darling Criterion: 0.75489

T.AD = (Anderson-Darling Criterion - mean)/sigma

Null Hypothesis: All samples come from a common population.

AD T.AD asympt. P-value version 1: 16.99 21.181 1.2532e-09 version 2: 17.00 21.233 1.2380e-09

¿ shapiro.test(qx.actual) shapiro test

Shapiro-Wilk normality test

data: qx.actual W = 0.60361, p-value = 7.479e-16

¿ shapiro.test(qx.estimate) shapiro test

Shapiro-Wilk normality test

data: qx.estimate W = 0.64699, p-value = 5.875e-15

Female lives

chisq.test(femalemakeham) pearson's chi square

Pearson's Chi-squared test

data: femalemakeham X-squared = 0.68372, df = 110, p-value = 1

Warning message: In chisq.test(femalemakeham) : Chi-squared

approximation may be incorrect

¿ cvf¡-cramer.test(qx.aF,qx.eF) ¿ cvf

1 -dimensional nonparametric Cramer-Test with kernel phiCramer (on equality of two distributions)

x-sample: 111 values y-sample: 111 values

critical value for confidence level 95 observed statistic 0.123093 , so that hypothesis ("x is distributed as y") is ACCEPTED . estimated p-value = 0.09090909

[result based on 1000 ordinary bootstrap-replicates]

ks.test(qx.aF,qx.eF,alternative = "two.sided",exact=NULL) Kolmogorov two sided

Two-sample Kolmogorov-Smirnov test

data: qx.aF and qx.eF D = 0.54955, p-value = 5.551e-15 alternative hypothesis: two-sided

Warning message: In ks.test(qx.aF, qx.eF, alternative = "two.sided", exact = NULL) : p-value will be approximate in the presence of ties ¿

ks.test(qx.aF,qx.eF,alternative = "less",exact=NULL) kolmogorov one sided

Two-sample Kolmogorov-Smirnov test

data: qx.aF and qx.eF $D^- = 0.036036, p - value =$

$0.8658 alternative hypothesis : the CDF of x lies below that of y$

Warning message: In ks.test(qx.aF, qx.eF, alternative = "less", exact = NULL) : p-value will be approximate in the presence of ties

ad.test(qx.aF,qx.eF)

Anderson-Darling k-sample test.

Number of samples: 2 Sample sizes: 111, 111 Number of ties: 10

Mean of Anderson-Darling Criterion: 1 Standard deviation of Anderson-Darling Criterion: 0.75489

T.AD = (Anderson-Darling Criterion - mean)/sigma

Null Hypothesis: All samples come from a common population.

AD T.AD asympt. P-value version 1: 18.219 22.810 2.6114e-10 version 2: 18.300 22.855 2.6124e-10

¿ shapiro.test(qx.aF) shapiro test

Shapiro-Wilk normality test

data: qx.aF W = 0.5653, p-value ¡ 2.2e-16

¿ shapiro.test(qx.eF) shapiro test

Shapiro-Wilk normality test

data: qx.eF W = 0.62384, p-value = 1.914e-15

7.2   Cubic spline results

Male lives

¿ chisq.test(malesspline) pearson's chi square

Pearson's Chi-squared test

data: malesspline X-squared = 0.2618, df = 110, p-value = 1

Warning message: In chisq.test(malesspline) : Chi-squared approximation may be incorrect

cvm¡-cramer.test(qx,spline.qx) cramer von mises test ¿ cvm

1 -dimensional nonparametric Cramer-Test with kernel phiCramer (on equality of two distributions)

x-sample: 111 values y-sample: 111 values

critical value for confidence level 95 observed statistic 0.03262741 , so that hypothesis ("x is distributed as y") is ACCEPTED . estimated p-value = 0.7362637

[result based on 1000 ordinary bootstrap-replicates]

¿ ks.test(qx,spline.qx,alternative = "two.sided",exact=NULL) Kolmogorov two sided

Two-sample Kolmogorov-Smirnov test

data: qx and spline.qx D = 0.13514, p-value = 0.2629 alternative hypothesis: two-sided

Warning message: In ks.test(qx, spline.qx, alternative = "two.sided", exact = NULL) : p-value will be approximate in the presence of ties ¿

ks.test(qx,spline.qx,alternative = "less",exact=NULL) kolmogorov one sided

Two-sample Kolmogorov-Smirnov test

data: qx and spline.qx $D^- = 0.09009, p-value = 0.4062 alternative hypothesis : the CDF of x lies below that of y$

Warning message: In ks.test(qx, spline.qx, alternative = "less", exact = NULL) : p-value will be approximate in the presence of ties

Anderson-Darling k-sample test.

Number of samples: 2 Sample sizes: 111, 111 Number of ties: 8

Mean of Anderson-Darling Criterion: 1 Standard deviation of Anderson-Darling Criterion: 0.75489

T.AD = (Anderson-Darling Criterion - mean)/sigma

Null Hypothesis: All samples come from a common population.

AD T.AD asympt. P-value version 1: 1.3977 0.52689 0.20187 version 2: 1.4300 0.56324 0.19448

shapiro.test(qx) shapiro test

Shapiro-Wilk normality test

data: qx W = 0.6148, p-value = 1.252e-15

¿ shapiro.test(spline.qx) shapiro test

Shapiro-Wilk normality test

data: spline.qx W = 0.57542, p-value ¡ 2.2e-16

Female lives

¿ chisq.test(femalesspline) pearson's chi square

Pearson's Chi-squared test

data: femalesspline X-squared = 0.1419, df = 110, p-value = 1

Warning message: In chisq.test(femalesspline) : Chi-squared approximation
may be incorrect

¿ cramer.test(qx.aF,spline.Fqx) cramer von mises test

1 -dimensional nonparametric Cramer-Test with kernel phiCramer (on
equality of two distributions)

x-sample: 111 values y-sample: 111 values

critical value for confidence level 95 observed statistic 0.01955044 , so that
hypothesis ("x is distributed as y") is ACCEPTED . estimated p-value =
0.8631369

[result based on 1000 ordinary bootstrap-replicates]

¿ ks.test(qx.aF,spline.Fqx,alternative = "two.sided",exact=NULL)

Kolmogorov two sided

Two-sample Kolmogorov-Smirnov test

data: qx.aF and spline.Fqx D = 0.063063, p-value = 0.9801 alternative
hypothesis: two-sided

Warning message: In ks.test(qx.aF, spline.Fqx, alternative = "two.sided",
exact = NULL) : p-value will be approximate in the presence of ties ¿
ks.test(qx.aF,spline.Fqx,alternative = "less",exact=NULL) kolmogorov one
sided

Two-sample Kolmogorov-Smirnov test

data: qx.aF and spline.Fqx $D^- = 0.036036, p-value = 0.8658 alternative hypothesis : the CDF of x lies below that of y$

Warning message: In ks.test(qx.aF, spline.Fqx, alternative = "less", exact = NULL) : p-value will be approximate in the presence of ties

Anderson-Darling k-sample test.

Number of samples: 2 Sample sizes: 111, 111 Number of ties: 11

Mean of Anderson-Darling Criterion: 1 Standard deviation of

Anderson-Darling Criterion: 0.75489

T.AD = (Anderson-Darling Criterion - mean)/sigma

Null Hypothesis: All samples come from a common population.

AD T.AD asympt. P-value version 1: 0.43821 -0.74420 0.81405 version 2: 0.40300 -0.79107 0.84942

¿ shapiro.test(qx.aF) shapiro test

Shapiro-Wilk normality test

data: qx.aF W = 0.57947, p-value = 2.548e-16

¿ shapiro.test(spline.Fqx) shapiro test

Shapiro-Wilk normality test

data: spline.Fqx W = 0.54262, p-value ¡ 2.2e-16

7.3   Lee - Carter model results

¿ chisq.test(lcmales)

Pearson's Chi-squared test

data: lcmales X-squared = 0.11878, df = 110, p-value = 1

Warning message: In chisq.test(lcmales) : Chi-squared approximation may be incorrect

¿ cvm¡-cramer.test(qx,qx.estimate) ¿ cvm

1 -dimensional nonparametric Cramer-Test with kernel phiCramer (on equality of two distributions)

x-sample: 111 values y-sample: 111 values

critical value for confidence level 95 observed statistic 0.00527798 , so that hypothesis ("x is distributed as y") is ACCEPTED . estimated p-value = 0.999001

[result based on 1000 ordinary bootstrap-replicates]

¿ ks.test(qx,qx.estimate,alternative="two.sided",exact=NULL)

Two-sample Kolmogorov-Smirnov test

data: qx and qx.estimate D = 0.063063, p-value = 0.9801 alternative hypothesis: two-sided

Warning message: In ks.test(qx, qx.estimate, alternative = "two.sided", exact = NULL) : p-value will be approximate in the presence of ties ¿ ks.test(qx,qx.estimate,alternative="less",exact=NULL)

Two-sample Kolmogorov-Smirnov test

data: qx and qx.estimate $D^- = 0.063063, p - value = 0.6431 alternative hypothesis : the CDF of x lies below that of y$

Warning message: In ks.test(qx, qx.estimate, alternative = "less", exact = NULL) : p-value will be approximate in the presence of ties

¿ ad.test(qx,qx.estimate)

Anderson-Darling k-sample test.

Number of samples: 2 Sample sizes: 111, 111 Number of ties: 7

Mean of Anderson-Darling Criterion: 1 Standard deviation of Anderson-Darling Criterion: 0.75489

T.AD = (Anderson-Darling Criterion - mean)/sigma

Null Hypothesis: All samples come from a common population.

AD T.AD asympt. P-value version 1: 0.26008 -0.98017 0.96844 version 2: 0.29000 -0.94041 0.94905

¿ shapiro.test(qx)

Shapiro-Wilk normality test

data: qx W = 0.60361, p-value = 7.479e-16

¿ shapiro.test(qx.estimate)

Shapiro-Wilk normality test

data: qx.estimate W = 0.62263, p-value = 1.808e-15

7.4   Forecast results

ARIMA model without drift

Forecast method: ARIMA(0,1,0)

Model Information: Series: kt.1 ARIMA(0,1,0)

$\text{sigma}^2 estimated as 3.822 : loglikelihood = -18.8 AIC = 39.61 AICc = 40.18 BIC = 39.81$

Error measures: ME RMSE MAE MPE MAPE MASE ACF1 Training set -1.600605 1.854787 1.602039 -1014.901 1053.585 0.9004029 -0.1993468

Forecasts: Point Forecast Lo 80 Hi 80 Lo 95 Hi 95 11 -8.844482 -11.35006 -6.338900 -12.67644 -5.0125253 12 -8.844482 -12.38791 -5.301054 -14.26369 -3.4252769 13 -8.844482 -13.18428 -4.504687 -15.48163 -2.2073384 14 -8.844482 -13.85565 -3.833318 -16.50840 -1.1805687 15 -8.844482 -14.44713 -3.241831 -17.41300 -0.2759665

ARIMA model with drift

Forecast method: ARIMA(0,1,0) with drift

Model Information: Series: kt.1 ARIMA(0,1,0) with drift

Coefficients: drift -1.7792 s.e. 0.2865

sigma^2 $estimated as 0.7389 : loglikelihood = -9.67 AIC = 23.34 AICc =$
$25.34 BIC = 23.73$

Error measures: ME RMSE MAE MPE MAPE MASE ACF1 Training set
0.000894798 0.7688246 0.6201985 1034.89 1060.533 0.3485736 -0.3211898

Forecasts: Point Forecast Lo 80 Hi 80 Lo 95 Hi 95 11 -10.62373 -11.72531
-9.52215 -12.30845 -8.93901 12 -12.40298 -13.96084 -10.84511 -14.78553
-10.02042 13 -14.18222 -16.09021 -12.27423 -17.10024 -11.26420 14
-15.96147 -18.16463 -13.75831 -19.33091 -12.59203 15 -17.74072 -20.20392
-15.27751 -21.50786 -13.97357

Cubic smoothing spline

Forecast method: Cubic Smoothing Spline

Model Information: $beta[1] 4.317349$

$call spline f(x = kt.copy, h = 5)$

Error measures: ME RMSE MAE MPE MAPE MASE ACF1 Training set
-0.7271741 1.040821 0.7429266 -2.815897 63.81906 0.4175512 -0.2809474

Forecasts: Point Forecast Lo 80 Hi 80 Lo 95 Hi 95 11 -10.79321 -12.06094
-9.525471 -12.73204 -8.854373 12 -12.91971 -14.64217 -11.197254 -15.55398
-10.285441 13 -15.04621 -17.35648 -12.735949 -18.57946 -11.512969 14
-17.17272 -20.17233 -14.173109 -21.76022 -12.585213 15 -19.29922 -23.07108
-15.527362 -25.06778 -13.530660

7.5   Table of results

Table 2: Log transformation for male and female lives

linear	Male		Female		linear	Male		Female	
Age	**qx**	**y=lnqx**	qx	y=lnqx	**Age**	**qx**	**y=lnqx**	qx	y=lnqx
0	0.00761	-4.8783	0.00622	-5.0800	26	0.00141	-6.5642	0.00053	-7.5426
1	0.00052	-7.5617	0.00044	-7.7287	27	0.0014	-6.5713	0.00055	-7.5056
2	0.00035	-7.9576	0.00028	-8.1807	28	0.00139	-6.5785	0.00058	-7.4525
3	0.00026	-8.2548	0.00020	-8.5172	29	0.00143	-6.5501	0.00062	-7.3858
4	0.00021	-8.4684	0.00017	-8.6797	30	0.00141	-6.5642	0.00063	-7.3698
5	0.00018	-8.6226	0.00014	-8.8739	31	0.00147	-6.5225	0.00068	-7.2934
6	0.00016	-8.7403	0.00013	-8.9480	32	0.00148	-6.5157	0.00073	-7.2225
7	0.00015	-8.8049	0.00012	-9.0280	33	0.00155	-6.4695	0.00078	-7.1562
8	0.00015	-8.8049	0.00012	-9.0280	34	0.00163	-6.4192	0.00084	-7.0821
9	0.00015	-8.8049	0.00012	-9.0280	35	0.00172	-6.3654	0.00092	-6.9911
10	0.00015	-8.8049	0.00012	-9.0280	36	0.00181	-6.3144	0.00099	-6.9178
11	0.00017	-8.6797	0.00012	-9.0280	37	0.00197	-6.2297	0.00109	-6.8216
12	0.00019	-8.5685	0.00013	-8.9480	38	0.00211	-6.1611	0.00119	-6.7338
13	0.00023	-8.3774	0.00015	-8.8049	39	0.00224	-6.1013	0.00132	-6.6301
14	0.00030	-8.1117	0.00019	-8.5685	40	0.00245	-6.0117	0.00144	-6.5431
15	0.00042	-7.7753	0.00024	-8.3349	41	0.00266	-5.9294	0.00156	-6.4631
16	0.00064	-7.3540	0.00033	-8.0164	42	0.00289	-5.8465	0.00171	-6.3713
17	0.00085	-7.0703	0.00038	-7.8753	43	0.00314	-5.7635	0.00189	-6.2712
18	0.00114	-6.7767	0.00044	-7.7287	44	0.00343	-5.6752	0.00205	-6.1899
19	0.00129	-6.6531	0.00044	-7.7287	45	0.00372	-5.5940	0.00222	-6.1102
20	0.00133	-6.6226	0.00044	-7.7287	46	0.00404	-5.5115	0.0024	-6.0323
21	0.00144	-6.5431	0.00048	-7.6417	47	0.00438	-5.4307	0.0026	-5.9522
22	0.00143	-6.5501	0.00047	-7.6628	48	0.00477	-5.3454	0.0028	-5.8781
23	0.00142	-6.5571	0.00048	-7.6417	49	0.00514	-5.2707	0.00302	-5.8025
24	0.00142	-6.5571	0.00050	-7.6009	50	0.0056	-5.1850	0.00327	-5.7230
25	0.00139	-6.5785	0.00050	-7.6009	51	0.00599	-5.1177	0.00348	-5.6607

71

Table 3: Log transformation contd.

| linear | Male | | Female | | linear | Male | | Female | |
Age	qx	y=lnqx	qx	y=lnqx	Age	qx	y=lnqx	qx	y=lnqx
52	0.00649	-5.03749	0.00377	-5.58068	78	0.05436	-2.91213	0.03759	-3.28102
53	0.00695	-4.96901	0.00404	-5.51151	79	0.05975	-2.81759	0.04151	-3.18182
54	0.00740	-4.90628	0.00435	-5.43758	80	0.06653	-2.7101	0.04663	-3.06551
55	0.00806	-4.82084	0.00470	-5.36019	81	0.07308	-2.6162	0.05153	-2.96559
56	0.00854	-4.76299	0.00507	-5.28441	82	0.08083	-2.51541	0.05783	-2.85025
57	0.00924	-4.68421	0.00555	-5.19396	83	0.08898	-2.41934	0.06429	-2.74435
58	0.00996	-4.60918	0.00607	-5.1044	84	0.09848	-2.3179	0.07197	-2.63151
59	0.01077	-4.53099	0.00664	-5.01464	85	0.10877	-2.21852	0.08064	-2.51776
60	0.01182	-4.43796	0.00732	-4.91714	86	0.12108	-2.1113	0.0904	-2.40351
61	0.01254	-4.37883	0.00792	-4.83836	87	0.13331	-2.01508	0.10105	-2.29214
62	0.01383	-4.28092	0.00875	-4.7387	88	0.14718	-1.9161	0.11294	-2.1809
63	0.01492	-4.20505	0.00950	-4.65646	89	0.16261	-1.8164	0.12591	-2.07219
64	0.01635	-4.11353	0.01048	-4.55829	90	0.17815	-1.72513	0.1399	-1.96683
65	0.01763	-4.03815	0.01136	-4.47766	91	0.19505	-1.6345	0.1557	-1.85982
66	0.01902	-3.96226	0.01234	-4.39491	92	0.21389	-1.54229	0.17242	-1.75782
67	0.02063	-3.88101	0.01352	-4.30359	93	0.23373	-1.45359	0.19049	-1.65816
68	0.02234	-3.80138	0.01464	-4.224	94	0.2548	-1.36728	0.20869	-1.56691
69	0.02446	-3.71072	0.01604	-4.13267	95	0.26828	-1.31572	0.2258	-1.48811
70	0.02677	-3.62047	0.01777	-4.03024	96	0.28807	-1.24455	0.24562	-1.40397
71	0.02893	-3.54288	0.01923	-3.95128	97	0.30819	-1.17704	0.26612	-1.32381
72	0.03177	-3.44923	0.02125	-3.8514	98	0.3285	-1.11322	0.28716	-1.24772
73	0.03489	-3.35556	0.02325	-3.76145	99	0.34885	-1.05311	0.30859	-1.17574
74	0.03783	-3.27465	0.02562	-3.66438	100	0.36908	-0.99674	0.33023	-1.10797
75	0.04181	-3.17462	0.02825	-3.56666	101	0.38904	-0.94407	0.35189	-1.04444
76	0.04526	-3.09533	0.03092	-3.47635	102	0.40861	-0.89499	0.3734	-0.98511
77	0.04969	-3.00195	0.034087	-3.37904	103	0.42764	-0.84947	0.39457	-0.92996

72

Table 4: Makeham model results for actual(act) and estimate(est) mortality rates

| | Male | | Female | | | Male | | Female | |
| | act | est | act | est | | act | est | act | est |
Age	**act**	**est**	**act**	**est**	**Age**	**act**	**est**	**act**	**est**
0	0.00761	0.01037	0.00622	0.00750	26	0.00141	0.01133	0.00053	0.00805
1	0.00052	0.01038	0.00044	0.00751	27	0.0014	0.01142	0.00055	0.00810
2	0.00035	0.01040	0.00028	0.00752	28	0.00139	0.01152	0.00058	0.00816
3	0.00026	0.01041	0.0002	0.00752	29	0.00143	0.01162	0.00062	0.00822
4	0.00021	0.01043	0.00017	0.00753	30	0.00141	0.01174	0.00063	0.00829
5	0.00018	0.01044	0.00014	0.00754	31	0.00147	0.01186	0.00068	0.00837
6	0.00016	0.01046	0.00013	0.00755	32	0.00148	0.01199	0.00073	0.00845
7	0.00015	0.01048	0.00012	0.00756	33	0.00155	0.01214	0.00078	0.00854
8	0.00015	0.01050	0.00012	0.00757	34	0.00163	0.01230	0.00084	0.00863
9	0.00015	0.01052	0.00012	0.00758	35	0.00172	0.01246	0.00092	0.00874
10	0.00015	0.01054	0.00012	0.00759	36	0.00181	0.01265	0.00099	0.00885
11	0.00017	0.01057	0.00012	0.00761	37	0.00197	0.01285	0.00109	0.00897
12	0.00019	0.01060	0.00013	0.00762	38	0.00211	0.01306	0.00119	0.00911
13	0.00023	0.01062	0.00015	0.00764	39	0.00224	0.01329	0.00132	0.00925
14	0.0003	0.01066	0.00019	0.00766	40	0.00245	0.01355	0.00144	0.00941
15	0.00042	0.01069	0.00024	0.00768	41	0.00266	0.01382	0.00156	0.00958
16	0.00064	0.01073	0.00033	0.00770	42	0.00289	0.01411	0.00171	0.00977
17	0.00085	0.01077	0.00038	0.00772	43	0.00314	0.01443	0.00189	0.00997
18	0.00114	0.01082	0.00044	0.00775	44	0.00343	0.01478	0.00205	0.01019
19	0.00129	0.01086	0.00044	0.00778	45	0.00372	0.01515	0.00222	0.01043
20	0.00133	0.01091	0.00044	0.00781	46	0.00404	0.01555	0.0024	0.01069
21	0.00144	0.01097	0.00048	0.00784	47	0.00438	0.01599	0.0026	0.01097
22	0.00143	0.01103	0.00047	0.00787	48	0.00477	0.01646	0.0028	0.01128
23	0.00142	0.01110	0.00048	0.00791	49	0.00514	0.01697	0.00302	0.01161
24	0.00142	0.01117	0.0005	0.00795	50	0.0056	0.01752	0.00327	0.01197
25	0.00139	0.01125	0.0005	0.00800	51	0.00599	0.01812	0.00348	0.01237

Table 5: Makeham model results contd.

	Male		Female			Male		Female	
Age	**act**	**est**	**act**	**est**	**Age**	**act**	**est**	**act**	**est**
52	0.00649	0.01877	0.00377	0.01279	78	0.05436	0.07502	0.03759	0.05314
53	0.00695	0.01947	0.00404	0.01326	79	0.05975	0.08015	0.04151	0.05701
54	0.0074	0.02022	0.00435	0.01376	80	0.06653	0.08568	0.04663	0.06119
55	0.00806	0.02104	0.0047	0.01431	81	0.07308	0.09163	0.05153	0.06572
56	0.00854	0.02192	0.00507	0.01490	82	0.08083	0.09802	0.05783	0.07061
57	0.00924	0.02288	0.00555	0.01555	83	0.08898	0.10489	0.06429	0.07590
58	0.00996	0.02391	0.00607	0.01625	84	0.09848	0.11227	0.07197	0.08162
59	0.01077	0.02503	0.00664	0.01701	85	0.10877	0.12018	0.08064	0.08779
60	0.01182	0.02624	0.00732	0.01784	86	0.12108	0.12866	0.0904	0.09445
61	0.01254	0.02755	0.00792	0.01874	87	0.13331	0.13775	0.10105	0.10163
62	0.01383	0.02896	0.00875	0.01971	88	0.14718	0.14749	0.11294	0.10937
63	0.01492	0.03048	0.0095	0.02077	89	0.16261	0.15790	0.12591	0.11771
64	0.01635	0.03213	0.01048	0.02192	90	0.17815	0.16902	0.1399	0.12668
65	0.01763	0.03391	0.01136	0.02317	91	0.19505	0.18089	0.1557	0.13633
66	0.01902	0.03583	0.01234	0.02452	92	0.21389	0.19355	0.17242	0.14669
67	0.02063	0.03791	0.01352	0.02599	93	0.23373	0.20703	0.19049	0.15782
68	0.02234	0.04015	0.01464	0.02758	94	0.2548	0.22136	0.20869	0.16974
69	0.02446	0.04258	0.01604	0.02931	95	0.26828	0.23659	0.2258	0.18251
70	0.02677	0.04519	0.01777	0.03119	96	0.28807	0.25273	0.24562	0.19616
71	0.02893	0.04801	0.01923	0.03323	97	0.30819	0.26981	0.26612	0.21074
72	0.03177	0.05105	0.02125	0.03543	98	0.3285	0.28786	0.28716	0.22628
73	0.03489	0.05434	0.02325	0.03782	99	0.34885	0.30690	0.30859	0.24283
74	0.03783	0.05788	0.02562	0.04042	100	0.36908	0.32692	0.33023	0.26041
75	0.04181	0.06170	0.02825	0.04323	101	0.38904	0.34795	0.35189	0.27905
76	0.04526	0.06581	0.03092	0.04627	102	0.40861	0.36996	0.3734	0.29878
77	0.04969	0.07024	0.03408	0.04957	103	0.42764	0.39295	0.39457	0.31961

74

Table 6: Cubic smoothing spline results for mortality rates

cubic	Male			Female		
Age	crude	graduated	actual	crude	graduated	actual
0	0.00766	0.00061	0.00761	0.00625	0.00010	0.00622
1	0.00052	0.00069	0.00052	0.00044	0.00012	0.00044
2	0.00035	0.00077	0.00035	0.00028	0.00014	0.00028
3	0.00026	0.00084	0.00026	0.00020	0.00016	0.0002
4	0.00021	0.00092	0.00021	0.00017	0.00018	0.00017
5	0.00018	0.00099	0.00018	0.00015	0.00021	0.00014
6	0.00016	0.00106	0.00016	0.00013	0.00023	0.00013
7	0.00015	0.00112	0.00015	0.00012	0.00025	0.00012
8	0.00015	0.00118	0.00015	0.00012	0.00027	0.00012
9	0.00015	0.00122	0.00015	0.00012	0.00029	0.00012
10	0.00015	0.00126	0.00015	0.00012	0.00031	0.00012
11	0.00017	0.00129	0.00017	0.00012	0.00033	0.00012
12	0.00019	0.00132	0.00019	0.00013	0.00035	0.00013
13	0.00023	0.00134	0.00023	0.00015	0.00036	0.00015
14	0.00030	0.00135	0.0003	0.00019	0.00038	0.00019
15	0.00043	0.00135	0.00042	0.00024	0.00039	0.00024
16	0.00064	0.00136	0.00064	0.00033	0.00040	0.00033
17	0.00085	0.00135	0.00085	0.00038	0.00041	0.00038
18	0.00114	0.00135	0.00114	0.00044	0.00042	0.00044
19	0.00129	0.00134	0.00129	0.00044	0.00043	0.00044
20	0.00133	0.00133	0.00133	0.00044	0.00044	0.00044
21	0.00144	0.00133	0.00144	0.00048	0.00045	0.00048
22	0.00143	0.00132	0.00143	0.00047	0.00047	0.00047
23	0.00143	0.00131	0.00142	0.00048	0.00048	0.00048
24	0.00142	0.00131	0.00142	0.00050	0.00049	0.0005
25	0.00139	0.00131	0.00139	0.00050	0.00051	0.0005

Table 7: Cubic smoothing spline results contd.

cubic	Male			Female		
Age	crude	graduated	actual	crude	graduated	actual
26	0.00141	0.00132	0.00141	0.00053	0.00052	0.00053
27	0.00140	0.00133	0.0014	0.00055	0.00055	0.00055
28	0.00139	0.00135	0.00139	0.00058	0.00057	0.00058
29	0.00143	0.00137	0.00143	0.00062	0.00060	0.00062
30	0.00141	0.00141	0.00141	0.00064	0.00064	0.00063
31	0.00147	0.00145	0.00147	0.00068	0.00068	0.00068
32	0.00148	0.00151	0.00148	0.00073	0.00073	0.00073
33	0.00155	0.00158	0.00155	0.00078	0.00078	0.00078
34	0.00163	0.00166	0.00163	0.00084	0.00085	0.00084
35	0.00173	0.00175	0.00172	0.00092	0.00093	0.00092
36	0.00181	0.00186	0.00181	0.00099	0.00101	0.00099
37	0.00197	0.00198	0.00197	0.00109	0.00110	0.00109
38	0.00212	0.00212	0.00211	0.00119	0.00121	0.00119
39	0.00225	0.00228	0.00224	0.00132	0.00132	0.00132
40	0.00246	0.00246	0.00245	0.00145	0.00145	0.00144
41	0.00266	0.00266	0.00266	0.00156	0.00158	0.00156
42	0.00290	0.00288	0.00289	0.00171	0.00172	0.00171
43	0.00314	0.00313	0.00314	0.00189	0.00187	0.00189
44	0.00344	0.00340	0.00343	0.00205	0.00203	0.00205
45	0.00372	0.00370	0.00372	0.00223	0.00220	0.00222
46	0.00405	0.00403	0.00404	0.00240	0.00239	0.0024
47	0.00439	0.00438	0.00438	0.00260	0.00258	0.0026
48	0.00478	0.00476	0.00477	0.00281	0.00280	0.0028
49	0.00516	0.00518	0.00514	0.00302	0.00303	0.00302
50	0.00561	0.00561	0.0056	0.00327	0.00327	0.00327
51	0.00600	0.00607	0.00599	0.00349	0.00354	0.00348

Table 8: Cubic smoothing spline results contd.

cubic	Male			Female		
Age	crude	graduated	actual	crude	graduated	actual
52	0.00651	0.00656	0.00649	0.00377	0.00383	0.00377
53	0.00697	0.00708	0.00695	0.00405	0.00415	0.00404
54	0.00743	0.00762	0.0074	0.00436	0.00449	0.00435
55	0.00809	0.00821	0.00806	0.00472	0.00487	0.0047
56	0.00858	0.00883	0.00854	0.00509	0.00527	0.00507
57	0.00928	0.00950	0.00924	0.00557	0.00572	0.00555
58	0.01001	0.01023	0.00996	0.00609	0.00621	0.00607
59	0.01083	0.01102	0.01077	0.00667	0.00675	0.00664
60	0.01189	0.01189	0.01182	0.00735	0.00735	0.00732
61	0.01262	0.01285	0.01254	0.00796	0.00800	0.00792
62	0.01392	0.01390	0.01383	0.00879	0.00873	0.00875
63	0.01503	0.01505	0.01492	0.00955	0.00953	0.0095
64	0.01649	0.01633	0.01635	0.01053	0.01041	0.01048
65	0.01779	0.01773	0.01763	0.01143	0.01138	0.01136
66	0.01920	0.01927	0.01902	0.01241	0.01245	0.01234
67	0.02084	0.02097	0.02063	0.01361	0.01363	0.01352
68	0.02259	0.02283	0.02234	0.01475	0.01493	0.01464
69	0.02477	0.02488	0.02446	0.01617	0.01636	0.01604
70	0.02714	0.02714	0.02677	0.01793	0.01793	0.01777
71	0.02936	0.02962	0.02893	0.01942	0.01965	0.01923
72	0.03228	0.03234	0.03177	0.02148	0.02156	0.02125
73	0.03551	0.03536	0.03489	0.02353	0.02366	0.02325
74	0.03856	0.03870	0.03783	0.02596	0.02600	0.02562
75	0.04270	0.04241	0.04181	0.02866	0.02862	0.02825
76	0.04630	0.04654	0.04526	0.03140	0.03155	0.03092
77	0.05096	0.05117	0.04969	0.03467	0.03486	0.03408

Table 9: Cubic smoothing spline results contd.

cubic	Male			Female		
Age	crude	graduated	actual	crude	graduated	actual
78	0.05587	0.05636	0.05436	0.03831	0.03860	0.03759
79	0.06159	0.06221	0.05975	0.04239	0.04287	0.04151
80	0.06883	0.06882	0.06653	0.04775	0.04775	0.04663
81	0.07586	0.07631	0.07308	0.05289	0.05335	0.05153
82	0.08423	0.08477	0.08083	0.05956	0.05977	0.05783
83	0.09313	0.09429	0.08898	0.06642	0.06710	0.06429
84	0.10358	0.10494	0.09848	0.07465	0.07542	0.07197
85	0.11502	0.11680	0.10877	0.08403	0.08483	0.08064
86	0.12888	0.12994	0.12108	0.09467	0.09540	0.0904
87	0.14283	0.14438	0.13331	0.10643	0.10721	0.10105
88	0.15888	0.16015	0.14718	0.11970	0.12030	0.11294
89	0.17700	0.17723	0.16261	0.13437	0.13471	0.12591
90	0.19557	0.19556	0.17815	0.15042	0.15042	0.1399
91	0.21612	0.21507	0.19505	0.16885	0.16740	0.1557
92	0.23950	0.23576	0.21389	0.18869	0.18572	0.17242
93	0.26466	0.25773	0.23373	0.21054	0.20546	0.19049
94	0.29201	0.28104	0.2548	0.23300	0.22675	0.20869
95	0.31698	0.30582	0.26828	0.25860	0.24971	0.2258
96	0.34394	0.33220	0.28807	0.28317	0.27452	0.24562
97	0.37060	0.36034	0.30819	0.31048	0.30138	0.26612
98	0.39766	0.39045	0.3285	0.33816	0.33051	0.28716
99	0.42744	0.42279	0.34885	0.36287	0.36221	0.30859
100	0.45764	0.45764	0.36908	0.39680	0.39680	0.33023
101	0.49879	0.49536	0.38904	0.43126	0.43470	0.35189
102	0.50335	0.53638	0.40861	0.45669	0.47639	0.3734
103	0.53478	0.58121	0.42764	0.48926	0.52244	0.39457

Table 10: Estimated mortality rates using Lee Carter model

Age	yr2000	yr2001	yr2002	yr2003	yr2004
0	0.00781969	0.00777683	0.00776641	0.00772831	0.00765351
1	0.00055385	0.00054655	0.00054478	0.00053835	0.00052586
2	0.00038166	0.00037408	0.00037225	0.00036563	0.00035289
3	0.00028915	0.00028337	0.00028198	0.00027693	0.00026721
4	0.00023524	0.00022910	0.00022762	0.00022229	0.00021212
5	0.00020222	0.00019675	0.00019543	0.00019069	0.00018166
6	0.00018077	0.00017511	0.00017376	0.00016888	0.00015963
7	0.00016281	0.00015974	0.00015900	0.00015632	0.00015115
8	0.00016616	0.00016252	0.00016164	0.00015847	0.00015239
9	0.00016062	0.00015732	0.00015652	0.00015364	0.00014809
10	0.00018069	0.00017337	0.00017163	0.00016539	0.00015372
11	0.00018724	0.00018194	0.00018067	0.00017608	0.00016735
12	0.00022359	0.00021425	0.00021203	0.00020409	0.00018924
13	0.00026433	0.00025640	0.00025450	0.00024766	0.00023467
14	0.00034926	0.00033832	0.00033570	0.00032628	0.00030841
15	0.00047659	0.00046394	0.00046091	0.00044995	0.00042905
16	0.00074383	0.00071908	0.00071317	0.00069190	0.00065168
17	0.00095113	0.00092673	0.00092087	0.00089970	0.00085926
18	0.00122434	0.00120413	0.00119925	0.00118152	0.00114724
19	0.00139008	0.00136622	0.00136047	0.00133956	0.00129916
20	0.00138610	0.00137396	0.00137101	0.00136026	0.00133924
21	0.00147227	0.00146473	0.00146289	0.00145618	0.00144301
22	0.00143891	0.00143693	0.00143645	0.00143468	0.00143119
23	0.00141033	0.00141289	0.00141351	0.00141581	0.00142036
24	0.00140162	0.00140610	0.00140719	0.00141121	0.00141919
25	0.00135205	0.00136009	0.00136205	0.00136929	0.00138373

Table 11: Lee Carter model results contd.

Age	yr2005	yr2006	yr2007	yr2008	yr2009
0	0.00763080	0.00755538	0.00751710	0.00746612	0.00740350
1	0.00052210	0.00050973	0.00050352	0.00049532	0.00048535
2	0.00034908	0.00033666	0.00033048	0.00032238	0.00031263
3	0.00026431	0.00025485	0.00025014	0.00024397	0.00023654
4	0.00020911	0.00019934	0.00019452	0.00018825	0.00018077
5	0.00017898	0.00017033	0.00016606	0.00016052	0.00015391
6	0.00015691	0.00014814	0.00014385	0.00013830	0.00013172
7	0.00014960	0.00014455	0.00014203	0.00013873	0.00013474
8	0.00015057	0.00014467	0.00014174	0.00013791	0.00013331
9	0.00014643	0.00014104	0.00013835	0.00013484	0.00013061
10	0.00015032	0.00013949	0.00013426	0.00012756	0.00011972
11	0.00016477	0.00015643	0.00015233	0.00014700	0.00014067
12	0.00018493	0.00017120	0.00016458	0.00015611	0.00014623
13	0.00023083	0.00021847	0.00021241	0.00020455	0.00019522
14	0.00030315	0.00028621	0.00027792	0.00026719	0.00025447
15	0.00042286	0.00040280	0.00039292	0.00038006	0.00036472
16	0.00063987	0.00060191	0.00058338	0.00055944	0.00053117
17	0.00084727	0.00080840	0.00078921	0.00076422	0.00073438
18	0.00113696	0.00110328	0.00108644	0.00106428	0.00103749
19	0.00128706	0.00124743	0.00122763	0.00120161	0.00117016
20	0.00133289	0.00131186	0.00130123	0.00128714	0.00126990
21	0.00143901	0.00142571	0.00141896	0.00140996	0.00139891
22	0.00143012	0.00142657	0.00142475	0.00142233	0.00141933
23	0.00142175	0.00142642	0.00142881	0.00143202	0.00143600
24	0.00142164	0.00142985	0.00143407	0.00143974	0.00144679
25	0.00138817	0.00140311	0.00141082	0.00142120	0.00143416

Table 12: Lee Carter model results contd.

Age	yr2000	yr2001	yr2002	yr2003	yr2004
26	0.00137279	0.00138009	0.00138188	0.00138845	0.00140154
27	0.00135537	0.00136506	0.00136744	0.00137619	0.00139366
28	0.00133209	0.00134458	0.00134764	0.00135894	0.00138157
29	0.00136407	0.00137750	0.00138079	0.00139295	0.00141731
30	0.00136683	0.00137568	0.00137785	0.00138582	0.00140173
31	0.00143076	0.00143910	0.00144114	0.00144865	0.00146362
32	0.00147506	0.00147607	0.00147631	0.00147721	0.00147900
33	0.00157864	0.00157112	0.00156930	0.00156261	0.00154946
34	0.00167017	0.00166085	0.00165859	0.00165030	0.00163405
35	0.00179586	0.00177903	0.00177495	0.00176007	0.00173099
36	0.00189990	0.00187715	0.00187164	0.00185157	0.00181252
37	0.00208968	0.00205995	0.00205277	0.00202663	0.00197590
38	0.00227798	0.00223845	0.00222892	0.00219429	0.00212738
39	0.00242946	0.00238412	0.00237320	0.00233353	0.00225704
40	0.00267433	0.00262063	0.00260770	0.00256079	0.00247051
41	0.00284816	0.00280232	0.00279125	0.00275101	0.00267315
42	0.00307848	0.00303393	0.00302316	0.00298398	0.00290799
43	0.00338177	0.00332322	0.00330910	0.00325779	0.00315865
44	0.00363033	0.00358311	0.00357169	0.00353009	0.00344925
45	0.00391796	0.00387124	0.00385993	0.00381872	0.00373851
46	0.00426099	0.00421068	0.00419850	0.00415412	0.00406773
47	0.00462224	0.00456757	0.00455435	0.00450612	0.00441226
48	0.00505067	0.00498723	0.00497188	0.00491596	0.00480723
49	0.00536114	0.00531248	0.00530068	0.00525760	0.00517345
50	0.00572749	0.00569853	0.00569149	0.00566573	0.00561514
51	0.00603764	0.00602599	0.00602315	0.00601275	0.00599224

Table 13: Lee Carter model results contd.

Age	yr2005	yr2006	yr2007	yr2008	yr2009
26	0.00140557	0.00141910	0.00142608	0.00143547	0.00144718
27	0.00139904	0.00141718	0.00142654	0.00143919	0.00145499
28	0.00138856	0.00141216	0.00142440	0.00144094	0.00146169
29	0.00142484	0.00145029	0.00146348	0.00148134	0.00150374
30	0.00140662	0.00142311	0.00143162	0.00144310	0.00145743
31	0.00146823	0.00148373	0.00149172	0.00150249	0.00151592
32	0.00147955	0.00148138	0.00148231	0.00148357	0.00148512
33	0.00154547	0.00153219	0.00152545	0.00151646	0.00150541
34	0.00162911	0.00161273	0.00160441	0.00159334	0.00157974
35	0.00172220	0.00169316	0.00167849	0.00165905	0.00163529
36	0.00180076	0.00176198	0.00174247	0.00171668	0.00168529
37	0.00196066	0.00191056	0.00188543	0.00185229	0.00181208
38	0.00210735	0.00204175	0.00200899	0.00196593	0.00191392
39	0.00223417	0.00215940	0.00212213	0.00207322	0.00201423
40	0.00244356	0.00235562	0.00231187	0.00225453	0.00218553
41	0.00264980	0.00257323	0.00253492	0.00248451	0.00242349
42	0.00288516	0.00281015	0.00277254	0.00272295	0.00266279
43	0.00312897	0.00303176	0.00298322	0.00291941	0.00284231
44	0.00342492	0.00334484	0.00330461	0.00325148	0.00318690
45	0.00371434	0.00363469	0.00359461	0.00354162	0.00347711
46	0.00404169	0.00395587	0.00391268	0.00385557	0.00378603
47	0.00438397	0.00429074	0.00424382	0.00418178	0.00410624
48	0.00477449	0.00466667	0.00461246	0.00454084	0.00445373
49	0.00514801	0.00506388	0.00502137	0.00496501	0.00489611
50	0.00559978	0.00554872	0.00552279	0.00548824	0.00544579
51	0.00598598	0.00596512	0.00595448	0.00594026	0.00592271

Table 14: Lee Carter model results contd.

Age	yr2000	yr2001	yr2002	yr2003	yr2004
52	0.00647489	0.00647782	0.00647854	0.00648116	0.00648635
53	0.00696729	0.00696321	0.00696222	0.00695858	0.00695138
54	0.00737396	0.00737894	0.00738015	0.00738460	0.00739341
55	0.00807556	0.00807109	0.00807001	0.00806602	0.00805814
56	0.00868414	0.00865308	0.00864553	0.00861785	0.00856339
57	0.00949303	0.00943932	0.00942628	0.00937855	0.00928488
58	0.01051960	0.01039974	0.01037073	0.01026492	0.01005887
59	0.01151068	0.01135228	0.01131400	0.01117458	0.01090391
60	0.01261330	0.01244335	0.01240226	0.01225261	0.01196198
61	0.01355513	0.01333559	0.01328261	0.01308993	0.01271708
62	0.01511114	0.01482936	0.01476146	0.01451489	0.01403931
63	0.01646426	0.01612433	0.01604253	0.01574578	0.01517495
64	0.01780078	0.01748145	0.01740447	0.01712479	0.01658483
65	0.01918768	0.01884408	0.01876125	0.01846030	0.01787924
66	0.02066951	0.02030348	0.02021522	0.01989453	0.01927519
67	0.02303282	0.02248649	0.02235527	0.02188005	0.02096963
68	0.02483484	0.02426155	0.02412379	0.02362473	0.02266778
69	0.02724364	0.02659761	0.02644244	0.02588046	0.02480375
70	0.02979182	0.02908165	0.02891108	0.02829337	0.02711004
71	0.03228553	0.03149032	0.03129941	0.03060837	0.02928591
72	0.03543404	0.03455625	0.03434553	0.03358282	0.03212342
73	0.03894051	0.03796685	0.03773314	0.03688732	0.03526929
74	0.04193348	0.04094734	0.04071041	0.03985219	0.03820707
75	0.04624518	0.04517585	0.04491886	0.04398774	0.04220184
76	0.05021330	0.04902445	0.04873883	0.04770425	0.04572126
77	0.05478005	0.05357060	0.05327973	0.05222515	0.05019939

Table 15: Lee Carter model results contd.

Age	yr2005	yr2006	yr2007	yr2008	yr2009
52	0.00648794	0.00649325	0.00649597	0.00649961	0.00650412
53	0.00694918	0.00694184	0.00693809	0.00693307	0.00692686
54	0.00739611	0.00740512	0.00740973	0.00741592	0.00742358
55	0.00805573	0.00804769	0.00804358	0.00803808	0.00803128
56	0.00854682	0.00849167	0.00846360	0.00842614	0.00838001
57	0.00925645	0.00916205	0.00911413	0.00905035	0.00897202
58	0.00999673	0.00979183	0.00968865	0.00955214	0.00938583
59	0.01082252	0.01055486	0.01042050	0.01024322	0.01002794
60	0.01187454	0.01158693	0.01144251	0.01125187	0.01102029
61	0.01260527	0.01223865	0.01205524	0.01181386	0.01152175
62	0.01389710	0.01343215	0.01320033	0.01289606	0.01252909
63	0.01500466	0.01444921	0.01417303	0.01381133	0.01337631
64	0.01642322	0.01589442	0.01563050	0.01528382	0.01486527
65	0.01770533	0.01713622	0.01685217	0.01647902	0.01602850
66	0.01908977	0.01848286	0.01817986	0.01778172	0.01730089
67	0.02069899	0.01981937	0.01938385	0.01881535	0.01813452
68	0.02238308	0.02145707	0.02099817	0.02039871	0.01968016
69	0.02448365	0.02344324	0.02292808	0.02225559	0.02145018
70	0.02675828	0.02561512	0.02504915	0.02431042	0.02342582
71	0.02889315	0.02761789	0.02698720	0.02616469	0.02518083
72	0.03169004	0.03028308	0.02958737	0.02868016	0.02759518
73	0.03478891	0.03322973	0.03245897	0.03145411	0.03025268
74	0.03771779	0.03612683	0.03533870	0.03430949	0.03307632
75	0.04167041	0.03994152	0.03908453	0.03796489	0.03662255
76	0.04513153	0.04321414	0.04226439	0.04102426	0.03953856
77	0.04959577	0.04762946	0.04665328	0.04537633	0.04384302

Table 16: Lee Carter model results contd.

Age	yr2000	yr2001	yr2002	yr2003	yr2004
78	0.05962333	0.05838248	0.05808381	0.05700016	0.05491490
79	0.06610642	0.06460936	0.06424940	0.06294470	0.06043989
80	0.07397730	0.07224193	0.07182486	0.07031372	0.06741533
81	0.08148282	0.07954711	0.07908195	0.07739674	0.07416534
82	0.09037599	0.08819711	0.08767358	0.08577717	0.08214188
83	0.09977989	0.09733794	0.09675128	0.09462644	0.09055450
84	0.10985544	0.10731404	0.10670294	0.10448776	0.10023453
85	0.12132150	0.11852403	0.11785125	0.11541217	0.11072759
86	0.13441146	0.13145629	0.13074505	0.12816473	0.12320090
87	0.14723800	0.14415743	0.14341546	0.14072182	0.13553180
88	0.16120384	0.15810566	0.15735862	0.15464381	0.14940052
89	0.17759715	0.17426908	0.17346626	0.17054767	0.16490582
90	0.19269654	0.18944688	0.18866205	0.18580572	0.18027014
91	0.20854987	0.20552330	0.20479130	0.20212376	0.19693811
92	0.22824714	0.22499111	0.22420338	0.22133203	0.21574680
93	0.24703065	0.24395254	0.24320703	0.24048683	0.23518303
94	0.26733155	0.26435949	0.26363906	0.26100834	0.25586968
95	0.28253218	0.27921582	0.27841211	0.27547778	0.26974874
96	0.30172117	0.29846493	0.29767529	0.29479076	0.28915147
97	0.32102553	0.31785561	0.31708645	0.31427522	0.30877228
98	0.34030407	0.33724480	0.33650209	0.33378611	0.32846330
99	0.35940829	0.35648213	0.35577136	0.35317098	0.34806901
100	0.37819855	0.37542386	0.37474956	0.37228152	0.36743422
101	0.39654420	0.39393601	0.39330190	0.39098002	0.38641547
102	0.41432714	0.41189653	0.41130536	0.40913992	0.40487929
103	0.43143925	0.42919386	0.42864755	0.42664578	0.42270414

Table 17: Lee Carter model results contd.

Age	yr2005	yr2006	yr2007	yr2008	yr2009
78	0.05429261	0.05226234	0.05125257	0.04992982	0.04833860
79	0.05969392	0.05726515	0.05606013	0.05448465	0.05259412
80	0.06655286	0.06374717	0.06235655	0.06053986	0.05836215
81	0.07320401	0.07007748	0.06852830	0.06650497	0.06408033
82	0.08106069	0.07754538	0.07580417	0.07353069	0.07080732
83	0.08934378	0.08540846	0.08345991	0.08091644	0.07787083
84	0.09896778	0.09484339	0.09279716	0.09012204	0.08691237
85	0.10933201	0.10478703	0.10253150	0.09958212	0.09604249
86	0.12172005	0.11689055	0.11448987	0.11134660	0.10756801
87	0.13398133	0.12891778	0.12639666	0.12309148	0.11911175
88	0.14783088	0.14269388	0.14012986	0.13676186	0.13269631
89	0.16321557	0.15767965	0.15491408	0.15127884	0.14688686
90	0.17860802	0.17315189	0.17041892	0.16681894	0.16245774
91	0.19537685	0.19023773	0.18765522	0.18424471	0.18009939
92	0.21406438	0.20852357	0.20573757	0.20205663	0.19758003
93	0.23358204	0.22829826	0.22563488	0.22210898	0.21781000
94	0.25431609	0.24918045	0.24658684	0.24314815	0.23894739
95	0.26801736	0.26229653	0.25940890	0.25558204	0.25090971
96	0.28744525	0.28180089	0.27894793	0.27516288	0.27053510
97	0.30710548	0.30158533	0.29879147	0.29508095	0.29053820
98	0.32684937	0.32149865	0.31878718	0.31518250	0.31076369
99	0.34652054	0.34138171	0.33877456	0.33530536	0.33104752
100	0.36596172	0.36107047	0.35858625	0.35527777	0.35121268
101	0.38502771	0.38041407	0.37806852	0.37494225	0.37109712
102	0.40358296	0.39926998	0.39707531	0.39414804	0.39054433
103	0.42150407	0.41750859	0.41547385	0.41275813	0.40941208

Table 18: Lee Carter model parameters estimation

x	ax	bx	Years	kt
0	-4.87177	0.003428	2000	7.168738
1	-7.55746	0.008247	2001	5.559169
2	-7.96013	0.012461	2002	5.166803
3	-8.23835	0.012542	2003	3.726592
4	-8.4727	0.01645	2004	0.878915
5	-8.62828	0.017049	2005	0.008751
6	-8.75993	0.019772	2006	-2.89917
7	-8.80756	0.011817	2007	-4.38657
8	-8.80111	0.013756	2008	-6.37874
9	-8.82897	0.012919	2009	-8.84448
10	-8.80291	0.025708		
11	-8.71105	0.017861		
12	-8.59568	0.02652		
13	-8.37387	0.018929		
14	-8.10129	0.019775		
15	-7.76841	0.01671		
16	-7.35412	0.021034		
17	-7.07321	0.016158		
18	-6.77892	0.010347		
19	-6.65485	0.010762		
20	-6.61979	0.005472		
21	-6.54311	0.003194		
22	-6.54929	0.000856		
23	-6.55515	-0.00113		
24	-6.55521	-0.00198		
25	-6.57904	-0.00368		
26	-6.56658	-0.0033		
27	-6.57123	-0.00443		

Table 19: Lee Carter model parameters estimation contd.

x	ax	bx
28	-6.57875	-0.0058
29	-6.55293	-0.00609
30	-6.56582	-0.00401
31	-6.52293	-0.00361
32	-6.51528	-0.00042
33	-6.47168	0.002969
34	-6.41893	0.003479
35	-6.36334	0.005854
36	-6.31871	0.007492
37	-6.23357	0.00891
38	-6.16137	0.010886
39	-6.10287	0.011718
40	-6.01319	0.01262
41	-5.93203	0.010097
42	-5.84681	0.009072
43	-5.76558	0.010869
44	-5.67503	0.008149
45	-5.59376	0.007468
46	-5.50913	0.007395
47	-5.42767	0.007408
48	-5.34215	0.007873
49	-5.26662	0.005681
50	-5.18225	0.003158
51	-5.11535	0.001204
52	-5.03455	-0.00028
53	-4.96565	0.000365
54	-4.90309	-0.00042
55	-4.81733	0.000345

Table 20: ARIMA model Forecast mortality rates for the period 2008 - 2014

x/qx	2008	2009	2010	2011	2012	2013	2014
60	0.011284	0.011128	0.010974	0.010823	0.010673	0.010525	0.01038
61	0.011855	0.011658	0.011464	0.011273	0.011086	0.010902	0.01072
62	0.012948	0.0127	0.012457	0.012218	0.011984	0.011755	0.01153
63	0.013873	0.013579	0.013291	0.013009	0.012733	0.012463	0.012199
64	0.015343	0.01506	0.014783	0.01451	0.014243	0.013981	0.013723
65	0.016542	0.016238	0.01594	0.015647	0.015359	0.015076	0.014799
66	0.017849	0.017525	0.017206	0.016893	0.016586	0.016284	0.015988
67	0.018912	0.018451	0.018001	0.017562	0.017134	0.016716	0.016308
68	0.0205	0.020014	0.019539	0.019075	0.018623	0.018181	0.017749
69	0.022369	0.021824	0.021292	0.020773	0.020266	0.019772	0.019289
70	0.024435	0.023837	0.023252	0.022682	0.022126	0.021583	0.021053
71	0.026304	0.025638	0.024988	0.024355	0.023737	0.023135	0.022548
72	0.028834	0.028099	0.027383	0.026684	0.026004	0.02534	0.024693
73	0.031624	0.03081	0.030017	0.029244	0.028491	0.027757	0.027041
74	0.034484	0.033649	0.032834	0.032039	0.031263	0.030505	0.029765
75	0.038155	0.037246	0.036359	0.035493	0.034647	0.03382	0.033013
76	0.041234	0.040229	0.039247	0.038289	0.037354	0.036441	0.03555
77	0.045593	0.044556	0.043542	0.04255	0.041581	0.040633	0.039707
78	0.050154	0.049079	0.048026	0.046995	0.045985	0.044997	0.044029
79	0.054752	0.053473	0.052223	0.051001	0.049808	0.048641	0.047502
80	0.060848	0.059374	0.057935	0.056529	0.055157	0.053817	0.052509

Table 21: Cubic smoothing spline forecast mortality rates

x/qx	2008	2009	2010	2011	2012	2013	2014
60	0.011248	0.01106	0.010875	0.010694	0.010515	0.01034	0.010167
61	0.011809	0.011572	0.01134	0.011112	0.010889	0.01067	0.010456
62	0.012889	0.012592	0.012301	0.012017	0.011739	0.011468	0.011203
63	0.013804	0.013451	0.013106	0.012771	0.012444	0.012126	0.011816
64	0.015276	0.014937	0.014605	0.01428	0.013963	0.013652	0.013348
65	0.016471	0.016105	0.015748	0.015399	0.015057	0.014722	0.014395
66	0.017773	0.017383	0.017002	0.016628	0.016263	0.015906	0.015557
67	0.018803	0.01825	0.017714	0.017193	0.016687	0.016196	0.01572
68	0.020386	0.019803	0.019236	0.018685	0.01815	0.01763	0.017125
69	0.022241	0.021587	0.020953	0.020336	0.019738	0.019157	0.018593
70	0.024295	0.023576	0.022879	0.022203	0.021546	0.020908	0.020289
71	0.026147	0.025348	0.024574	0.023822	0.023094	0.022387	0.021702
72	0.028661	0.02778	0.026926	0.026097	0.025294	0.024516	0.023761
73	0.031432	0.030457	0.029511	0.028595	0.027706	0.026845	0.02601
74	0.034287	0.033286	0.032314	0.03137	0.030453	0.029562	0.028697
75	0.037941	0.036851	0.035792	0.034763	0.033763	0.032792	0.031848
76	0.040998	0.039792	0.03862	0.037483	0.036378	0.035306	0.034264
77	0.045349	0.044104	0.042893	0.041715	0.040568	0.039452	0.038366
78	0.049901	0.04861	0.047351	0.046125	0.044929	0.043763	0.042627
79	0.054451	0.052916	0.051424	0.049973	0.048561	0.047188	0.045854
80	0.060501	0.058733	0.057016	0.055347	0.053725	0.05215	0.05062